BURMESE ECLIPSE CALCULATIONS

Lars Gislén
University of Lund, Dala 7163, 24297 Hörby, Sweden.
Email: LarsG@vasterstad.se

Abstract: Two Burmese eclipse calculations, one lunar and one solar, are analysed using examples from a Burmese manuscript. The fundamental parameters are with some important exceptions the same as in the Suryasiddhanta, but the handling of, for instance, parallax in the solar eclipse is different and much simplified. Specific to Burma are also the shadow calculations.

Keywords: Burmese astronomy, eclipse calculations, eclipse parallax.

1 INTRODUCTION
The traditional astronomical calculation procedures in Southeast Asia are little known in the West. Although they clearly show a great influence from traditional methods in India, their actual implementation is very different. The procedures are laid out as a kind of computer program, a set of rules without explanation that once followed, generate a set of numbers that describe the astronomical phenomenon in question. Clearly, this set of rules has been established by some competent agent who knew very well their astronomical background but who intended the rules to be used by other agents whose only function was to compute by rote. This presents some difficulties for the analysis as many of the intermediate steps in the calculation are omitted and have to be reconstructed.

There are many clever calculation shortcuts, as will be seen below. The parallax calculation procedures, for instance, are extremely simplified but still quite adequate. In an earlier paper (Gislén and Eade, 2014) we explained the Burmese shadow calculations, and they will reappear in the present paper and will vindicate the conclusions we made there.

2 THE SOURCE
The main source for this investigation is a Burmese astrology dissertation (U Thar-Tha-Na, 1937).

3 PARAMETERS
Hindu and Southeast Asian astronomers use sidereal longitudes, i.e. longitudes measured eastward along the ecliptic from a fixed origin on the ecliptic. Modern astronomy uses tropical longitudes where this origin is slowly moving westward along the ecliptic due to precession and is determined as the point on the ecliptic where the Sun crosses the celestial equator on the vernal equinox. In Hindu astronomy the fixed origin for longitude is the point where the mean Sun, the mean Moon and the mean planets were assumed to gathered together at the start of the Kaliyuga epoch, which was midnight on 18 February 3102 BCE.

The [Old] Suryasiddhanta has 1577917800 days in 4320000 solar revolutions, the Kaliyuga. As is usual in Southeast Asian astronomy these numbers are divided by 5400 to have the more handy 292207 days in 800 years. In the Burmese Thandeikhta scheme (Irwin, 1909), used in the calculations, this is corrected by 28 extra days (4320000/193/800) in a Kaliyuga, making the mean solar theory precisely that of the modern Hindu Suryasiddhanta.

The longitude of the solar apogee, the point on the ecliptic where the solar longitude is such that the distance between the Sun and the Earth is largest, is assumed to be 4640'. The apogee value (mandocca) in modern Suryasiddhanta is 4636'.

The Moon is assumed to make 5775336 revolutions in a Kaliyuga or exactly 7219167° in 25 years. The mean daily motion of the Moon is then 790.5811288° ≈ 790° 35". These are Suryasiddhanta values.

The lunar apogee is as in Suryasiddhanta (without bija) and makes 488203 revolutions in 4230000 solar years or precisely 488203° in 200 years. The daily motion is then 6.682974624° = 6' 41".

The lunar node is as in Suryasiddhanta (with bija) and makes 232242 retrograde revolutions in a Kaliyuga or precisely 116121° in 100 years or 3.179143493° = 3' 11" per day.

The epoch of Thandeikhta is BE (Burmese Era) 1100.

The calculations are made for Amarapura (Mandalay) with a geographical latitude given as 21° 32'.

For the Moon, the lunar apogee, and the lunar node, the epoch is Kaliyuga 0 (i.e. midnight on 18 February 3101 BCE). The Moon has an extra epoch correction of −12' as compared with Suryasiddhanta. This could be a way to account for the difference in geographical longitude between India and Burma. The lunar apogee is assumed to have a Kaliyuga epoch value of 5140° and the lunar node has an epoch value of 10945°. The Suryasiddhanta values are 5400° and 10800° respectively.
4 UNITS AND NOTATION

The time unit used in Southeast Asia is the nadi (Burmese nayi) that is 1/60 of a day and night. The nadi is further divided into 60 vinadis (Burmese bizana). A hexadecimal notation is used: xx:yy, meaning xx + yy/60. For longitudes the same notation means arc minutes and arc seconds. We sometimes use the notation xx:yy:zz for signs, degrees, minutes. The Burmese calculations are presented in grids that we denote by A, B, C ... Within each grid we give the column number and the row number. Thus [B 2,8] denotes grid B, column 2, and row 5. The transcribed grids have labels specifying the different numbers, some of these labels can be identified as somewhat distorted Sanskrit or Pali words but most labels are obscure Burmese technical terms.

5 LUNAR ECLIPSE CALCULATION

5.1 Preliminaries

One of the model calculations made in the source is the partial lunar eclipse of 26 July 1934. The input for the calculation is the BE year (1296) and the sutin (102), the elapsed days of the Burmese year. The Burmese calculations are typically arranged such that all operations involve entire numbers. The Burmese division algorithm is rather involved and for this we have used Western notation instead without affecting the final result. In two cases we have used decimal notation for simplicity.

