Model Uncertainty in Fire Safety Engineering

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Model Uncertainty in Fire Safety Engineering

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Abstract
The objective of this licentiate dissertation was to evaluate the predictive capability of smoke transport models quantitatively, in terms of model error and the uncertainty in the model error. The result is an adjustment model that can be used to take model error into account in future model predictions, thereby increasing the predictive capability. To exemplify the evaluation procedure an analysis is presented on model predictions from multiple scenarios.

The results of the analysis show that the predictive capability can be questioned for the two-zone models analysed, and the models should not be used uncritically. The analysis also shows that the model error can be quantified and taken into account, to increase the accuracy of the model predictions.

If uncertainty is not taken into account it is impossible to ensure that quantitative design criteria are fulfilled, which can lead to unsafe designs or unnecessarily expensive buildings. The choice of model can have substantial effects on the result. To be able to evaluate the magnitude of the model uncertainty comparison must be made with the other types of uncertainties present in the calculations. It is possible to do this quantitatively if the statistical method presented in this dissertation is used.

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Summary

Traditionally, fire safety has often been addressed with methods based on prescriptive recommendations. The opportunity to use an alternative analytical approach has led to the development of fire safety engineering, beginning with structural fire safety design in the 1960’s and today including fire safety design and fire risk analysis in general. The prediction of reality using model calculations and dealing with the errors and uncertainties associated with the calculations are two key tasks for a professional practitioner using an analytical approach. In fire safety engineering, smoke transport models are commonly used to predict the conditions caused by a fire. This is done despite the fact that knowledge of the errors and uncertainties associated with the models is lacking and there are insufficient means available to take them into account.

The licentiate dissertation “Model Uncertainty in Fire Safety Engineering” is part of the project “Design Based on Calculated Risk”, which is financed by The Swedish Fire Research Board (BRANDFORSK) and The Development Fund of the Swedish Construction Industry (SBUF). The objective of this part of the project was to evaluate the predictive capability of smoke transport models quantitatively, in terms of model error and uncertainty in the model error. The result is an adjustment model that can be used to take model error into account in future model predictions and thereby increase the predictive capability of the model. To exemplify the results of this study, model predictions of the smoke temperature and smoke layer height by the computer model CFAST 2.0 are evaluated by means of multi-scenario analysis. A single-scenario analysis is also carried out on smoke temperature predictions by the models FAST 3.1, FASTLite 1.0 and FPETool 3.2.

The analysis shows that the predictive capability of the two-zone models can be questioned and that the model results should not be used uncritically, without consideration of the model error. In the analysis of the scenarios it is concluded that the smoke transport model CFAST 2.0 overpredicts the temperature and underpredicts the smoke layer height. Whether or not this can be considered as a conservative prediction in a specific application depends on how hazardous conditions are defined in that situation. The analysis also shows that the model error can be quantified and taken into account, thus increasing the accuracy of the model predictions. A general all-scenario adjustment factor can not be derived, due to the variation in the predictive capability with the type of scenario. For a prediction in a specific scenario the adjustment model can be used to derive a conservative estimate of the model output to be used in a deterministic analysis. The adjusted temperature can also be expressed as a distribution, if the prediction is to be used in a probabilistic uncertainty analysis.

Even if the model error is taken into account, there will still be some bias and uncertainty in the adjusted predictions, but substantially less then before. If uncertainty is not taken into account, it is impossible to ensure that quantitative design criteria are fulfilled, which can lead to unsafe designs or unnecessarily expensive buildings. The choice of model can have severe effects on the result. To be able to evaluate the severity of the model uncertainty in relation to the total uncertainty in the assessment, a comparison with the other types of uncertainties included in the calculations, e.g. the uncertainty in input data, uncertainty in other predicted variables, etc., must be performed. This is possible quantitatively if the statistical method presented in this dissertation is used.
Sammanfattning (Summary in Swedish)


Licentiatavhandlingen ”Model Uncertainty in Fire Safety Engineering” är en del av projektet ”Dimensionering efter beräknad risk”, som finansieras av Styrelsen för svensk brandforskning (BRANDFORSK) och Svenska Byggbranschens Utvecklingsfond (SBUF). Målsättningen med det här delprojektet är att kvantitativt analysera modellfel och modellomsäkerhet i beräkningsresultat från brandgasspridningsmodeller. Resultatet av analysen är en modell som kan användas för att korrigera fel i framtidiga brandgasspridnings beräkningar och därmed öka precisionen och minska osäkerheten i beräkningarna. För att exemplifiera metodiken har en omfattande analys av beräkningsresultat från två-zons modellen CFAST 2.0 utförts och även en mindre analys av beräkningar med modellerna FAST 3.1, FASTLite 1.0 och FPETool 3.2.


Även om modellberäkningarna kan korrigeras, så kommer det fortfarande att finnas osäkerhet i det justerade värdet. Om modellfel och modellomsäkerhet inte beaktas i beräkningarna så är det omöjligt att bedöma om kvantitativa dimensioneringskriterier är uppfyllda, vilket kan leda till att säkerheten underdimensioneras eller att byggkostnaden ökar i onödan. Valet av modell kommer då att ha stor inverkan på resultatet. För att kunna utvärdera effekten av fel och osäkerheter modell beräkningar måste en jämförelse göras med andra delar av den totala osäkerheten i beräkningarna, tex osäkerhet i indata och osäkerhet i andra beräkningar. Detta är möjligt om metoden för att kvantifiera modellomsäkerhet som presenteras i denna avhandling används.
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1 Introduction

1.1 Background

Risk assessment, i.e. risk analysis and risk evaluation, has for decades been an important tool in helping decision-makers to make rational decisions in, for example, the nuclear, offshore and chemical process industries. Risk assessment is often based on an engineering approach to the problem. Risk analysis has been used together with fire safety engineering principals during the last few years with increasing frequency in many fire safety applications, especially fire safety design and fire risk assessment.

Fire hazards associated with life safety in buildings have traditionally been dealt with by “deemed to satisfy” provisions or prescriptive regulations. There has been no room for an engineering approach. During recent years, this trend has started to change and modern performance-based regulations allow the use of design methods that are based on engineering calculations.

Demands from building contractors used to be “do as little as possible to meet the regulations”. The correlation between fire risk assessment and business interruption, i.e. loss of profit in economic terms, was not clear to practitioners although research results indicated such a connection (Ramachandran, 1995; Watts, 1995). Today, the possibility of minimising building and risk costs with a fire safety engineering approach is well-known (Jönsson and Lundin, 1998; Mattsson, 1997). The benefits of an engineering approach have been more widely recognised, at the same time as regulations, methods and knowledge have made such an approach possible.

The effect has been that the discipline of fire safety engineering has grown and developed rapidly. Two major areas of application in which an engineering approach has been adopted are in fire safety design and fire risk assessment. There are many similarities between design and risk assessment. The basic equations and mathematical prediction models used by practitioners to estimate the consequence of fire are based on the same research.

The demand for efficient engineering approaches has increased in both these areas and the use of sophisticated prediction models is a step in this direction. Due to the complex nature of fire, mathematical prediction models used in fire safety engineering are often simplified and based on a number of assumptions. Even when very sophisticated models are available, a trade-off is often necessary between accuracy, cost and time for design engineers. Many years of research have made it possible to model a wide range of fire phenomena with fire and smoke transport models.

One problem that has been partly overlooked is the quality of predictive models. Due to a lack of knowledge of the predictive capability of such models, the credibility of model predictions in fire safety applications has been questioned. Within the fire safety engineering community there is a need for guidelines on how to take variability and inaccuracy in the model output, i.e. model uncertainty, into account.
The equations used to calculate the conditions in a building fire are often complex and therefore solved numerically by computers. The output from a single computer model is often only part of the total assessment. The output from smoke transport models is often used as input in design and risk assessment calculations and such models are therefore referred to as sub-models.

Figure 1 presents a fire safety design situation in which the result from a smoke transport model is used as input in a design calculation. In structural fire safety engineering applications the temperature exposure resulting from the fire is required as input. Although the final design will be affected by many factors, the accuracy of the computer model can have a significant influence on the results if the smoke temperature \( T_g \) is a sensitive input parameter. Since the smoke temperature is predicted by a model, there are many sources of uncertainty associated with this variable.

The design objective:
To calculate the minimum required concrete cover, \( l \), to maintain load-bearing capacity for 30 minutes.

Examples of sub-models:
- temperature exposure
- heat transfer in concrete beam
- material response to temperature
- load-bearing capacity

Figure 1. Example of design application; design of a load-bearing structure, exposed to fire.

A number of design guidelines (BSI, 1997; Buchanan, 1994; FEG, 1996; ISO/PDTR 13387-1, 1998) have been developed to help the engineer to address fire safety design problems. The many input and output variables used in fire safety engineering calculations and the relations between them are thoroughly discussed in design guidelines. General recommendations are given regarding the type of models which are suitable and some assistance is given in choosing design values. The model recommendations are often very general and are focused on the model output to be used, with no suggestion of any specific model or guidelines on how to take into account the predictive capability of the model.

Since both assumptions and simplifications are always present in models, it is impossible to predict real conditions exactly, even for a laboratory-controlled experiment. Models might give an accurate estimate of reality, but include a model error. The quality and usefulness of the model for decision-makers will depend on its accuracy. The increased use of prediction models in practical applications justifies the question: How good or accurate are model predictions?

Many models that can be used to predict fire and smoke spread are available for commercial use (Friedman, 1992) and since development is continuing, the number will
increase. Several qualitative studies have shown that the predictive capability of the models varies considerably (Beard, 1997; Hostikka et al., 1998). Qualitative analysis of the error in model predictions can often be insufficient in a quantitative assessment of a problem. No general method has so far been established for the quantification of the prediction capability of smoke transport models.

General recommendations have both advantages and disadvantages. One advantage is that if the guidelines are general they do not need to be updated when new and better models become available. Another advantage is that the designer can pick the most suitable or appropriate model for the specific problem. Together with increased flexibility, however, must come increased responsibility. Knowledge is necessary of the limitations of and errors associated with the equations or computer models used. The engineer has an obligation to use this knowledge. The predictive capability of models varies and there is no label on models stating the quality of the results.

