On the analysis of the exterior Calderón operator for a non-spherical geometry

Kristensson, Gerhard; Stratis, Ioannis; Wellander, Niklas; Yannacopoulos, Athanasios

2017

Document Version:
Peer reviewed version (aka post-print)

Link to publication

Citation for published version (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
On the analysis of the exterior Calderón operator for a non-spherical geometry

Gerhard Kristensson* (1), Ioannis G. Stratis (2), Niklas Wellander (3) (1), and Athanasios N. Yannacopoulos (4)

(1) Department of Electrical and Information Technology, Lund University, P.O. Box 118, SE-221 00 Lund, Sweden.
(2) Department of Mathematics, National and Kapodistrian University of Athens, Panepistimioupolis, GR 15784, Zographou, Athens, Greece.
(4) Department of Statistics, Athens University Economics and Business, Patision 76, Athens, GR 10434, Greece.

Let $\Omega$ be an open, bounded, domain in $\mathbb{R}^3$ with simply connected Lipschitz boundary $\Gamma$. The outward pointing unit normal is denoted $\hat{\nu}$, and denote the exterior of the domain $\Omega$ by $\Omega_e = \mathbb{R}^3 \setminus \overline{\Omega}$. See Figure 1 for a typical geometry.

Consider the following exterior problem:

1) $(E_{sc}, H_{sc}) \in H_{loc}(\text{curl}, \Omega_e) \times H_{loc}(\text{curl}, \Omega_e)$
2) $\nabla \times E_{sc}(x) = ik H_{sc}(x), \ \ n \in \Omega_e$
3) $\hat{x} \times E_{sc}(x) - H_{sc}(x) = o(1/x)$ as $x \to \infty$
4) $\hat{x} \times E_{sc}(x) + E_{sc}(x) = o(1/x)$

where $x = |x|$. This problem has a unique solution [1].

The exterior Calderón operator $C_e$ is defined as $C_e : \hat{x} \times E_{sc} \mapsto \hat{x} \times H_{sc}$, from the Sobolev space $H^{-1/2}(\text{div}, \Gamma)$ onto itself, where the fields $E_{sc}$ and $H_{sc}$ satisfy Problem (E). The exterior Calderón operator is strongly related to the scattering problem for a PEC scatterer. The norm of this operator, which is finite in the space $H^{-1/2}(\text{div}, \Gamma)$ but not in $L^2(\Gamma; C^3)$, quantifies the largest amplification factor of the surface current for a given incident field on a PEC surface.

In this paper, we prove:

• There exists an intrinsic orthonormal basis on $\Gamma$. This basis generalizes the concept of vector spherical harmonics on a spherical surface to a general Lipschitz surface, and constitutes a natural basis for the analysis of the exterior scattering problem.

• Expressed in the expansion coefficients of this intrinsic basis, we find a representation map of the exterior Calderón operator. We prove that this map is invertible and we also give a simple expression of its inverse.

• As an operator in $H^{-1/2}(\text{div}, \Gamma)$, the norm of the Calderón operator is finite. We find a simple way of computing this norm as the largest eigenvalue of a quadratic form using the representation map.

• The connection between the transition matrix (T-matrix) for a PEC obstacle and the corresponding Calderón operator.

References