First the thawana, the elapsed days since the Thandeikhta epoch BE 1100, is calculated (Irwin, 1909: 16) by:

\[(y - 1100) \cdot 292207 + 17742 + \text{int}(y - 1100/193)/800 = a + b/800\]

\[292207 = 1577917800/5400 = 681 \cdot 41\] is the number of uncorrected days in 4320000/5400 = 800 solar years. It is a standard number in Southeast Asian calendar computations (Eade, 1995).

\[\text{int}(x)\] means the integer part of the number x. This term gives an extra correction of 28 days in a Kaliyuga.

\[y\] is the Burmese year, and \(a\) the integer part of the division and \(b\) the remainder. The New Year thawana then is \(a + 1\).

The New Year weekday is given by the remainder of \((\text{thawana} - 2)/7\) with \(1 =\) Sunday. \(800 - b\) is the kyammat, the mean solar longitude at the New Year midnight (24 hours) expressed in 800 parts of a mean solar day. The Kaliyuga year is the Burmese year + 3739.

Thus we have

[A 1.1] 1296, the Burmese year.
[A 1.2] 1296 + 3739 = 5035, the Kaliyuga year.

\[196 \cdot 292207 + 17742 + 1)/800 = 71612 + 715/800\]

[A 1.3] 800 - 715 = 85, the New Year kyammat.
\(\text{thawana} = 71612 + 1 = 71613\)

[A 1.4] \((71613 - 2)/7\) = 10230 + 1/7, the New Year weekday = 1, Sunday.

The source has 5932:53, either a typo or the result of different rounding.

5.2 Mean Solar Longitude

The result is 12290510:3 by getting the kyammat at midnight on the New Year is 85. This to we add the kyammat resulting from the elapsed days from the New Year to the eclipse day, 102, the sutin. Each day corresponds to a kyammat of 800. Thus we have in total 85 + 102-800 = 81685. Now the mean daily motion of the Sun in arc minutes is 21600-800/292207 = 59.8′. By multiplying the kyammat by 21600 and dividing by 292207 we arrive at the mean solar longitude in arc minutes, skipping whole rotations (21600) if possible: 6038:10 [A 1.6].

5.3 Mean Lunar Longitude

Multiply the Kaliyuga year by 7219167, the motion of the mean Moon in arc minutes in 25 years and divide by 25. Skip whole rotations. The result is 1033:48. Subtract the epoch correction of 12′ to get 1021:48. This is the New Year mean position.

For the mean daily motion we use 79058′ in 100 days. The correct value is 79058.11287. This introduces an error 79058.11287 - 79058 = 1/9 in 100 days. The kyammat gives a contribution 85-79058/800/100 = 84.0. The sutin gives a contribution 102-79058/100 = 80639:9.6. The extra correction gives 102 (1/9)/100 = 0.6.6. In total 80723:16, with rounding. Adding the New Year longitude and skipping whole rotations we finally get 16945:4 [A 1.7].

5.4 Lunar Apogee

Multiply the Kaliyuga year by 488203, the motion of the lunar apogee in 200 years and divide by 200. The result is 12290510:31. Add the epoch value 5140 and skip whole rotations to get 5250:31. For the mean daily motion during 101 days we use 675. The correct value is 6.682974624-101 = 674.980437. The error is 674.980437 - 675 = -0.019563 = -1/51 in 101 days. The kyammat gives a contribution 85-675/800/101 = 0.43. The sutin gives a contribution 102-675/101 = 681:41. The extra correction gives -102 (1/51)/101 = -0:1. In total 682:23. Adding the New Year longitude and skipping whole rotations we finally get 5932:54 [A 1.8]. The source has 5932:53, either a typo or the result of a different rounding.

5.5 Lunar Node

Multiply the Kaliyuga year by 116121, the motion of the lunar node in 100 years and divide by
100. The result is 5846692:21. Add the epoch value 10945 and skip whole rotations to get 4037:21. For the mean daily motion during 100 days we use 318. The correct value is 317.9143493. The error is 317.9143493 – 318 = –0.0857 ≈ −1/12 in 100 days. The kyammat gives a contribution 85:318/800/100 = 0:20. The sutin gives a contribution 102:318/100 = 324:22. The extra correction gives −102:1(1/12)/100 = −0:5. In total 324:37. Adding the New Year longitude and skipping whole rotations we finally get 4361:58 [A 1,9].

5.6 True Longitudes
The true longitudes are calculated from

\[ \lambda = \lambda_M + e \sin(\omega - \lambda_M) \]  

(1)

where \( \lambda \) is the true longitude, \( \lambda_M \) the mean longitude, \( \omega \) the apogee longitude and \( e \) the eccentricity.

This expression uses a constant eccentricity, a difference from Suryasiddhanta that uses an eccentricity dimension that varies with the anomaly. For the Sun the Suryasiddhanta dimensions of the epicycle vary between 14° and 13:40° (Burgess, 2000: 70). The Burmese calculation uses a fixed dimension that corresponds to 13:44°. For the Moon the Suryasidhanta epicycle dimension varies between 32° and 31:40° (ibid.) while the Burmese dimension is fixed at 31:44°.

The second term of the formula, the equation, is taken from Table 1 (Solar chaya) with argument in steps of 10°. The anomaly \( \alpha = \omega - \lambda_M \) is reduced to lie in the interval \([0, 90°]\) using the symmetry of the sine function, i.e.

If \(0° < \alpha \leq 5400°\) do nothing Quadrant 0
If \(5400° < \alpha \leq 10800°\) replace \( \alpha \) with \(10800° - \alpha\). Quadrant 1
If \(10800° < \alpha \leq 16200°\) replace \( \alpha \) with \( \alpha - 10800°\). Quadrant 2
If \(16200° < \alpha \leq 21600°\) replace \( \alpha \) with \(21600° - \alpha\). Quadrant 3

The equation is positive for anomaly in quadrants 0 and 1, otherwise negative.