1.2 Objectives and purpose

The aim of this dissertation is to report on the findings of a study of model uncertainty in smoke transport models. The scope of the study was to analyse the predictive capability of models used in fire safety engineering applications. Emphasis has been placed on deterministic smoke transport models. The objective was to develop a methodology to take model uncertainty into account in model output explicitly. The approach taken was based on a statistical quantitative analysis of model predictions and experimental measurements to quantify the model error and the uncertainty in the model error, i.e. the model uncertainty.

The purpose of the work was to derive a method to explicitly take the predictive capability of smoke transport models into account in fire safety engineering. Another purpose was to develop a tool that could be used to assess uncertainty in engineering analysis.

The following steps were included in the quantitative analysis:

- The study of the uncertainties in model predictions from smoke transport models
- The development of a method to quantify the model error and model uncertainty
- The development of a model to adjust future model predictions for model error and to take model error into account.

To illustrate the method, a multi-scenario analysis was carried out on model predictions from the smoke transport model CFAST 2.0 (Peacock et al., 1993a). A comparison of the model error for several smoke transport models in a single-scenario analysis is also presented.
There is no generally accepted protocol for addressing uncertainties in a model confidence study with a statistically quantified approach. The quantitative approach must be regarded as one of several recognised tools. The list below gives a variety of tools which can be used in model evaluation (ASTM, 1996).

- Comparison with standard tests
- Comparison with large-scale simulations
- Comparison with documented fire experience (expert judgement)
- Comparison with previously published full-scale test data
- Comparison with proven benchmark models

The most suitable method depends on factors such as data available, the available resources, area of application, etc.

The need for proper evaluation is obvious. Model predictions can not be used uncritically in engineering applications purely on the basis of their existence nor can the assessment of predictive capability be based on faith alone. The purpose of this work is to elucidate this area, by studying one of the suggested approaches above. This dissertation presents a method of quantifying and taking the model error in account for smoke transport models, based on comparison with previously published full-scale test data.

1.3 Overview of the dissertation

After this brief introduction Chapter 2 gives a general presentation of uncertainty in engineering calculations and the different sources are divided into classes. Depending on the type of uncertainty and the interest in it, the results of calculations can be presented in different ways. It is necessary to recognise the additional sources of uncertainty which are introduced when a problem is assessed with analytical methods in contrast to detailed recommendations. Uncertainty can be taken into account on different levels of detail, depending on different aspects related to the problem being studied. When calculations are used as the basis for rational decisions, it is necessary to have explicit knowledge of the uncertainty involved.

In Chapter 3 a short presentation is given of the two major areas in which fire safety engineering has proven to be a necessity. A brief description is given of the applications, regulations, design methods and design criteria involved in fire safety engineering. Model prediction is an important tool in the analytical assessment of a problem. Uncertainties in the analytical design and analysis methods are discussed in general and with focus on the use of prediction models. Predictive capability is of great importance for the final result. Attention was directed to the differences in uncertainty between traditional methods and the analytical methods used in an engineering approach.

In Chapter 4, different types of prediction models are introduced through general and fire and smoke transport models. When model predictions are used, model error and model uncertainty are always introduced due to assumptions and simplification. The models and their sources of error and uncertainty are evaluated. Finally, the possibility of adjusting the output from deterministic smoke transport models is discussed.
Statistical methods used to quantify the model error and model uncertainty were studied in Chapter 5 and applied to a specified range of model output, i.e. scenario configuration or scenario. A statistical model was developed based on the study of model error and model uncertainty in a single-test and multiple tests defined as a scenario configuration. The statistical method is based on the comparison of predicted and measured data from well-defined scenarios.

In this study a multi-scenario analysis of the smoke transport model CFAST 2.0 was carried out to exemplify the statistical method. The scenarios used in the analysis are presented in Chapter 6. The analysis is based on previously published data from various sources. No additional full-scale experiments were carried out. The statistical method was also used to compare a number of different smoke transport models in a single-scenario analysis.

In Chapter 7 the results from the single- and multi-scenario analysis are presented together with examples on how the results can be used in assessment of uncertainty and evaluation of predictive capability. Chapter 8 contains discussion and the conclusions are given in Chapter 9. A number of appendices are included, presenting statistical model, written as a Matlab file, and various simulation results.

The dissertation has been written for engineers who are familiar with fire safety engineering applications and smoke transport. Basic knowledge of engineering statistics is also required.
2 Uncertainties in calculations

Input parameters for mathematical models, the accuracy of mathematical models and the competence of the engineer who makes decisions are all included in engineering design and analysis and will have an influence on the final result. If decisions are to be based on the results of engineering calculations it is necessary to understand how uncertainties are introduced into the engineering calculations, how they effect the results and how they can be dealt with, so that failure does not occur. Blockley (1980) presented a list of different failure types for structural engineering, which can be generalised and also recognised in the area of fire safety engineering. The failure types defined by Blockley can be categorised in different uncertainty classes. Energistyrelsen (1996) used the following classes to define uncertainty in engineering applications that can lead to undesired events, i.e. failure:

- Resources
- Assumptions and decisions
- Mathematical models
- Input data

The classes should be seen as examples, since other types of categorisation can be used (Rowe, 1994). The classes are organised in a hierarchy where the first, uncertainty in resources, is the most general and the last, uncertainty in input data, is the most specific one. It can be concluded that the uncertainty classes are of different nature, although the classes are not necessarily independent. In the high hierarchy classes are of implicit nature, while the uncertainties in the specific classes can be dealt with more explicitly.

The relation between the classes is illustrated in Figure 2. Methods of dealing with an uncertainty depend on which class it belongs to. Section 2.1 gives a brief description of the different classes.

Figure 2. The hierarchy of classes of uncertainty. (Energistyrelsen, 1996)
2.1 Classes of uncertainty

2.1.1 Uncertainty in resources

This class of uncertainty is very general and difficult for the engineer to estimate at the time the modelling is being performed. Nevertheless, this class of uncertainty effects the results. Factors such as the state of the art of the tools that are available, quality control of projects, management routines and policy in the company, uncertainty in methods of assessing a problem, the quality of the research results in the particular area, can be found in this class. These factors are not related to the engineer’s abilities in a specific situation, but rather the limitations set by others. The resources available, for example time and money, are factors that significantly affect decisions and thus results. The effect on the results from the factors mentioned above is difficult to predict since there are no tools available to directly link this kind of uncertainty to the result in qualitative terms. Uncertainty in resources often affects the result implicitly, and it is important to consider the uncertainties in this class to ensure high quality in engineering applications.

Tools that can be used by management to consider these uncertainties can be the adoption of a quality control system, continuous planning for education of engineers, keeping up to date with the latest knowledge and tools, supporting development and research, etc. Knowledge of the competence of the staff is also necessary, to be able to assign the right people to the right task.

The parameters in classes higher up in the hierarchy effect the uncertainty of the parameters in the more specific classes. If these uncertainties are effectively dealt with, they will affect uncertainties in the classes with lower hierarchy. For example, a trade-off between money and accuracy can lead to the purchase of a cheaper model. The uncertainty in resources can affect the selection of a prediction model and thereby affects the uncertainty in the mathematical model, see Section 2.1.3.

2.1.2 Uncertainty in assumptions and decisions

This class is related to the uncertainty in assumptions and decisions made by an engineer when solving a problem. Issues such as defining the scope and limitations of the analysis, the engineer’s ability to describe the process or system, the choice of analysis method, the choice of mathematical model, and identification and screening of hazards are included in this class. Uncertainties in this class are also difficult to predict and deal within a quantitative way.

Company policies and routines are important means of reducing uncertainty in this class. One problem faced by engineering firms is that the variation in the design results depends on which engineer performs the calculation. It is impossible to totally standardise assumptions and decisions made by individuals, but steps can be taken to align them. The objective of a firm must be similar or almost identical prediction results regardless of the engineer, if the same model is used.

The room for the individual engineer to make the wrong decision or mistakes can be decreased, but there must be flexibility for the engineer to make his/her own adjustments for the specific problem. It may thus not be possible to specify such adjustments in general guidelines.
The implementations of well established design guidelines and standards for risk analysis may be a step in the right direction to reduce this kind of uncertainty.

Education and ability to understand company policies is essential. The introduction of an Engineering Certificate may be one way of ensuring a minimum level of education, experience and competence among engineers. The purpose of the certificate would be to ensure the quality of engineering work. If a suitable certificate were to be required to perform a given task, the uncertainty in this class would be reduced.

2.1.3 Uncertainty in mathematical prediction models

Even when using a computer model according to the recommendations for a problem that is within the explicit limits of applicability for the model, the result will be uncertain due to the errors in the model. If the errors are not constant for the range of applicability there will be an uncertainty in the model error, i.e. a model uncertainty. Uncertainties in mathematical prediction models are normally thought of as an uncertainty due to lack of knowledge.

In engineering applications mathematical models are often involved. The complexity of the models can vary from simple summation to advanced modelling of physical or chemical processes. In fire safety engineering applications, results from a number of different mathematical models are often used and linked together. For example, it is possible to model the time elapsed before critical conditions are reached, detection time and movement time of humans. This report focuses on the uncertainty in smoke transport models, which are mathematical models used to predict the time elapsed before critical conditions are reached. One of the objectives of the study presented in this report has been to develop a statistical model that quantifies the uncertainty in this type of model.

The uncertainty in different prediction models is likely to vary which links this uncertainty class both to the assumptions and decisions made by the engineer and to company policies. The engineer and the company are likely to decide what type of model that should be used and also how the uncertainty in the calculations is to be handled.

In a round robin code assessment exercise a well-specified scenario and instructions on which model to use were given to a number of researchers in different organisations that which used different models and different types of models (Hostikka et al., 1998). Some of the researchers had been instructed to use the same model, in order to evaluate the influence of the modeller. Although the task was well specified, different assumptions and simplifications had to be made to suit the different models. The conclusions from the modelling exercise were that the results varied greatly and depended on which model was used. The results obtained from the same model also varied, which indicates that the uncertainty arising from uncertainties in assumptions and decisions is not negligible.

The model uncertainty and model error in smoke transport models are discussed further in Chapter 4.
2.1.4 Uncertainty in input data
The input parameters in the mathematical models are often subject to uncertainty. The uncertainty can be propagated through the mathematical model and its effect on the results evaluated. The quality of the model output obviously depends on the quality of the input data, but also on the uncertainty in the mathematical model itself, as discussed in Section 2.1.3. Input data can often be based on measurements of real world conditions. Uncertainty in input data can be quantified by statistical analysis of measured data with expert judgement (EAL, 1997). Uncertainty in input data can be due to natural variation or lack of knowledge, see Section 2.2.

