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2017

Document Version:
Peer reviewed version (aka post-print)

Link to publication

Citation for published version (APA):
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Let \( \Omega \) be an open, bounded, domain in \( \mathbb{R}^3 \) with simply connected Lipschitz boundary \( \Gamma \). The outward pointing unit normal is denoted \( \hat{\nu} \), and denote the exterior of the domain \( \Omega \) by \( \Omega^e = \mathbb{R}^3 \setminus \overline{\Omega} \). See Figure 1 for a typical geometry.

Consider the following exterior problem:

1) \( (\mathbf{E}_{sc}, \mathbf{H}_{sc}) \in H_{loc}(\text{curl}, \Omega^e) \times H_{loc}(\text{curl}, \Omega^e) \)

2) \[
\begin{align*}
\nabla \times \mathbf{E}_{sc}(x) &= ik \mathbf{H}_{sc}(x) \\
\nabla \times \mathbf{H}_{sc}(x) &= -i k \mathbf{E}_{sc}(x)
\end{align*}
\]

3) \[
\begin{align*}
\hat{x} \times \mathbf{E}_{sc}(x) - \mathbf{H}_{sc}(x) &= o(1/x) \\
\text{or} \\
\hat{x} \times \mathbf{H}_{sc}(x) + \mathbf{E}_{sc}(x) &= o(1/x)
\end{align*}
\]

4) \( \hat{\nu} \times \mathbf{E}_{sc} \in H^{-1/2}(\text{div}, \Gamma) \)

where \( x = |x| \). This problem has a unique solution [1].

The exterior Calderón operator \( C^e \) is defined as \( C^e : \hat{\nu} \times \mathbf{E}_{sc} \mapsto \hat{\nu} \times \mathbf{H}_{sc} \), from the Sobolev space \( H^{-1/2}(\text{div}, \Gamma) \) onto itself, where the fields \( \mathbf{E}_{sc} \) and \( \mathbf{H}_{sc} \) satisfy Problem (E). The exterior Calderón operator is strongly related to the scattering problem for a PEC scatterer. The norm of this operator, which is finite in the space \( H^{-1/2}(\text{div}, \Gamma) \) but not in \( L^2(\Gamma; \mathbb{C}^3) \), quantifies the largest amplification factor of the surface current for a given incident field on a PEC surface.

In this paper, we prove:

- There exists an intrinsic orthonormal basis on \( \Gamma \). This basis generalizes the concept of vector spherical harmonics on a spherical surface to a general Lipschitz surface, and constitutes a natural basis for the analysis of the exterior scattering problem.
- Expressed in the expansion coefficients of this intrinsic basis, we find a representation map of the exterior Calderón operator. We prove that this map is invertible and we also give a simple expression of its inverse.
- As an operator in \( H^{-1/2}(\text{div}, \Gamma) \), the norm of the Calderón operator is finite. We find a simple way of computing this norm as the largest eigenvalue of a quadratic form using the representation map.
- The connection between the transition matrix (T-matrix) for a PEC obstacle and the corresponding Calderón operator.

References