For the Sun we have the fixed value of the apogee \( \omega = 4640° \) and with \( \lambda_M = 6038° \) [A 6.1] and get an anomaly of 4640 – 6038 = –1398. As this is negative we add a whole turn, 21600°, to get 21600 – 1398 = 20202.

This anomaly is in quadrant 3, [A, 1,10] and the reduced value is 21600 – 20202 = 1398°. Each 10° interval corresponds to 600° so the entry point in the table is 1398/600 = 2.198/600. Interpolation in the table gives the equation

\[ 45 + (66 – 45):198:600 = 51:56 \]  

[A 2,1].

### Table 1: Solar chaya.

<table>
<thead>
<tr>
<th>Arg/10</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>0</td>
<td>23</td>
<td>45</td>
<td>66</td>
<td>85</td>
<td>101</td>
<td>113</td>
<td>123</td>
<td>129</td>
<td>131</td>
</tr>
</tbody>
</table>

The table difference 66 – 45 = 21 [A 1,11] is saved for the computation of the true daily motion of the Sun below. As we are in quadrant 3, the equation is subtractive and we get the true solar longitude

6038°:10 = 51:56 = 5986:14 [A 2,2].

For the Moon we have \( \omega = 5932:53 \) [A 1,8] and \( \lambda_M = 16945:4 \) [A 1,7]. The Moon’s mean longitude is given an extra correction of the solar equation (including sign) divided by 27, in our case –51:56/27 = −1:55. We have no explanation for this correction.

The corrected mean longitude is then 16945 – 1:55 = 16943:9 [A 2,3]. Thus the corrected anomaly is 5932:53 – 16943:9 + 21600 = 10589:44. This is in quadrant 1 [A 2,4], and the reduced anomaly is 10800 – 10589:44 = 210:16 ≈ 210.

With the same procedure as for the Sun, but with a lunar chaya (Table 2), we get a difference of 53 [A 2, 5], and an equation of +18:34.

The true longitude is then 16943:9 + 18:34 = 16961:43 [A 2,6].

The motion of the node is retrograde and its longitude is 21600 – 4361:58 = 17238:2 [A 2,7].

5.7 True Daily Motions
The mean daily motion of the Sun is 59:8 [A 2,8].

Using the table difference in [A 1,11] we compute the difference correction to the mean motion: 59:8:21/600 = 2:4.

The correction is positive for the anomaly in [90°, 270°], quadrants 1 and 2, otherwise negative. The true daily motion of the Sun is thus 59:8 – 2:4 = 57:4 [A 2,10].

The correctness of this procedure can be seen by taking the time derivative of the true longitude equation: \( d\lambda/dt = d\lambda_M/dt + d(e \sin \alpha)/d\alpha \cdot d\alpha/dt \). Here, \( d\lambda_M/dt \) is the true daily motion in longitude, \( d\lambda_M/dt \) the mean daily motion in longitude, \( d(e \sin \alpha)/d\alpha \) is the chaya table difference, and \( d\alpha/dt \) the mean daily change in anomaly.

As the apogee of the Sun is fixed, its motion in anomaly is the same as the mean motion.

The mean daily motion of the Moon is 790:35 [A 2,9]. For the Moon we use the table difference

### Table 2: Lunar chaya.

<table>
<thead>
<tr>
<th>Arg/10</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>0</td>
<td>53</td>
<td>104</td>
<td>152</td>
<td>195</td>
<td>232</td>
<td>262</td>
<td>285</td>
<td>298</td>
<td>303</td>
</tr>
</tbody>
</table>
ence [A 2.5] and a mean motion of the anomaly that is the difference between the mean lunar motion and the motion of the apogee 790.35 – 6:41 = 783.54:5. The correction to the mean motion is then 783.54:53/600 = 69:15. As we are in quadrant 1 the correction is positive. True daily lunar motion 790.35 + 69:15 = 859.50 [A 2.11].

The daily motion of the node is 3:11 (retrograde) [A 3.1].

The true daily motion in elongation is 859.50 – 57:4 = 802:46 = 48166 [A 3.2]. The last number comes from multiplying 802:46 by 60.

5.8 Precession
The precession is identical with the one used in Hindu astronomy. It is a zigzag function with a period of 7200 years and an amplitude of 27° = 162° = 1800/9:10. The precession is positive from AD 412 to AD 2212.

Add the epoch value 88 to the Kaliyuga year and divide by 1800: (5035 + 88)/1800 = 2 + 1523/1800. The number 2 [A 3, 3] tells us that the precession is positive and has the value 1523/9:10 = 1370:42 [A 3.4]. The tropical true solar longitude is thus 5986:14 + 1370:42 = 122° 37′ = 4:2:37 [A 3.5]. The tropical true lunar longitude is thus 16961:43 + 1370:42 = 305° 32′ = 10:5:32 [A 3.6].

5.9 Length of Day and Night
Again a table is used for this calculation. Table 3 gives the varying excess day times in vinadis over the mean half day of 15 nadis. For zodiacal signs above 6 the excess is negative. The solar tropical longitude is 4:2:37. Interpolating in the table gives 86 – (86 – 48):2:37/30 = 82 = 1:22.

A half day is then 15 + 1:22 = 16:22 [A 3.7].
A half night is 30 – 16:22 = 13:38 [A 3.8].
A quarter day is 8:11 [A 3.9].
A quarter night is 6:49 [A 3.10].