2.2 Quantitative presentation of uncertainty
The classes described in Section 2.1.3 and Section 2.1.4 are often merged together in quantitative analysis of uncertainty. They are referred to as uncertainty in model predictions (IAEA, 1989).

In quantitative uncertainty analysis two different types of uncertainties are normally recognised. They are of different natures, but can be analysed using the same type of statistical methods. Both uncertainty types can be expressed as random variables and can be described by statistical distributions and are called Type A and Type B (IAEA, 1989).

- Type A uncertainties are characterised as natural stochastic variations and are sometimes referred to as aleatory uncertainties. This type of uncertainty represents natural random variation. An example of this kind of variation is wind speed during a year or the number of people in a shopping centre during a day.

- Type B uncertainties are characterised as uncertainties in knowledge and are sometimes called epistemic uncertainties. Typical knowledge uncertainty in engineering applications maybe, for example, lack of knowledge of certain parameters, uncertainty in mathematical models due to simplification and assumptions in the model made by the model developer.

If a model is used to predict real world conditions, the accuracy of the prediction depends on the characteristics of the problem and the model chosen. Since there is a variety of different types of models, it is important to consider what kind of problem the model is intended for, the type of information that is given and how uncertainties are treated. When the uncertainties in the model output are evaluated it is important to be aware of how uncertainties in the input data and the prediction model affect the results and how uncertainties are dealt within the modelling exercise.

The degree to which uncertainties can be taken into account differs depending on whether the model is deterministic or probabilistic. The choice of model and treatment of uncertainties must depend on the problem modelled and the influence of the uncertainties on the results.

The following sections describe how uncertainties of type A and type B are expressed quantitatively in the model output. The suitability of the different types of models depends on the kind of uncertainty present and the kind of information that is needed.
Uncertainties in calculations

about the uncertainty. This has to be carefully considered by the engineer carrying out the modelling. Figure 3 to 7 are adapted from “Evaluating the Reliability of Predictions Made Using Environmental Transfer Models” (IAEA, 1989).

2.2.1 Deterministic prediction, no account taken of uncertainty
The model output in Figure 3 is expressed as a single value, i.e. a point estimate. No variation limits are given, which indicates that the uncertainty has not been taken into account.

![Deterministic prediction](image)

*Figure 3. Applicable when both type A and type B uncertainties are negligible.*

This is a simple, deterministic type of model, which may be time-dependent, or not. Time-dependent models are often used in fire safety engineering applications since fire development is a time-dependent phenomenon. The output from computational prediction models often consists of point-type estimates of several variables which can be expressed as a vector. The types of models used in fire safety engineering applications are discussed in Chapter 4.

The prediction made by a deterministic model can be dependent on several different input variables, but if there are uncertainties the uncertainty in the result will not be apparent.

A model with this type of output is suitable for situations where both type A and type B uncertainties are negligible, e.g. in a well-defined controlled laboratory experiment. This is very seldom the case when modelling is used in real-world applications, where uncertainty is almost always present. If uncertainty is present and a simple deterministic model is used, a separate adjustment must be made to include the uncertainty in the result, see Section 4.5.

2.2.2 Deterministic prediction, account taken of type B uncertainties
At a specific point in time in a well-defined fire scenario there is only one “true” mean temperature. If the prediction of the true mean temperature is based on a simple calculation of the average value from twenty thermocouple measurements, \( T_i \), see Eq. [2.1], the mean will vary since it depends on where the thermocouples are located in the room. This uncertainty is a typical uncertainty of type B.

\[
T_{\text{mean}} = \frac{\sum_{i=1}^{20} T_i}{20}
\]

[2.1]
The point-type (deterministic) prediction, $T_{\text{mean}}$, of the true deterministic mean temperature will vary due to insufficient resolution of the model. If the number of measuring points were to be increased, the uncertainty would be reduced, but would not disappear. Figure 4 shows the output from a model of a deterministic value, where there is an uncertainty in the prediction.

![Figure 4: The type A uncertainty is negligible while type B uncertainties are not.](image)

The distribution represents the variation in the prediction, but the prediction itself is of point-type. In the example, it is assumed that there is no uncertainty in the measurements, $T_i$. The uncertainty originates from the “poor” representation of the real conditions by the model in Eq. [2.1], and is therefore classified as a type B uncertainty. If the configuration of thermocouples in the fire room is changed, the measurements, $T_i$, are likely to change and, therefore, also the model prediction of the mean temperature. Figure 5 describes the probability of a certain predicted mean temperature on the basis of different thermocouple configurations in the fire room.

![Figure 5: Effect on average temperature due to type B uncertainty.](image)

It is important to realise that the result is a single deterministic value, but the value is uncertain.
2.2.3 Probabilistic prediction, no account taken of type B uncertainty

Figure 6 shows the results obtained from a probabilistic model where the model output is uncertain. The input parameters are stochastic due to natural variation. In many real world situations parameters vary and it is inappropriate to treat them as deterministic variables.

![Figure 6. Type B uncertainty is negligible while Type A is not.](image)

The time required for people to evacuate a warehouse can be used as an example. Evacuation time is a function of the number of people in the building. The number of people in a warehouse will vary during the day and therefore the evacuation time will be different depending on when evacuation takes place. If the model prediction is defined as evacuation time from the warehouse, with no specification of time of the day the evacuation is to take place, the model result from a probabilistic model will have the appearance of a curve like the graph in Figure 6. The result is given as a complementary cumulative distribution function (CCDF profile) and answers the question: How high is the probability that the evacuation time will exceed y?

This type of model output is necessary to show the effect of type A uncertainty on the result. The prediction is not a single deterministic variable. It is a variable that varies within an interval specified by the CCDF profile.

2.2.4 Probabilistic prediction, account taken of type B uncertainty

In a situation where there is a need to study the effect of uncertainties of type A and type B, model output can be presented as in Figure 7. This can occur in a situation when a stochastic process is modelled and the value of a variable contains an uncertainty of type B.

![Figure 7. Neither type B nor type A uncertainty is negligible.](image)
The uncertainty of type A is modelled probabilistically and is represented by a CCDF profile. The type B uncertainty is also represented by a distribution, but is treated differently to display the presence of different types of uncertainty. To illustrate the combined effect of the uncertainties in a two-dimensional illustration certain quantiles from the type B distribution are used. In Figure 7 the dashed lines represents the 5% and 95% quantiles. In the reference prediction, the 50% quantile from the type B distribution is used. If there is no need to separate the different types of uncertainties they can be modelled together and merged into a single CCDF profile (IAEA, 1989).

In the discussion above it has been assumed that the modelling is accurate and all the uncertainty is to be found in the input variables. If model uncertainty is to be taken into account the strategy adopted will depend on the type of model used, the type of uncertainty present and if the different types of uncertainties have to be separated in the results. This is discussed further for deterministic models in Section 4.5.

2.3 When to take uncertainty into account

Uncertainties can be dealt with and expressed in different ways and depending on the situation at hand. In the report “Uncertainty in Quantitative Risk Analysis” (Energistyrelsen, 1996) three different ways of dealing with the problem are suggested:

- No treatment of uncertainties
- A rough estimate of uncertainties
- Extensive analysis of uncertainties.

2.3.1 No treatment of uncertainties
The following “acceptable” reasons for not dealing with uncertainties in the calculations were presented (Energistyrelsen, 1996):

- The problem to be analysed is well defined (laboratory controlled experiment)
- An accepted model is used with accepted input data (design variables)
- The possibility of an unwanted consequence is unacceptable
- Conservative estimates and “worst case” conditions are used.

Unacceptable reasons are:
- There is no specific request or requirement for an uncertainty analysis
- Lack of resources in terms of time, money and competence.

Quantitative analysis and calculations can give the impression of presenting a single true value, if an uncertainty analysis is not carried out. It is very important that decision-makers are informed of this.

2.3.2 A rough estimate of uncertainties
A rough estimate consists of a quantitative and/or qualitative analysis of relevant uncertainties. The uncertainties that have been included in the analysis must be explicitly described and the effect on the result presented. Sensitivity analysis can be used to identify the most important variables (NKB, 1997).

2.3.3 Extensive analysis of uncertainties
An extensive analysis includes detailed quantitative analysis of all the important uncertainties. The analysis should describe the importance of the different uncertainties,
the type of uncertainties and quantify the uncertainty in the result by propagating the uncertainty in the calculations.

Models seldom include a feature to address model uncertainty originating from the model itself. Therefore the engineer must address this separately in his/her analysis. Different methods for propagating uncertainties are available and depend on how the uncertainty is expressed and the type of model used (Magnusson, 1997). In simple analytical expressions propagation can be performed with the Monte Carlo technique with software such as @Risk (Palisade, 1996) or with the reliability index-$\beta$ method also available as software for computers (Strurel, 1995). If the model is more complex or only available as a computer program a separate computer program might have to be written to perform an efficient uncertainty analysis. An alternative is to create a meta-model of the original model (Frantzich, 1998), to be able to use the simpler analysis methods. The result of an uncertainty analysis is often presented as an interval with a certain confidence level, as presented in Section 2.2.4.
3 Fire safety engineering approach

Although fire safety engineering is a relatively young branch of engineering, compared with traditional fields such as structural or civil engineering, fire safety issues have been addressed for a very long time. Traditionally, preventive actions against fire damage have been strongly influenced by of rule of thumb and experience from earlier fire accidents. Today, decisions are often influenced by analysis and prediction in contrast to history and tradition.

One of the underlying factors is that the development of fire safety engineering has made it possible to assess fire safety with analytical tools. Another important factor is how the requirements of fire safety are expressed in codes and regulations. A general trend in society today is that codes seem to be changing from prescribing how to achieve the objectives of the code to prescribing what should be achieved. The development in the area of fire safety began with structural fire protection in the 1960’s, and during the last decade has reached the area of life safety. The reason is that more flexible and cost-efficient results can be achieved, but also that it is necessary to keep up with rapid developments in society. The effect of these changes is increased flexibility, but also increased responsibility for the professional practitioner.

3.1 Definition of fire risk

Fire safety can be seen as the opposite to fire risk. The objective of most fire safety engineering applications is related to design to prevent or assess the risk of fire. An instinctive emotional interpretation of risk is often something related to an unwanted consequence or loss, but it seems somewhat vague to use it as a definition in an engineering assessment. Even if the word risk is very well-known, it is a term with a broad meaning and there is no generally agreed definition which covers all areas of application. It is difficult to find a definition that suits all purposes, but rather avoiding the definition of a confusing term, a clear definition is given here.

Kaplan and Garrick (1981) argue that the question “What is risk?” can be divided into three questions: “What can happen?” “How likely is it to happen?” “If it does happen, what are the consequences?” They conducted that risk can be described as a triplet \( (S_i, L_i, X_i) \), where \( S_i \) denotes the \( i \)th sub-scenario, \( L_i \) the probability or likelihood of sub-scenario \( i \) occurring and \( X_i \) the consequences of the \( i \)th sub-scenario. Depending on the definitions (Kaplan, 1997), the sub-scenarios can be organised in an event tree where an initial event is the source of the events leading to possible outcomes \( S_i \), see Figure 8.
Hazard is defined as the source of the risk. Hazard identification is the first step in the event tree modelling process and answers the question “What can go wrong and why?” (Kolluru et al., 1996). An initial event or failure triggers an accident and depending on subsequent events, for example failure in fire safety systems, different sub-scenarios are defined. In Figure 8 the consequences $X_i$ can represent a single consequence or several consequences.

Event trees offer an effective way of structuring and analysing a problem and provide a useful tool to deal with the uncertainties in fire safety engineering. The complexity of the necessary analysis depends on the scope of the problem. The complexity of the analysis of the risk can vary from simple deterministic consequence analysis of one or several sub-scenarios to a complete quantification of all the sub-scenarios where the probability and consequence of each sub-scenario are treated as stochastic variables.

If an analysis is performed of the consequences $X_i$ in one or more sub-scenario a consequence analysis results. This can be used to identify the “worst case”, i.e. to obtain an idea of the variation in the possible consequences of a specified hazard or to quantify the consequences in a specified sub-scenario. The information on the risk is limited since the likelihood and perhaps not all sub-scenarios are analysed. This type of analysis is useful when no treatment of uncertainty is required or when a rough estimate of uncertainties is required, see Section 2.3. The uncertainty is dealt with by using guidelines and well-established methods of quantifying the consequences.

Quantitative risk analysis (QRA) is a more extensive form of consequence analysis. In a QRA the likelihood of each sub-scenario is also analysed and a quantitative risk measure can be calculated. This methodology is preferably used when a detailed quantitative analysis of the uncertainties is required, see Section 2.3.3. The different types of uncertainties that can be identified with an event tree can be dealt with by different methods.

A quantitative definition of fire risk is defined here and will be used in the discussion of quantitative approaches to fire risk and fire safety within fire safety engineering. Fire
risk has traditionally not been dealt with explicitly in regulatory systems, but the possibility of using analytical tools in practical engineering applications like consequence and risk analysis is improving. Jönsson and Lundin (1998) demonstrated a number of benefits of risk analysis in design applications, e.g. as a tool for verification and cost-benefit analysis. Frantzich (1998) presented examples of how to assess uncertainties in fire safety engineering applications, by using uncertainty analysis techniques and event-tree modelling, based on the Kaplan-Garrick definition of risk.

The capability of quantitative approach is very dependent on the aspects related to the area of application. Examples of such aspects are: how the requirements are specified, the available design or assessment methods, the required analytical tools, the type of uncertainty present and how to deal with the uncertainty, etc. An attempt to shed some light on these aspects is made in the following sections.

### 3.2 Areas of application

It is well-known that fire can cause direct damage to humans, property and the environment and also be the triggering event for other serious accidents. Fire safety design and fire risk assessment are areas in which the benefits of an engineering approach have been recognised and where the regulatory system permits assessment of fire risk with engineering methods. A short overview of these areas is given below.

#### 3.2.1 Fire safety design

Building design is an example of an application where fire safety must be taken into consideration. The main objective of fire safety design is normally to protect and prevent injury to occupants and fire service personnel and damage to the property and the environment. The overall fire safety design in a building can be divided into a number of different sub-systems. To concretise the design objective sub-groups can be specified. In the Swedish building code (Boverket, 1998) the following division is made:

- Escape in event of fire
- Protection against the outbreak of fire
- Protection against the spread of fire inside a fire compartment
- Protection against the spread of fire and fire gases between the fire compartments
- Protection against the spread of fire between buildings
- Load-bearing capacity in the event of fire
- Fire fighting facilities

To provide adequate, safety the engineer must design a fire safety system for the building. The solution or fire safety strategy can include a number of sub-systems, e.g. alarm systems, a smoke-exhaust system and extinguishing systems such as sprinklers. The building code normally defines a minimum level of fire safety for the occupants. It is not uncommon that the owner or insurance company to have higher demands on property protection.

Several large engineering firms have recognised the opportunities given by an analytical assessment of fire safety issues and have formed specific engineering groups dealing with fire safety.
Other areas where fire safety design is of interest is in the chemical process industry and in the offshore industry.

### 3.2.2 Fire risk assessment

Fire can cause severe damage to an industry or to society and can also initiate a chain of events which will have disastrous effects in, e.g. a factory, a ship, an aeroplane etc. Fire risk assessment is a way of analysing and evaluating the fire risk and is often used to provide information which can be used to control and reduce the risk or to verify that risk levels are acceptable according to specified levels. These actions are defined as risk management (IEC, 1995). Fire risk management is a rational way of dealing with fire risk, instead of regarding accidents as just coincidences. Legislation often requires the oil and chemical industries to make explicit analyses of the risk to which the surroundings are exposed. These risk analyses are important tools in land use planning, design of an insurance strategy for industries, etc. Fire constitutes a serious risk source and can also lead to serious damage.

The tools used to predict the consequences in applications of fire safety design and fire risk assessment are the same and the requirement to address the uncertainties are also the same. The similarities between design and risk assessment are many. In this dissertation, fire safety design is used as an example of an application, but the situation is almost exactly the same in assessment applications.

In the following sections, the types of codes and regulations that make it possible to use an engineering approach to address fire safety are presented. The available methods and the uncertainty introduced when an engineering approach is used instead of a traditional approach are discussed.

### 3.3 Codes and regulations

The need for fire safety in society is obvious and is addressed in several different codes and regulations. For example, one third of the contents of the Swedish building code (Boverket, 1998) deals with fire safety and in regulations controlling the transport of hazardous materials and land use planning fire safety forms an extensive part. The final requirements on which the engineering design is based are often a combination of requirements from different codes and regulations plus additional demands. Insurance companies, the building owner and/or tenants can also add additional requirements. The regulations and codes can be divided into two types where the objectives are expressed differently. These types are called prescriptive codes and performance-based codes and are described in Sections 3.3.1 and 3.3.2. The type of code directly influences the design method that can be used.

#### 3.3.1 Prescriptive codes

Prescriptive codes are characterised by being deemed to satisfy provisions, and do not explicitly express the required level of safety. The flexibility is often limited since a prescriptive regulation is a more or less detailed instruction for how to solve a problem. Prescriptive codes address the important issues and ensure that they are solved in a way such that the requirements of society are met implicitly. In other words: “if you do it by the book, you are home free”. Traditionally, fire safety regulations have been formulated in a prescriptive way and there has been no room for engineering analysis.
During the last decade, a different type of code has been introduced, which opens up the possibility of fire safety engineering in many new areas.

### 3.3.2 Performance-based codes

Performance-based codes are conceptually different from prescriptive codes. The general idea of a performance-based code or regulation is to regulate what has to be achieved, not how. These regulations express the requirements in terms of explicit performance objectives. The method of solving a particular problem is left to the designer and his/her engineering skills. Several methods are available to meet the performance objectives specified in a code.

In many counties, such as Australia, Japan, New Zealand, Norway, Sweden and the UK, the potential of performance-based regulations has been recognised (SFPE, 1998). The traditionally prescriptive nature of building regulations in these counties has changed to performance-based codes. This development is continuing and a number of counties, for example Canada, Denmark, South Africa and the USA, are planning to introduce performance-based codes. The current status of performance-based codes and their application can be found in the proceedings of the Second International Conference on Performance-Based Codes and Fire Safety Design Methods (SFPE, 1998).

### 3.3.3 Goals, performance objective and performance criteria

Performance objectives are requirements which must be met in order to achieve the fire safety goals stated in the code. Examples of principal fire safety requirements in the Swedish legislation concerning buildings (BVF, 1997) are:

- the outbreak of fire is prevented,
- the spread of fire and fire gases in the building is limited,
- persons in the building can escape from the building or be rescued in some other way,
- consideration is given to the safety of the personnel of the rescue service.

These fire safety goals are expressed qualitatively; they define the purpose of the code and express its intent. Performance objectives are more specific and specify conditions which must be met in order to fulfil the objectives. The objectives can be seen as an interpretation of how the goals may be accomplished. Example of objectives are; no structural damage, no life loss to persons not immediately connected with initial fire development, separating occupants from fire effects for a specified length of time, and containing the fire to the room of origin (Watts, 1997).

If a prescriptive design approach is chosen, the approved solutions will fulfil these goals by definition. If the detailed regulations are followed the solution will be considered safe enough.

If an analytical design method is used, the quantitative performance objectives must be interpreted in terms of performance criteria, i.e. design criteria. Examples of performance criteria are; specified level of toxic gases, smoke layer height, radiation, visibility, etc. The criteria are defined depending on which safety goals that are of concern. If a criterion is tenable to humans, the level chosen to be the performance criterion is referred to as the critical level or the hazardous conditions.
The development from prescriptive regulations to performance-based codes puts more pressure on the authorities responsible for the regulations. Well-defined performance objectives and performance criteria are necessary to use an engineering approach. Poorly defined objectives lead to an unnecessary source of uncertainty.

### 3.4 Fire safety design methods

Design methods are necessary to meet the requirements. Performance-based codes do not place any specific demands on the selection of design method. The only requirement is that the design meets the performance objective. Prescriptive codes, on the other hand, are very precise regarding the design method. The method is often part of the code and is presented as a combination of detailed instructions and “deemed-to-satisfy-provisions” or is an approved document which is referred to by the code. The type of code will thus affect the kind of method that can be used in the application.