5.10 Noon Shadow
The equinoctial shadow for a location with geographical latitude \( \varphi \) is given by

\[
S_{\text{eq}} = G \tan \varphi
\]

where \( G \) is the height of the gnomon, in Burmese astronomy taken as 420 units or 7:0. The noon shadow at another date is

\[
S_{\text{noon}} = G \tan (\varphi - \delta)
\]

where \( \delta \) is the declination of the luminary. The Burmese method is to state the equinoctial shadow for the particular location, for Amara-pura 165 (\( S_{\text{eq}} = 420 \tan (21:32) = 165 \)), and then a table of the quantity \( |G \tan (\varphi - \delta) - G \tan \varphi| \) for different zodiacal signs. For Amara-pura we have Table 4.

Using the table with the Sun we have the tropical true longitude 4:2:37. Interpolating in the table gives 159 – (159 – 92):2:37/30 = 153. Subtracting this from the equinoctial shadow gives 165 – 153 = 12. Thus the solar noon shadow is 0:12 [A 3.11].


The reason for calculating both the solar and lunar noon shadow is, as we will see, that this eclipse starts before sunset.

5.11 Conjunction Time
The longitude of the Earth’s shadow will be exactly 180° away from the Sun. In our case we get 5986:14 + 10800 = 16786:14 [B 1.1].

The longitude of the Moon is 16961:43, thus it has moved 16961:43 – 16786:14 = 175:29 too far. This is converted into time using the daily motion in elongation, 175:29:3600/48166 = 13:7 nadis before midnight (24 hours) [B 1.2]. This is 30 – 13:7 = 16:53 \( \text{after noon} \) [B 1.3]. The Moon will move 859:50-13:7/60 = 187:58 backwards during this time and have an opposition longitude of 16961:43 – 187:58 = 16773:45 [B 1.4].

In the same way we can calculate the opposition position of the node, remembering its retrograde motion, and get 17238:44 [B 1.5]. The Moon has not yet reached the ascending node and its latitude is south. The number 0 [B 1.6] is the quadrant relative to the ascending node: (17238:44 – 16773:45)/5400 = 0: 464:59. The number 464:59 is the distance of the Moon from the ascending node.

5.12 Latitude of the Moon
The inclination of the lunar orbit in many Indian texts is taken as 4.5°. In the neighbourhood of the node we can approximate a spherical triangle by a planar one and get the lunar latitude by multiplying the distance from the node by

\[
\text{Table 3: Excess day time.}
\begin{array}{|c|c|c|}
\hline
\text{Sign} & \text{Excess} & \text{Sign} & \text{Excess} \\
\hline
0 & 6 & 6 & 0 \\
1 & 5 & 7 & 11 \\
2 & 4 & 8 & 10 \\
3 & 3 & 9 & 102 \\
\hline
\end{array}
\]

\[
\text{Table 4: Shadow table.}
\begin{array}{|c|c|c|c|}
\hline
\text{Sign} & \text{Excess} & \text{Sign} & \text{Excess} \\
\hline
0 & 6 & 0 & 6 \\
1 & 5 & 92 & 7 \\
2 & 4 & 159 & 8 \\
3 & 3 & 183 & 9 \\
\hline
\end{array}
\]
tan(4.5°) = 0.0787 ≈ 1/12.7. Some Southeast Asian astronomical schemes use the small angle approximation of the tangent function tan(4.5°) = tan(9°/2) = 9/2 – π/180 = 9/2 – 3/180 = 9/2 – 1/60 = 1/13.3 (Faraut, 1910; Gislén and Eade, 2001; Wisandarukorn, 1997). The Burmese scheme uses the factor 1/13 that may be a rounded value of either of these previous numbers. In the present case the distance from the node is 464.59, thus the latitude is 464.59/13 = 35.46 [B 1,7]. There can be an eclipse only if the distance of the Moon from the node is less than about 10°. The resulting difference in latitude between the above schemes is less than about 1 arc minute.

5.13 Diameters and Radii
As in Hindu astronomy, the diameter of the luminaries is assumed to be proportional to their true daily motions with a mean diameter of 31°. The diameter of the shadow is taken as 2.5 times the lunar diameter, a value also used in other Southeast Asian schemes (Gislén and Eade, 2001).
Diameter of the Moon 31·859.50/790.35 = 33.43 [B 1,8].
Diameter of the shadow 33.43·2.50 = 84.18 [B 2,1].
Sum of the radii (84.18 + 33.43)/2 = 59:1 [B 2,2].
Difference of radii (84.18 – 33.43)/2 = 25.18 [B 2,3].
Crescent 33.43/13 = 2.53 [B 2,5].

5.14 Eclipse Duration
(59:1 – 35:46)2 = 12538681 [B 3,5].
(35:46-60)2 = 4605316 [B 3,6].
(25:18-60)2 = 2304324 [B 3,7].
This is smaller than the number on the row above, thus the eclipse is partial.
√((59:1-60)2 – (35:46-60)2) = 2816.37

See Figure 1 for the geometry.

We convert this value to time using the daily motion in elongation 2816.30·48166 = 3:30 [B 2,6], the preliminary half duration.

5.15 Correction for the Change in Latitude During the Eclipse
The relative daily motion of the Moon relative to the node is 859.50/13 = 663.1.
This motion during 3:30 nadis is 663:1–3:30/60 = 50.2.
The change in latitude is 50.2/13 = 3.85.
The Moon is approaching the ascending node. Thus its south latitude decreases during the eclipse.
The latitude at the start of the eclipse is 35.46 + 3:52 = 39:38 (south) [B 2,7].
The latitude at the end of the eclipse is 35:46 – 3:52 = 31:54 (south) [B 2,8].
(39:38-60)2 = 5654884 [B 3,8].
(31:54-69)2 = 3966396. This number is written below grid B.
The formula R = r – β, used for the ‘eclipsed part’ above, is only correct at the conjunction. The correct formula to use is R + r = √(β2 + Δ2), where Δ is the Moon’s distance from the conjunction longitude. The numbers in [B 3,1] and [B 3,2] are purely nominal.