A common misunderstanding is that when a performance-based code is introduced, the former prescriptive method, i.e. standard method, becomes obsolete and calculations must be used to show that the performance objective is fulfilled. This is completely wrong. When the type of regulation changes, there is normally no change in the level of safety required by society. The old method is thus still applicable and can be used, i.e. a prescriptive method can be used to meet the objectives of a performance-based code. The relation between the type of code and method is presented in Table 1.

<table>
<thead>
<tr>
<th>Type of regulation</th>
<th>Design method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prescriptive code</td>
<td>Standard method</td>
</tr>
<tr>
<td>Performance-based code</td>
<td>Standard method(s)</td>
</tr>
<tr>
<td></td>
<td>Analytical methods</td>
</tr>
<tr>
<td></td>
<td>Testing</td>
</tr>
</tbody>
</table>

In Sections 3.4.1 - 3.4.3 the design methods that can be used in fire safety design and fire risk assessment are presented.

#### 3.4.1 Standard method - approved document

Trial-and-error, experience, “deemed to satisfy provisions”, detailed requirements and common sense have influenced the development of methods used in the traditional approach to dealing with fire safety, e.g. the prescriptive building code. In some countries, e.g. in New Zealand and the UK, the standard method was separated from the prescriptive code and published in approved documents (BIA, 1995; HMSO, 1992). Today, the detailed requirements, with a few exceptions, can still be used as a design method, to comply with the goals of performance-based codes. In the standard method, buildings are divided into classes and the requirements for each class are prescribed in terms of acceptable solutions and specific requirements in the code or in approved documents.
A problem with the standard method is that the detailed demands are not always based on scientific knowledge or real needs for performance, but rather on praxis, rules of thumb or sometimes even guesswork. In areas where there is a lack of knowledge and engineering principles this might be the only way to ensure that the demands of society are fulfilled.

Prescriptive demands for a building can for example, be a minimum exit width, maximum walking distance to escape route, detailed specification of material and surface covering for walls and ceilings, maximum number of occupants in a building, etc.

### 3.4.2 Calculations - analytical approach

An analytical approach to fire safety problems is based on engineering principles and scientific research. In a design situation two somewhat different types of analytical methods can be used.

**Design by design calculations**

The first method is design in a traditional meaning. Part of the system is designed with analytical equations so that the requirements are met. To be able to derive such equations, the uncertainties must be well-known and explicit performance criteria must be given in context to these uncertainties. When these conditions are met, it is possible to create expressions to calculate the required escape width for an assembly hall, using input data, e.g. human characteristics, occupant density, etc. If the design equations are generally expressed the input data must be given as design variables, where account has been taken of the uncertainty in the input in the specific situation. There will be no requirements on a separate uncertainty analysis, since the conditions in Section 2.3.1 are met. These types of design equations are commonly used in structural engineering and have been developed for fire safety design of structural elements (Magnusson and Pettersson, 1980/81).

For applications concerning the early phase of fire development, for example life safety in buildings and detection and suppression of fire, this type of design equation has not yet been derived. A first approach has been presented by Frantzich et al. (1997) but more research must be carried out before this method can be used in practice in fire safety engineering.

**Design by verification calculations**

To ensure compliance with the performance-based code, predictions of the conditions in the design can be made and compared with quantitative performance criteria. The predictions can be made with analytical tools using manual calculations or more complex fire modelling with computers. To assist the engineer to approach fire safety with an analytical method a number of guidelines have been published. Equations and analytical expressions which can be used are presented in a number of engineering guidance documents (BSI, 1997; Buchanan, 1994; FEG 1996). Key questions are “What kind of fire should be fire be analysed?” and “How should the sub-scenario be specified?” A single analysis is far from sufficient and a number of appropriate design scenarios must be evaluated (Olsson, 1999).
Guidance is available on how to specify the design scenarios used in the evaluation of the design in standards such as the ISO documents (ISO/PDTR 13387-2, 1998). Although the standards and guidance documents exist, the engineer subjectively chooses most of the input in a fire scenario. The following definition of Fire Scenario is taken from the Australian Fire Engineering Guidelines (1996).

"For prescribed conditions associated with the ignition, growth, spread, decay and burnout of a fire in a building or a part of building, a fire scenario is defined by specifying the calculated (or otherwise determined) times of occurrence of critical events relevant to each of the sub-systems under investigation".

The sub-system, mentioned in the definition above, is the part of a total system, e.g. a building, where the fire safety can be subject to evaluation. The verification calculations in a design application can be limited to the performance of a single sub-system or a check of whether the performance objective in a code is fulfilled or not, and how the sub-system interacts with the total fire safety in the building.

### 3.4.3 Testing
Full-scale testing is often very expensive and is not often used to verify the performance of a design. The main application of testing in Fire Safety Engineering is to test products according to standard tests, in order to obtain approval for a certain fire resistance rating. In design and risk analysis the characteristics of the products are often expressed by this rating, for example, insulation and integrity for 30 minutes. Further information is seldom required of the specific product in a design application. A difficulty in testing is ensuring that the conditions defining the standard test are used. It is the performance of the product in the real environment that is important and this must be checked in the testing facility. Tests are also performed to verify calculations. One example is the “hot-smoke test” which is used to verify the performance of smoke ventilation systems.

### 3.5 Selection of design method
All the design methods presented in Section 3.4 can be used to fulfil the goals and performance objectives in a performance-based code. An engineering approach is to use experience based on earlier engineering work, and learning-by-doing in combination with calculation is often an efficient way. The balancing of factors such as time, money, selection of method and tools etc., is the real art of professional engineering. Knowing which to use to achieve the most cost-efficient solution or the solution that requires the most or the least engineering effort is a very valuable engineering asset. The knowledge or “feeling” for which solutions can be obtained is also very valuable. Knowing how to master the method chosen and being aware of its limitations can not be considered valuable, but rather a necessity. The different methods demand different levels of competence and equipment. This report will not describe the contents of the methods in great detail. The reader is referred to the engineering guidelines and approved documents referenced to earlier in this chapter for detailed descriptions of the methods.

That which has the greatest influence on the building design is most likely to be the demands of the prospective user and/or owner of the building. This defines the scope of the design situation. It is obvious that if the building can not be used according to the
tenant's needs, the building will not be built. Although fire safety is an important part of the planning of a building, the design process consists of much more than fire safety performance objectives and whether or not the objectives will be met with an analytical approach or not.

The fire safety engineer is only one of many engineers from different disciplines who must co-operate in the design process. All of the engineers have their specific objectives. The final design solution is likely to be a compromise between the professional practitioners. Calculations might show that the fire safety performance objectives can be fulfilled with a thinner wall than prescribed in the standard method, but, on the other hand, higher demands can be required, e.g. from the acoustic engineer.

It is generally recognised that the earlier the fire safety engineer is involved in a design process, the greater the possibility of influencing the design. The cost of changes will increase with time, and fire safety issues often affect the architect’s original plan (Jönsson et al., 1994).

A fire safety engineer normally charges by the hour. The effort of the engineering exercise is clearly associated with a cost, so the engineer does not have unlimited time available for analysis. To justify the use of an analytical approach, which is normally more time consuming than the standard method, there must be obvious benefits or needs. Depending on the situation such benefits can be shown with a cost-benefit analysis (Jönsson and Lundin, 1998), or be required if the building is designed in a flexible way which is not covered by the standard method.

In the early stages of the design process the standard method is often used as reference, to see what amount of and what kind of fire protection is needed. The building layout develops during the design process and thus also the explicit definition of performance criteria that must be met. If the design solution does not comply completely with an existing solution in the approved documents, or if it is recognised that savings can be made through some kind of trade-off, the scope of the engineering analysis can be defined. A trade-off is an alternative solution employed to meet the objective in the code for a specific problem. This may involve the use of a larger and more open building layout in exchange for sprinkler installation or other fire safety systems such as smoke ventilation, etc.

There are obvious reasons why the standard method will never be completely replaced. One reason is that there are many things that do not need to be analysed, since many aspects of fire safety design are well understood (Bukowski, 1997). Another reason is there is a lack of methodology and scientific knowledge with which to derive quantitative design criteria and mathematical models for predicting the effect of certain actions on fire safety. Examples of such factors are presented in the following list:

- influence and performance of the fire department
- quantified safety criteria for fire department personnel in rescue operations
- organisational effects on fire safety
- extinguishers and fire safety equipment effects on fire safety
- human behaviour and decision-making during evacuation
- measurement of performance and damage on environment due to fire
These aspects are difficult to take into account in a quantitative analysis. They must be included in a qualitative analysis or in a standard method. To be able to extend engineering analysis to these areas, additional research is needed. The inability to predict the performance and to transform performance objectives into quantitative performance criteria are limitations of the analytical approach.

Methods must be developed to verify and document new standard methods, so that experience from engineering analysis can be taken into account in an efficient way. On a local level practising professionals use their experience and conclusions drawn from engineering analysis in earlier projects. There is, however, a lack of such efforts on a global level, by professional societies and national organisations.

On the basis of the discussion in this section, it can be concluded that practical design can be regarded as a combination of the methods presented in Section 3.4. Information on the specific situation is necessary to determine to what extent analytical methods are effective in practise. In the next section, the aspects of uncertainty when using analytical methods to fulfil performance objectives are discussed.

### 3.6 Uncertainties in the engineering approach

The types of uncertainty in the standard method and the analytical method differ greatly. One difference arise from the type of uncertainty class from which the uncertainty in the design result originates, see Section 2.1. The major difference in uncertainties arises from how the uncertainties are assessed in the different methods. Since a fire safety engineering approach to a design problem is likely to be a combination of both the standard method and the analytical method, it is important to be aware of their differences and to know how to assess them.

In this section the main sources of uncertainty in the analytical method are discussed and short comments are given on the uncertainty in the standard method. No quantitative comparison of the uncertainties involved in the different methods is made. The quantitative study in this dissertation is limited to the uncertainty in one type of prediction model, i.e. smoke transport models, used in fire safety engineering applications.

#### 3.6.1 Standard method

In the standard method the number of assumptions, simplifications, selection of values, decisions, etc. made by the engineer is very limited. The uncertainty in the result of the standard method can not be controlled by the engineer, but is determined by the design method itself, i.e. the people who developed the method. The dominating type of uncertainty in a design based on the standard method will therefore originate from the class uncertainties in resources, see Section 2.1.1. The uncertainties must be dealt with when the method itself is developed, and can not be assessed when the method is used in an engineering application.

A standard method is normally valid for a whole class of buildings, for example, assembly halls or hospital wards. There, however, can be considerable differences in design and architecture between buildings of the same class. The fire safety level is therefore expected to vary within a building class. To expect identical hospital wards on a national level would be very unrealistic. Thus there will be an uncertainty in the
resulting safety level if the standard method is used (Kristiansson, 1996). Over time, the
method is calibrated so that an acceptable, but implicit, safety level will be achieved.
In the standard method, a major source of error is due to the fact that the method does
not take specific aspects into account. This can not be avoided since buildings are dealt
within a class which must be general for the specific type of building.

3.6.2 Analytical method
Two generally recognised driving forces behind the development of performance-based
regulations are economics and flexibility. Another important driving force is the
possibility of obtaining an explicit uniform and more homogeneous safety level within a
specific type of building.

With the analytical method it is possible to design a building in a more specific way,
and uncertainties due to the use of classification are reduced compared with the standard
method. Although additional uncertainties are introduced through the numerous
decisions made by the engineer and by the uncertainties associated with model
predictions of real world conditions. The uncertainties are a combination from all of the
classes of uncertainty presented in Section 2.1.

A source of uncertainty that contributes to the uncertainty associated with the analytical
method is the interpretation of the performance criteria. The performance criteria in the
performance-based building codes are not always given in explicit quantitative terms
and the performance objectives are of a more qualitative nature. To use analytical tools,
the engineer must interpret the conditions according to his/her ability. The interpretation
is likely to vary among individual engineers, if no well-specified recommendations are
available.

When engineers use different criteria to assess the same problem the design result will
differ and it is difficult to make a comparison. The reason why the criteria are not
specified in the code is that the appropriate method of defining or transforming the
performance objectives into quantitative criteria varies between different situations. A
possible compromise may be recommendations for the common criteria in codes of
practice and engineering guidelines, which can be updated if and when there is a need.
Examples of these recommendations can be found in both building codes (Boverket,
1998) and in engineering guidelines (BSI, 1997; FEG, 1996).

Another aspect is that although a proper variable can be identified, our knowledge of
human response to different levels of toxic substances is uncertain. It is difficult to
measure lethal or hazardous doses of substances since full-scale tests are out of the
question. Values from animal tests are thus normally used. The more the problem is
broken down, the more uncertainties are identified. There is an uncertainty in the dose
response to humans, and other conditions critical to humans must be treated as uncertain
variables, with conservative estimates if no better data are available.

The use of a design scenario is often elementary in the verification of performance
objectives. If the design scenario is well defined, tools are available to predict the
conditions which can be caused by a fire. These predictions are often based on results
from fire models, which are discussed further in Chapter 4. According to Section 2.1 the
uncertainty in the results is related to the class uncertainty in mathematical models and
the uncertainty in the input data. The results are compared with a performance criterion and a decision can then be made as to whether or not the objectives are fulfilled. How the result is presented depends on how the uncertainty is expressed in the codes, and to what extent uncertainty is present. If no uncertainty is present or if the uncertainty is expressed as deterministic conservative values, the model output is deterministic, see Section 2.2.1. If uncertainty in input data and/or uncertainty in prediction models is present, the output is presented as a distribution according to Section 2.2.4. Separation of the different types of uncertainty is not normally necessary, since all forms of uncertainty must taken into account.

Tools that can be used to simplify the assessment of uncertainty and simplify the actions are presented in Section 2.3. An example is to deal with uncertain variables by deriving conservative values. Conservative values give deterministic representation of uncertain variables. A quantile between the 80% and 99% quantile of the distribution is often used if a high value of the variable is considered hazardous. A conservative value is seen as a value on the safe side, even if there is no guarantee that the value of the variable represented can actually be higher. This tool can be used to represent uncertain input data with a deterministic value and it is possible to make conservative adjustments, to model uncertainty. Neither of these tools for dealing with uncertainties is available today. The engineer must perform a separate uncertainty analysis to ensure that the performance objective is fulfilld. In Figure 9 the danger of neglecting uncertainty in model calculations is illustrated for a simple comparison with performance criterion. The safety in a system is the subject of evaluation. In the figure, the performance or exposure is predicted with a deterministic value, without regard to uncertainties. The real exposure is uncertain and is represented by a distribution. A comparison between the deterministic prediction and a performance criterion in terms of an accepted exposure gives the impression that the performance criterion is met, i.e. the critical exposure level is not exceeded. The figure shows that variation in the real exposure can exceed the critical level and thus failure can occur. If a simple deterministic analysis without regard to uncertainty is carried out, the decision-maker will not be aware of this.

![Figure 9. Comparison of performance criteria and model prediction.](image-url)

If the difference between two design solutions is minor and limited to a single sub-scenario, a simple comparison can be made. If one of the alternatives is obtained by using the standard method, the performance in the sub-scenario relevant for comparison can be quantified with engineering analysis and argued to represent acceptable safety.
This acceptable level can then be used to compare or verify that the fire risk is not higher in the alternative design solution. By using a relative comparison, uncertainties in defining performance criterion from performance objectives or qualitatively acceptable safety objectives are reduced. On the other hand, it is assumed in this approach that the design of the reference solution was performed correctly and that a design according to the standard method in this specific situation will lead to an acceptable solution. The safety level in prescriptive designs is known to vary greatly (Kristiansson, 1996). This type of evaluation is a relative comparison of the effect of the trade-off rather than a verification of a specified safety or performance level. An example of a situation when this is used is when it is necessary to use a longer distance to exit that prescribed.

The previous comparison example can be interpreted as a verification of consequences on a sub-scenario level. For a wide range of trade-offs this is sufficient, as long as uncertainties outside the single sub-scenario are affected by the trade-off. A problem occurs when a trade-off is made which requires a more extensive uncertainty analysis to ensure compliance with the code. There is little or no guidance regarding this in the code itself. The performance criteria are limited to single sub-scenarios and a broader analysis is therefore difficult.

The analytical method that is practically applicable is the type referred to as design based on verification calculations. The design scenario should represent the sub-scenario where it is appropriate to measure the performance, to be able to judge whether the fire risk exceeds the goals in the building code. Verification of performance in the design scenario should be sufficient to provide a measure of the safety level in the building. This is impossible. The connection between design scenario and risk is not that strong. Risk is a measure that takes into account consequences and the likelihood of all the sub-scenarios, while the design scenario only gives information on the consequences of one sub-scenario. A single sub-scenario analysis can only provide an indication of the risk in the whole system under very well-specified conditions.

The selection of design scenario is crucial for the outcome of a verification exercise, since the consequences of different sub-scenarios are likely to vary, see Figure 10. Choosing the appropriate design scenario is not commented on in the codes and is only discussed in loose qualitative terms in engineering guidance documents (ISO/PDTR 13387-2, 1998). It is left almost completely to the engineer to decide the design scenario and the effect of this is an extremely high uncertainty in the design process. In Figure 10 a fire risk in a building is modelled with the event tree technique. The consequences in the different sub-scenarios represent number of persons that do not have sufficient time to leave the building before hazardous conditions occur. Only one single consequence has been taken into consideration for each sub-scenario.
A common error in engineering analysis is to neglect the probability of failure for fire safety systems and use the sub-scenario in which every safety system is operating as design scenario. It is then impossible to make a relevant comparison of how the risk is affected by different types of fire safety systems. An example is when sprinkler systems are installed. It is obvious that a design with a sprinkler system will be superior to any other solution, if the only scenario studied is that when the sprinkler extinguishes the fire. On the other hand, if a sprinkler system can be installed to gain approval to increase the number of occupants in a fire compartment, the consequences if the system should fail to operate are likely to be much worse than in a solution without sprinkler.

When the effect of a trade-off is evaluated is often not sufficient to only study one single sub-scenario. The effect of the fire safety system installed to compensate for the trade-off from the approved solution can not only be compared between the two design alternatives when the system works. The over-all objectives of the building code are clearly that the trade-off is acceptable only if the fire risk is not increased. The contribution to the total fire risk in the system is of course likely to be very small from the sub-scenario when the safety system works. Comparison for a trade-off must be made in the scenarios contributing most to the risk. Verification or comparison on a sub-scenario level will not do this.

If the design scenario is defined as the worst case the sprinkler system will not give any benefit at all, which is obviously not a fair approach either. In numerous references it is pointed out that a multi-scenario analysis is required to be able to measure differences in performance between different solutions. Only qualitative vague guidance is given on how to choose the sub-scenario and if no quantitative guidance is given, the verification can hardly be improved. This is the same as using a consequence analysis of all the sub-scenarios instead of a single sub-scenario analysis. There is still too insufficient information for the decision-maker. Consideration must be taken of the likelihood and consequences of each sub-scenario in this type of verification, by using a risk measure as a performance criterion and risk analysis as a design tool.
An alternative approach is to design for an acceptable risk, which is verified with a quantitative risk analysis. The risk can be expressed in different ways (CPQRA, 1989) but is normally an overall measure of the sub-scenarios with unwanted consequences.

To use this approach the design criteria must be expressed as a quantified acceptable risk and not as a performance criterion on the sub-scenario level, as today. Since no quantified level of acceptable risk is given, one method of approach is to use comparative measures. If the risk in the approved document solutions is quantified, this can be used as an acceptance criterion. A major disadvantage is that the risk level in these solutions has been shown to vary considerably (Kristiansson, 1996). There are also arguments that the risk in designs made according to standard methods are not acceptable by the society (Brannigan, 1995), e.g. the public may believe that the risk of disastrous fires is zero since they are rare events.

One way to circumvent this problem is to specify the design criterion as an acceptable level of risk explicitly in the building code. There will however be problems in getting authorities and people without technical education to accept this (Bukowski, 1997).