5.16 Corrected Duration Times
We repeat the duration calculations with the new set of latitudes. The duration from the start to the middle eclipse is then 3:16 [B 3,3] and from the middle eclipse to the end, 3:43 [B 3,4]. In the Suryasiddhanta scheme this iteration is repeated several times.

5.17 Eclipse Times
Start of the eclipse 16:53 – 3:16 = 13:37 after noon [C 1,1]. This is before sunset as the half day length is 16:22.

5.18 Shadow Calculations
The Burmese method of calculating shadows for times other than noon/midnight is based on the following model. At noon the deviation from the
noon shadow has to be zero. At set/rise the deviation is infinite. A mathematical function with the required behaviour is 
\[
(H \cdot M) / (D - H)
\]
or the equivalent expression
\[
(D \cdot M) / (D - H) - M
\]
where \( H \) is the time from noon/midnight, \( D \) is the length of a half day/night, and \( M \) is a suitably chosen multiplier. Actually this multiplier is a rather complicated function of \( H \), the geographical latitude of the location and the tropical longitude of the luminary. Ignoring the rather weak geographical latitude dependence, we arrive at a multiplier, that depends on two arguments: \( H \) and the tropical longitude of the luminary. A table with such multipliers is given in the present and several other Burmese sources (Gislen and Eade, 2014); see Table 5 above.

When using the table, only the zodiacal sign is used and \( H \) is rounded to the nearest \( nadi \); there is no interpolation involved. The total shadow, \( S \), will then be
\[
S = S_{noon} + (D \cdot M) / (D - H) - M \quad (4)
\]
Grid D treats the shadow calculations. Column [D 1] shows a solar shadow calculation as the Sun has not yet set at the start of the eclipse and the Moon is below the horizon. [D 2] and [D 3] are lunar shadow calculations. We have

### Table 5: Multipliers.

<table>
<thead>
<tr>
<th>Nadis</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capricorn</td>
<td>61</td>
<td>12</td>
<td>168</td>
<td>216</td>
<td>251</td>
<td>280</td>
<td>307</td>
<td>325</td>
<td>338</td>
<td>345</td>
<td>349</td>
<td>350</td>
<td>325</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aquarius</td>
<td>63</td>
<td>13</td>
<td>176</td>
<td>220</td>
<td>250</td>
<td>280</td>
<td>305</td>
<td>321</td>
<td>331</td>
<td>335</td>
<td>337</td>
<td>337</td>
<td>310</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pisces</td>
<td>79</td>
<td>13</td>
<td>187</td>
<td>232</td>
<td>267</td>
<td>291</td>
<td>309</td>
<td>320</td>
<td>324</td>
<td>335</td>
<td>325</td>
<td>312</td>
<td>301</td>
<td>288</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aries</td>
<td>89</td>
<td>162</td>
<td>236</td>
<td>283</td>
<td>312</td>
<td>330</td>
<td>339</td>
<td>340</td>
<td>335</td>
<td>328</td>
<td>317</td>
<td>303</td>
<td>296</td>
<td>267</td>
<td>268</td>
<td></td>
</tr>
<tr>
<td>Taurus</td>
<td>178</td>
<td>304</td>
<td>363</td>
<td>395</td>
<td>408</td>
<td>415</td>
<td>402</td>
<td>391</td>
<td>375</td>
<td>313</td>
<td>338</td>
<td>318</td>
<td>296</td>
<td>309</td>
<td>251</td>
<td></td>
</tr>
<tr>
<td>Gemini</td>
<td>571</td>
<td>599</td>
<td>582</td>
<td>563</td>
<td>590</td>
<td>509</td>
<td>484</td>
<td>456</td>
<td>430</td>
<td>409</td>
<td>375</td>
<td>349</td>
<td>322</td>
<td>301</td>
<td>270</td>
<td>229</td>
</tr>
<tr>
<td>Cancer</td>
<td>455</td>
<td>522</td>
<td>539</td>
<td>530</td>
<td>515</td>
<td>494</td>
<td>471</td>
<td>450</td>
<td>427</td>
<td>402</td>
<td>372</td>
<td>351</td>
<td>326</td>
<td>302</td>
<td>267</td>
<td>241</td>
</tr>
</tbody>
</table>

\[ D \cdot M = 818 \times 310 = 253580 \ [D 2,4]. \]
\[ D - H = 818 - 787 = 31 \ [D 2,5]. \]
\[ S_{noon} = 5:59 = 359 \ [A 3,11]. \]
\[ S = S_{noon} + (D \cdot M) / (D - H) - M = 359 + 8180 - 310 = 8229 = 137:9 \ [D 2,6]. \]

End eclipse:
The time is 4:14 from sunset [D 3,1].
The time is 20:36 from solar noon, \( H = 30 - 20:36 = 9:24 = 564 \) from midnight [D 3,2].
Zodiacal sign = 10, \( H = 9 \) gives \( M = 331 \ [D 3,3]. \)
\[ D \cdot M = 818 \times 331 = 270758 \ [D 3,4]. \]
\[ D - H = 818 - 564 = 254 \ [D 3,5]. \]
\[ S_{noon} = 5:59 = 359 \ [A 3,11]. \]
\[ S = S_{noon} + (D \cdot M) / (D - H) - M = 359 + 1066 - 331 = 1094 = 18:14 \ [D 3,6]. \]

### 6 SOLAR ECLIPSE CALCULATION

The solar eclipse calculation example is for the annular eclipse on BE 1295, sutin 128, 21 August 1933. By coincidence this eclipse is also calculated for Calculuta (Burgess, 2000: 392g). The procedures for calculating the longitudes are the same as for a lunar eclipse but the kyammat is reduced by 400 in the calculations in order to get the longitudes at noon. In Burma the eclipse was partial.

For grid A, we only give the numbers without a detailed derivation.

### 6.1 Fundamental Data

- [A 1.4] The New Year weekday, 0, Saturday.
- [A 1.5] The sutin, 128.

### 6.2 Mean Longitudes


### 6.3 True Longitudes

- [A 1.10] Quadrant 3 for the solar equation.
- [A 2.1] Solar equation, 98:54, negative.
- [A 2.2] Solar true longitude, 7462:33.
6.4 True Daily Motions

[A 2.10] Mean lunar daily motion 790:35.
[A 3.3] True daily motion in elongation, 42179.

6.5 Precession, Day Length and Noon Shadow

[A 3.9] Quarter day, 7:56.

6.6 Parallax of the Middle Eclipse

The difference in longitude between the Sun and the Moon at noon is 7462:33 – 7458:54 = 3:39. This is the distance the Moon still has to travel. We convert this into time using the daily motion in elongation [A 3.3]: 3:39·3600/42179 = 0:19 nadis [B 1,1]. This is the time of the true conjunction after noon. The time after midnight is 30 + 0:19 = 30:19 [B 1,2].

The procedure used to calculate the parallax in longitude (time) is to take the true conjunction time after noon and express it in vinadis, v. The parallax in nadis, n, is then computed from the formula

\[ n = (7·v)/ (600 + |v|) \] (5)

|v| being the positive value of v.

The correct parallax is a complicated function of the latitude of the luminary, the time of day, and geographical latitude. However, the formula above gives a surprisingly good approximation to the true parallax at least for locations not too far from the equator (see below). For times before noon the parallax correction in longitude is negative. Applying the formula with 19 vinadis above, we arrive at the longitudinal parallax (in time measure) of 0:13 [B 1,3]. Thus the true apparent conjunction time, corrected for parallax, is at 0:19 + 0:13 = 0:32 [B 1,4] after noon.

During the time from noon the Moon moves 0:32·760:32/60 = 6:46 [B 1,5]. The movement of the node is in the same way 0:2, backwards [B 1,6]. The longitude of the Moon is then

<table>
<thead>
<tr>
<th>( \lambda_N )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0</td>
<td>4</td>
<td>8</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
</tbody>
</table>

7465:40 [B 1,7], and of the node 17317:20 [B 1,8].

The descending node is at 17317:20 – 1080 = 6517:20. The Moon has not reached the descending node; its longitude is still north. Relative to the ascending node it is in quadrant 1 [B 1,9]. The distance from the descending node is 51:40, giving a latitude of 51:40/13 = 3:58 (north) [B 1,10].

The procedure for calculating the parallax in latitude is as follows. The time in nadis of the apparent conjunction after noon is multiplied by 6 in order to convert it to degrees (360° = 60 nadis). The result is then added to the true tropical longitude of the Sun. The result is an approximation to the latitude of the nonagesimal, \( \delta_N \). The table is based on the formula

\[ \delta_N = \sin 24° \cdot \sin \lambda_N \] (6)

In our case we have \( \lambda_N = 147° 12′ + 6:0:32 = 150° 24′ \). Reducing to the first quadrant we get by interpolation \( \delta_N = 11° 50′ [B 1,11], \) positive.

The parallax in latitude is then

\[ \pi = 49′ \cdot \sin (\delta_N - \varphi) \] (6)

where \( \varphi = 21° 32′ \) is the actual geographical latitude. The value, 49′, is the effective horizontal parallax of the Moon and the Sun used in Southeast Asian astronomy. This is also the mean value used in Suryasiddhanta (Burgess, 2000(V): 11).

We have \( \delta_N - \varphi = 11° 50′ - 21° 32′ = -9° 42′ [B 1,12] \). The parallax is taken from a new table (Table 7); we get by interpolation 7:45, negative [B 1,13].

The latitude corrected for parallax is then 3:58 – 7:45 = -3:47 (south) [B 1,14].

6.7 Diameters and Radii

[B 2.2] True diameter of the shadow (Moon), 29:49.

Table 7: Parallax table.

<table>
<thead>
<tr>
<th>( \delta_N - \varphi )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi )</td>
<td>0</td>
<td>8</td>
<td>17</td>
<td>24</td>
<td>31</td>
<td>37</td>
<td>42</td>
<td>46</td>
<td>48</td>
<td>49</td>
</tr>
</tbody>
</table>
6.8 Preliminary Data for the Beginning and End of the Eclipse

[E 1.1] Square of sum of radii, \((29:59-60)^2 = 3236401\).
[E 1.2] Square of middle apparent latitude \((34:70-60)^2 = 51529\).
[E 2.1] Square root of the difference: 1785.
[B 2.6] Convert to time \(1785-60/42179 = 2:32\), preliminary half duration.