One way forward is to look at the solution instead of the problem. The problem is a risk communication problem, which is not impossible to solve. In risk analysis, acceptable levels of risk must be specified to be able to evaluate the performance. There is no such thing as zero risk or absolutely safe buildings. A zero accident level of safety, i.e. a vision of zero casualties from fire accidents, is unrealistic from an engineering point of view. Many unfortunate events can cause fatalities, but it is unrealistic to look at them as unpredictable accidents. It is impossible to use the zero vision to derive design criteria or acceptable risk limits, which is a necessity in practical engineering work in modern society. The criteria must be derived realistically. One way to exemplify an acceptable risk limit is to perform risk analysis on buildings that are built according to the regulations, but there are disadvantages with this approach also, see discussion earlier in this section.

If the acceptable risk is not given tools required to deal with uncertainty in the design problem can not be developed. The only alternative to simplify the assessment and analysis of uncertainty in the analytical methods is by regulatory specification of how to deal with the uncertainties. On what basis can such a specification be made?

A number of approaches for risk and uncertainty analysis for the design problem have been outlined by Magnusson et al. (1997).

1. Analyse a single sub-scenario with a single limit state described by an analytical expression, derived using a suitable method, including an uncertainty analysis included.

2. Analyse a single sub-scenario with a computer program including an uncertainty analysis.

3. Analyse a whole event tree with each sub-scenario described by an analytical expression and without explicit treatment of uncertainties (possibly including a sensitivity analysis of branch probabilities).
4. Use the same analytical expression as in (3) but include an uncertainty analysis. The main categories of uncertainty are branch probability uncertainties, parameter and model uncertainties.

5. Use integrated computer programs to analyse a whole event-tree.

By applying these approaches according to the situation, uncertainty can be dealt with appropriately. These steps are necessary to develop simplistic tools and conservative design variables for a defined problem area, e.g. design equations for a class of buildings (Frantzich et al., 1997).

It is very important that the engineer can recognise whether the uncertainty should be analysed separately, or if it is covered in the method used. The engineer must identify the uncertainty and deal with it in an appropriate way, depending on the situation and the requirements stated in Section 2.3. Simplified methods to deal with the uncertainties, e.g. design equations and conservative values, are not yet available. There is also lack of knowledge about the uncertainties in the models used to predict fire and smoke spread. Until now, no generally agreed protocol for the quantification of the predictive capability of these is available. The information on accuracy and uncertainty is useful when selection of model is done and can also be used to adjust the model predictions, if the model error is known and quantified. Explicit knowledge of the model error is necessary to ensure high quality and provide confidence in the engineering solutions. This dissertation presents a method of deriving this information based on a comparison between model predictions and experimental measurements.
4 Modelling

Modelling of real world conditions is an important tool in most engineering disciplines. Model predictions of real world conditions are never exact, since models are based on simplifications, approximations and assumptions. With knowledge of the range of applicability and the predictive capability within that range, it is possible to assess real world problems with an analytical approach based on engineering principles. In the previous chapter assessment tools with varying degrees of complexity were discussed. It was recognised that model predictions of the consequences from a fire are commonly used in fire safety engineering applications. It was also recognised that the uncertainty in the model output must be dealt with, according to the specific requirements related to the type of assessment.

Today, most calculations are performed on computers and therefore modelling is often synonymous with computer modelling. In this chapter the way in which calculations are performed within the model is not of interest. No distinction is made between simple models based on a single equation and a very complex computer model. Instead, the object of interest is how the model output is presented when the output can be adjusted based on knowledge of the accuracy and variability in the predictive capability of the model. Finally, methods taking the remaining uncertainty in the model output into account in an uncertainty analysis for a design problem are presented. In this dissertation smoke transport models will be used to exemplify the methodology, mainly CFAST 2.0 (Peacock et al., 1993a) but a number of other models will also be compared.

The ability to address the uncertainty in the model output varies depending on the model type and how the output is to be presented. There may also be different requirements on how the uncertainty in the model output should be presented, depending on how the information is going to be used. Model output is often used as input to other models in engineering applications, and therefore it is important that information on the uncertainty in the model output is considered.

4.1 Model type and model output

There are several different ways to classify models. Two commonly used ways are to look at the model type and to study the model output. Two types of models are termed physical models and mathematical models. Physical models are based on laboratory-scale simulations of a specific situation. This type of model is not considered in this study, since it is not common in fire safety engineering. The model type most commonly used in fire safety engineering is the mathematical type. Mathematical models can be empirical, i.e. based on presentation and extrapolation of experimental data, or they can be based on underlying analytical or numerical equations. These can also be combined into semi-empirical models.

The output can be given as a single value or, if the model output is time-dependent, the result is often plotted as a function of time in a diagram. If the model is probabilistic, information on uncertainty can be presented. In this section, a brief description of different types of models and their output is given.
4.1.1 Deterministic models

The basic type of mathematical model is the deterministic model. The result is presented in the same way as described in Section 2.2.1, where no account is taken of uncertainty. The model calculates a single value, i.e. a point-type estimate, and consists of one or more equations that describe the model output as a function of several deterministic input variables. The schematic relation between input and output for a deterministic model is shown in Figure 11.

![Figure 11](image1.png)

This type of model is used to describe a physical or chemical phenomenon or processes without taking uncertainties into account explicitly. If deterministic models are used in engineering applications it is necessary to assess the uncertainties involved separately. Whether the model prediction is conservative or not, depends on how the engineer deals with the uncertainties separately.

4.1.2 Deterministic models with multi-variable output

Deterministic models can be a combination of several different models that use the same input data. In, such a model is illustrated in Figure 12.

![Figure 12](image2.png)
These kinds of models are often very useful in engineering applications, where several variables have to be calculated. Time is often an important input variable. If only a single time point is of interest, time is normally a single input variable.

### 4.1.3 Time-dependent deterministic models

In many situations, it is of interest to study the model output as a function of time. The two deterministic types of models mentioned above can be time dependent.

![Figure 13. Illustration of a time-dependent deterministic model.](image)

Figure 13 shows an example of a simple time-dependent deterministic model with one output variable. The input variables, \( x \), can often change with time and are expressed as \( x(t) \).

### 4.1.4 Probabilistic models

In many engineering applications, no single value can represent the real world. When uncertainties are involved the result is often expressed as a range of possible solutions. The model input can have both type A and type B uncertainties and this will effect the kind of uncertainty in the model results. The engineer must be aware of how the uncertainty is dealt within the model in order to be able to understand the results. It is possible to obtain deterministic results from a probabilistic model, for example, conservative values or confidence limits.

![Figure 14. Illustration of a probabilistic model.](image)
Figure 14 illustrates a probabilistic model, with stochastic or deterministic input and stochastic output variables. No time-dependency is indicated which is adequate when a specific time point is used or if the model output is independent of time. Probabilistic models, as well as deterministic models, can be of multi-variable type, but these are not discussed here. If the mathematical model is used as a sub-model, it is possible to propagate the uncertainty through the engineering calculations to the effect on the final results. To do so, it is necessary for the rest of the engineering calculations to be probabilistic or for a separate uncertainty analysis to be carried out. If the probabilistic model output is to be used in a deterministic calculation, information on the uncertainty and its effect on the result will easily be lost. Difficulties encountered in representing uncertainty in deterministic calculations are discussed further in Section 4.5.

Even if the model output from a sub-model is probabilistic, the effect of the uncertainty on the final results can be lost if the other models used in the engineering applications are deterministic.

**4.1.5 Time-dependent probabilistic models**

A time-dependent stochastic function presents an uncertain value of the predicted variable as a function of time. An example is presented in Figure 15, where temperature (T) is calculated as a function of time (t). The conditions will vary with time and the development is visualised with a time-temperature graph, similar to the one presented in Figure 13.

![Figure 15. Illustration of a time-dependent probabilistic model](image-url)
The difference is that in this type of model, uncertainties can be included in the predictions. Every prediction, as a function of time, contains information on the uncertainty due to the uncertainty in the input variables. Even if the time of interest is determined exactly, the predicted temperature is given as a range of values. \( T_{0.95}(t) \) is the 95% confidence limit of the temperature as a function of time and \( T_{0.05}(t) \) the corresponding 5% limit. If a conservative estimate of the time-temperature relation is required in a design situation, where the temperature is considered hazardous, the function \( T_{0.95}(t) \) can be used. This type of model output contains much information and can be difficult to interpret, but in situations where uncertainty is present this type of model is convenient.

### 4.2 Fire models

The objective of fire modelling is usually to predict the real conditions in a fire enclosure, as accurately as possible. The conditions which are of interest depend greatly on the situation. Visibility, concentration of toxic species and thermal exposure are important conditions related to life safety. Many kinds of model output can be use to characterise these conditions.

The results from the computer models are used as input data in various fire safety engineering calculations. Examples of interesting data are temperature in the smoke layer, smoke movement, smoke layer height, radiation, activation time for sprinkler etc.

Extensive overviews of the fire and smoke transport models available have been presented in a number of papers and reports (Walton, 1995; Beard, 1997; Friedman, 1992). A general conclusion is that most of the models are deterministic in nature, and many of them are of multi-variable type. The lack of probabilistic models makes it difficult to include uncertainty in the calculations. This must be performed separately from the modelling, for example, in an uncertainty analysis. Within the wide range of models available for different specific applications, three different sub-groups can be recognised.

#### 4.2.1 One-zone models

So-called one-zone models have been used to predict post-flashover conditions in a fire room. The models have mainly been employed in structural engineering. The one-zone model assumes that the room is a homogeneous zone in which the temperature and the other conditions are identical everywhere within the zone at every point in time. This is not a good approximation of the early stages of fire development.

![Figure 16. Illustration of a one-zone model.](image)
4.2.2 Two-zone models
A more suitable assumption for pre-flashover modelling is the two-zone model. This type of model is commonly used in fire safety engineering applications and is the type of model which is considered in this dissertation.

In two-zone models the fire room is divided into two control-volumes. These are the upper smoke layer and the lower layer of “ambient” air. The zones are connected via a plume and openings to connecting rooms or the outside. The zones in a room are separated by an interface and it is assumed that there is no exchange of mass across the interface, except in the plume. The temperature within a zone is constant, so there is no temperature profile within a zone. The fire is modelled as a mass- and energy-producing source. Energy is released from the fire as radiation and hot smoke. The transport of mass and heat to the upper layer is carried out in the plume. The plume entrains ambient air due to buoyancy, which will increase the mass flow and lower the temperature as the mass in the plume rises. The smoke is assumed to fill the room from the top and the interface will drop until steady-state conditions occur or the room is totally smoke filled. Such a fire normally develops with time, and the equations for conservation of mass, energy and momentum are solved for each time step.