During this time the Moon moves relative to the node \(2:32\) \((760:32 + 3:11)/60 = 32:15\).
This corresponds to a change in latitude of 32:15/13 = 2.28. During the eclipse the lunar latitude will decrease as the Moon is approaching the descending node.
The true latitude at the beginning is 3:58 + 2:28 = 6:26 (north) [B 2,7].
The true latitude at the end is 3:58 – 2:28 = 1:30 (north) [B 2,8].
Preliminary time of the beginning 0:32 – 2:32 = -2:0, before noon [B 2,9].
Preliminary time of the end 0:32 + 2:32 = 3:4, after noon [B 2,10].

![Figure 2: Parallax in longitude.](image)

\[\lambda = 180^\circ\]
\[\lambda = 0^\circ\]

\[\text{Time from noon/hours}\]

6.9 Latitude Parallax Corrections

Repeating the procedures for parallax in latitude we have:
Nonagesimal longitude at the beginning 147° 12′ – 6:2:0 = 135° 12′.
Nonagesimal declination 16° 26′ [B 2, 11].
Nonagesimal longitude at the end 147° 12′ + 6:3:4 = 165° 36′.
Nonagesimal declination 5° 45′ [B 2, 12].
Argument for beginning parallax 16° 26′ – 21° 32′ = -5° 6′ (south) [B 2,13].
Argument for end parallax 5° 45′ – 21° 32′ = -15° 47′ (south) [B 2,14].
Parallax correction at the beginning 4:4 (south) [B 3,1].
Parallax correction at the end 13:12 (south) [B 3,2].
Apparent latitude at the beginning 6:26 – 4:4 = 2:22 (north) [B 3,3].
Apparent latitude at the end 1:30 – 13:12 = -11:42 (south) [B 3,4].

6.10 Time Corrections

[E 1.3] Square of latitude at the beginning \((2:22\cdot 60)^2 = 20164\).
[E 1.4] Square of latitude at the end \((11:42-60)^2 = 492804\).
[E 2.3] \(\sqrt{3236401 - 20164} = 1793\). Beginning of the eclipse.
[E 2.4] \(\sqrt{3236401 - 492804} = 1656\). End of the eclipse.
[B 3,7] Conversion to time from the true conjunction 1793-60/42179 = 2:33.
[B 3,8] Conversion to time from the true conjunction 1656-60/42179 = 2:21.
[B 3,9] 30:19 – 2:33 = 27:46, time from midnight
[B 3,10] 30:19 + 2:21 = 32:40, time from midnight

As counted from noon the times in [B 3,7] and [B 3,8] become
[B 3,12] 2:21 + 0:19 = 2:40 = 160

Using the formula (1) to compute the longitudinal parallax correction we get
[B 3,13] 1:17
[B 3,14] 1:28

The total duration of the beginning phase is then

[C 1,1] Beginning of the eclipse after midnight 30:0 – 3:31 = 26:29.
[C 1,2] Middle eclipse 30:32.
[C 1,3] End of the eclipse 30:0 + 4:8 = 34:8.
[C 2,2] Middle time from noon 0:32.
[C 2,3] End time from noon 4:8.
[C 3,1] Western time of the beginning 10:35:36
[C 3,2] Western time of the middle 12:12:48
[C 3,1] Western time of the end 1:39:12

Modern calculations give 10:23 a.m., 12:7 a.m., and 1:43 p.m. respectively, local true solar time.

6.11 Grid D, Shadow Calculations

The tropical longitude of the Sun is 147° 12′, zodiacal sign 4.
Half day \(D = 15:52 = 952\).
[D 1,1] The time of the beginning from sunrise 15:52 – 3:31 = 12:21
[D 1,2] The time of the beginning before noon 3:31.
[D 1,3] \(H = 3:31 = 211\), sign 4, 4 \textit{nadir} gives
M = 563.
[D 1] D·M = 952·563 = 535976.
[D 1.6] Shadow 3·47.
[D 2.1] The time of the middle eclipse from noon 0:32
[D 2.2] The time of the middle eclipse after noon 0:32.
[D 2.3] H = 0:32 = 32, sign 4, 1 nadi gives M = 571.
[D 2.4] D·M = 952·571 = 543292.
[D 2.6] Shadow 1·27.
[D 3.1] The time of the end from noon 4:8
[D 3.2] The time of the beginning after noon 4:8.
[D 3.3] H = 4:8 = 248, sign 4, 4 nadi 2 gives M = 563.
[D 3.4] D·M = 952·563 = 535976.
[D 3.6] Shadow 4·25.

7 THE BURMESE SOLAR ECLIPSE PARALLAX CORRECTIONS

The Burmese parallax corrections represent a large simplification as compared with the analogous Suryasiddhanta corrections and it is therefore interesting to compare them with parallaxes computed using modern astronomy. How accurate are they?

The parallax in latitude and longitude of the Moon in general depends on the observer’s latitude, φ the longitude of the Sun, λ, that in a solar eclipse is equal to that of the Moon, the time from noon h, and the obliquity ε of the ecliptic. We here neglect the dependence on the distance of the Moon from the Earth.

7.1 The Parallax in Longitude

As shown above, for the parallax in longitude the Burmese use a very simple function:

\[ n = 7·v / (600 + |v|) \]  

where v is the time in vinadis after noon, and n the parallax in nadi. Since 1 vinadi = 1/150 hour and the mean elongation motion is 730’ per day we can rewrite this formula as

\[ \delta \lambda = 85.2·h/(4 + |h|) \]  

where now h is the time after noon in hours and \( \delta \lambda \) the longitudinal parallax in arc minutes. The Burmese formula is explicitly independent of the longitude of the Sun.

We compare the value of this function with the corresponding modern function for parallax in longitude (Meeus, 2000: 98–100) being a function of the geographical latitude φ, the solar longitude λ, and the time from noon h.

We use φ = 23.2°, the geographical latitude of Ujjain, but our result is quite insensitive of this choice. All latitudes of Burmese sites are quite close to 20°, and the manuscript has 21° 32’. The graph of the parallax function versus the time in hours from noon (Figure 2) for a fixed solar longitude is a sigmoid curve. Different solar longitudes give similar parallel curves but displaced slightly upwards or downwards. For simplicity we only show the extreme cases, the bottom black curve being the parallax for solar longitude 0° and the upper black curve the parallax for solar longitude 180°. The red curve is the Burmese longitudinal parallax, which is a kind of average of the modern values and quite closely approximates the modern parallax for solar longitudes 90° and 270°. The horizontal axis in this diagram is the time in hours from noon, and the vertical axis is the parallax in arc minutes. Negative hours are times before noon.

The error of the Burmese longitudinal parallax is at most about 10’, which corresponds to about 20 minutes in time.

7.2 The Parallax in Latitude

The Burmese parallax in latitude depends, as does the modern parallax, on the solar longitude λ, the observer’s geographical latitude φ and the time h from noon. We have again taken the observer’s latitude as 23.2° and compare parallaxes for λ = 0°, 30°, 60°, 90°, 180°, 210°, 240°, 270° (see Figures 3a–h). Graphs for other longitudes that are multiples of 30° are identical to the others but with the sign of the time axis reversed. The black curves show the modern parallax and the red curves the Burmese one.

The Burmese algorithm has a maximum discrepancy of about 5’. The main part of this discrepancy is due to the Burmese use of a horizontal parallax value of 49’ instead of the modern value of 57’.

8 CONCLUSIONS

A general impression one gets from the calculations above is that they are of surprisingly good quality. There are very few computational or typographical errors. The computational methods show a delicate balance between the need for accuracy and clever approximation shortcuts for simplifying the calculations. This is especially evident in the solar eclipse calculation and in the corrections for parallax.

The shadow calculations seem to be a characteristic of Burma and give a very simple and interesting solution to a complicated mathematical problem.

9 ACKNOWLEDGEMENTS

I am grateful to two anonymous referees for their helpful comments on the original version of this paper.
Figures 3a–3h: Parallax in latitude.
10 REFERENCES

ThaNa.pdf. Richmond, –266.


11 APPENDICES

11.1 Appendix 1

Partial lunar eclipse grids Burmese Era 1296 sutin 102, 26 July 1934.

See Tables 8A–D.

Table 8 A–D: Transcription of lunar eclipse panels A – D.

<table>
<thead>
<tr>
<th>A</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1296</td>
<td>51:56</td>
<td>3:11</td>
</tr>
<tr>
<td>2</td>
<td>5035</td>
<td>5986:14</td>
<td>48166</td>
</tr>
<tr>
<td>3</td>
<td>85</td>
<td>16943:9</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1370:42</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>102</td>
<td>53</td>
<td>122:37</td>
</tr>
<tr>
<td>6</td>
<td>6038:10</td>
<td>16961:43</td>
<td>305:32</td>
</tr>
<tr>
<td>7</td>
<td>16945:4</td>
<td>17238:2</td>
<td>16:22</td>
</tr>
<tr>
<td>8</td>
<td>5932:53</td>
<td>59:08</td>
<td>13:38</td>
</tr>
<tr>
<td>9</td>
<td>4361:58</td>
<td>790:35</td>
<td>8:11</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>57:04</td>
<td>6:49</td>
</tr>
<tr>
<td>11</td>
<td>21</td>
<td>859:50</td>
<td>0:12 5:59</td>
</tr>
</tbody>
</table>

Table 9 A–E: Transcription of solar eclipse panels A – E.

<table>
<thead>
<tr>
<th>A</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1295</td>
<td>98:54</td>
<td>760:32</td>
</tr>
<tr>
<td>2</td>
<td>5034</td>
<td>7462:33</td>
<td>3:11</td>
</tr>
<tr>
<td>3</td>
<td>292</td>
<td>7739:07</td>
<td>42179</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>128</td>
<td>23</td>
<td>1369:48</td>
</tr>
<tr>
<td>6</td>
<td>7561:27</td>
<td>280:13</td>
<td>147:12</td>
</tr>
<tr>
<td>7</td>
<td>7447:47</td>
<td>7468:54</td>
<td>15:52</td>
</tr>
<tr>
<td>8</td>
<td>3664:02</td>
<td>18317:22</td>
<td>14:08</td>
</tr>
<tr>
<td>9</td>
<td>3242:38</td>
<td>59:08</td>
<td>7:56</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>790:35</td>
<td>7:04</td>
</tr>
<tr>
<td>11</td>
<td>16</td>
<td>57:33</td>
<td>1:07</td>
</tr>
</tbody>
</table>

11.2 Appendix 2

Annular solar eclipse grids Burmese Era 1295, sutin 128, 21 August 1933.

See Tables 9 A–E.
Dr Lars Gislén is a former lector in the Department of Theoretical Physics at the University of Lund, Sweden, and retired in 2003. In 1970 and 1971 he studied for a Ph.D. in the Faculté des Sciences, at Orsay, France. He has been doing research in elementary particle physics, complex systems with applications of physics in biology and with atmospheric optics. During the last fifteen years he has developed several computer programs and Excel sheets implementing calendars and medieval astronomical models from Europe, India and Southeast Asia (see http://home.thep.lu.se/~larsg/).