The output from a two-zone model of a fire room is multi-variable and is often time dependent. In this study the uncertainty in model output from the two-zone model CFAST 2.0 (Peacock et al., 1993a) has been analysed. The output used in the analysis was smoke layer temperature and smoke-free height, also called smoke layer height or interface height. Examples of these variables are shown in Figure 18 and Figure 19.
4.2.3 Computational fluid dynamics models
A much more complex model type uses a technique called Computational Fluid Dynamics (CFD), to simulate fires in compartments. This type of model is also often called a field model. In such models, the fire compartment is divided into many very small cells, as shown schematically in Figure 20. The equations governing mass, energy and momentum are solved in each cell for each time step. The calculations are very time consuming and this technique is still not used very much in practical fire safety engineering. However, several computer codes are commercially available (Beard, 1997).

Hanna (1991) suggests that there is an optimum degree of model complexity which minimises the error in the prediction. This indicates that it is not certain that a better prediction will be obtained simply by using a CFD model. The accuracy depends on whether the engineer is able to cope with a more complex model. The use of this type of model will certainly increase when the user interface is improved and the computational time reduced.

4.3 Evaluation, verification or validation of models?
According to an international survey (Friedman, 1992) there were over 72 different fire and smoke transport models available some years ago, and the number has certainly
increased during recent years. Several of the models in the survey were two-zone models, which are recognised as being commonly used tools in fire safety engineering applications. It can be difficult to choose between different types of models and between the models within a certain type. Questions such as; “Which model is most suitable for the problem?” and “Is the quality of the model prediction satisfactory?” are likely to be asked when choosing a model. This problem is not specific for fire models. Some kind of evaluation procedure, quality assessment or guidelines on how to choose the right model are necessary.

It has been difficult in many engineering disciplines to develop prediction tools with good predictive capability, e.g. prediction of the dispersion of dense gases or the dispersion of radioactive substances. The quality of a model prediction is related to the model error, i.e. the accuracy of the model, and the variation in the model error when the model is applied to a class of problems, i.e. the model uncertainty. Even if a model has a substantial model error it can be a very useful tool, as long as decision-makers are aware of the uncertainties.

Britter (1993) gives several qualitative recommendations on how to evaluate models. Work carried out by the ISO Sub-Committee 4 has resulted in evaluation guidelines (ISO/PDTR 13387-3, 1998). Similar guidelines have also been published by other organisations (ASTM, 1996). In both of these guidelines it is clear that it is not possible to evaluate the use and limitations of fire models per se. Instead, the guides provide a methodology for the evaluation of fire models for a specific application (ASTM, 1996).

It can be seen from the literature that the nomenclature in this area is vague and confused. The same scope and objectives are described with different definitions. The terms used to describe the action, process or assessment of the model accuracy or quality are referred to as model evaluation, model verification and model validation. There are no clear consistent definitions in the literature. Beard (1997) suggests that the term “validated” implies that the model has been “proven correct” and should not be used in the context of model evaluation. A “correct” model is impossible, according to the definition of modelling, since simplifications and approximations are natural part of a model. Evaluation and verification seem to be used synonymously for assessing the quality of, or for determining the applicability of, a model, while validation is seen as one of the parts in the process where the model error can be of interest. In most contexts validation means comparison with experimental data, which might be more appropriate.

In “The evaluation of technical models used for major-accident hazard installations” Britter (1993) suggests that the following questions must be answered to evaluate a model:

- Does the model assess the right problem and produce the right answers?
- Are the model inputs easily obtained?
- Is it clear what the model's limitations are?
- Is it clear what accuracy might be expected in the model results?
- What level of extrapolation is implicit in the model apart from data used for model development, and what are the likely effects?
- Is it financially expensive?
- Is it computationally expensive?
Answering the questions is one way to ensure that the correct tool is used and that the quality of the model prediction is acceptable. The questions are normally part of the optimisation of the available resources carried out by in the planning stage.

Gass and Joel (1981) discussed predictive capability in terms of model confidence. Model confidence is not used as an attribute of the model, but rather of the model user. Model confidence is loosely defined as the interaction between the model predictions and the decisions made by the model user and also how the attitude towards the model effects these decisions. The user’s confidence in a model is a dominating factor effecting the choice of model for an application. Gass and Joel define criteria for model confidence and also suggest how to qualitatively evaluate model confidence. One important aspect is whether the real-world approximation of the model is suitable for the problem area in question, i.e. a type of model evaluation. Gass and Joel conclude that:

- Long-term use of model should not be the only evidence for a high degree of confidence.
- It is necessary to have model developers, sponsors and users to recognise the need to support proper professional practices that will improve the utility of modelling capability.
- We can accept decisions based on model output on faith alone.

Britter (1993) suggested that a validation procedure or a protocol should be designed to determine how well a model predicts certain scenarios and to allow communication of this information to the user. He also recognised that is not always possible to find experimental results suitable for validation and suggested three similar techniques to be used in model evaluation.

- Comparison between prediction and experiment
- A code comparison exercise
- A benchmark exercise

The choice of evaluation technique is both a question of cost and available data. Sometimes, models can be compared to other models instead (Cole and Wicks, 1995; ASTM, 1996). Efforts are being made to introduce a Fire Data Management System (Portier et al. 1997) to organise and present data obtained from tests to simplify model verification, but no results possible to use in practical application have been presented.

Several qualitative studies have shown considerable difference in model predictions when different fire models have been used to assess the same problem (Luo et al., 1997; Hostikka et al., 1998), but there is lack of quantitative analysis. Methods to quantify the uncertainty in models are necessary to be able to address the uncertainty in a proper way in fire safety engineering.

Code comparison is no absolute assessment of the model. It provides a measure of the lack of consensus between model developers. The exercise can be used to improve the understanding of the relevant physical processes dominating the scenario analysed. The
method can also be used to evaluate the predictive capability of the model outside the range of available experimental data (Britter, 1993).

Benchmark exercises can be designed in many different ways. Normally, different users and different models are used to assess the same problem and the results compared. A number of such exercises have been carried out in many areas, e.g. dispersion modelling (Amendola et al., 1992; Cooke et al., 1997). The variation in the results is often substantial, but it is recognised that the variation is strongly linked to the uncertainty associated with the decisions made by the modeller. Similar exercises have been carried out in the field of fire and smoke transport modelling (Hostikka et al., 1998) and the results showed a substantial variation in the result also here.

In this dissertation a focus is placed on the technique based on comparison between predictions and measurements. If sufficient data is available a comparison can be made if the following restrictions are adhered to:

- an appropriate model is selected for comparison with observed results
- the model is used correctly and the output is correctly interpreted
- the model output data are consistent with the form of the observed data, e.g. the data are based on the same averaging time
- observations are representative of the scenario we wish to model
- observations have not been used in the development of the model
- the spatial locations of the observations and prediction are coincident

The purpose of this procedure is to be able to determine, reduce and minimise the model error in model predictions.

The main objective of this work was to find a procedure which could be used to quantify the model error on the basis of comparison with experiments, and to take the error into account in calculations, once it has been quantified. No effort was made to improve the prediction model itself. In all the validation and verification guidelines referred to, the comparisons with experiments are qualitative. A number of papers have presented validations of the smoke transport model CFAST 2.0. The results are reported as “overpredicted the temperature in the upper layer” and “gives conservative estimates of fire environment” by Luo et al. (1997), favourable, satisfactory, well predicted, successful or reasonable, systematic deviation exists or show obvious similarities by Peacock et al. (1998b). Some quantitative attempts have also been made (Peacock et al., 1998b), with quantitative specification of the interval for the variation of the error during the fire scenario. These results can hardly be used in quantitative assessment of uncertainties in design or analysis applications.

Model accuracy in quantitative terms is not considered nor the uncertainty in the model accuracy. If the model were always to make the same relative or absolute misprediction, the error would be very easy to compensate for. Lundin (1997) has shown that the model error in output from smoke transport models is more complex and requires statistical methods for quantification. In the following sections, a method of dealing with model error in smoke transport models is presented.
4.4 Model error and model uncertainty

The assessment of the model error in a deterministic model was carried out by comparing the quantitative difference between model predictions and experimental measurements. The general definition of model error in evaluation procedures is expressed as in Eq. [4.1].

\[
\varepsilon = x_{\text{predicted}} - x_{\text{measured}} \quad [4.1]
\]

Figure 21 shows an interpretation of Eq. [4.1] in a simple fire safety engineering calculation.

The model error in the predictions of a fire scenario will not be the same at each point in time in a scenario. Neither will the error be the same for two different scenarios at the same time. A constant model error would be idealistic from an error-correction point of view, but does not seem to be an adequate assumption (Lundin, 1997). Model uncertainty is therefore defined as the variation in model error. The term model uncertainty is similar to the term risk in the sense that there is no generally agreed definition (Mosleh et al., 1995). In this study the definition presented above will be used exclusively.

The commonly used types of prediction model in fire safety engineering applications produce model output in terms of a variable as a function of time, see Section 4.2.2. The time-dependent variable is normally compared with performance criteria, representing critical exposure criteria. Since the conditions are transient, the point in time at which the criteria are exceeded, i.e. time to critical condition, is of interest. When this point in time occurs during the fire development varies from situation to situation and can not be generally determined. Fires develop rapidly and realistic performance objectives do normally not require that fire development be totally prevented. The performance criteria are rather defined as how long a time the critical conditions may not be exceeded. An example is the life safety objectives in the Swedish building code (Boverket, 1998): “Occupants in a building must be able to evacuate before critical conditions occur”.

A relevant example of a model prediction in fire safety engineering applications, is the point in time when the smoke temperature or the smoke layer height in a design fire scenario exceeds a critical level. The error in the prediction of the temperature will then propagate to become an error in the predicted time, see Figure 22.
Figure 22. The model error in prediction of the design criterion will generate model error in the prediction of time to reach critical conditions.

4.4.1 What causes the model error?
There are many sources of model error. Both the model prediction and the measurements are sources of error. Assumptions and simplifications of the real world introduce errors as do external factors that have not been taken into account and may not be possible to measure.

The experimental setup must be similar to the situation modelled, otherwise, important factors which implicitly affect the error may change. Jones et al. (1996) compared predictions from CFAST 2.0 with full-scale fire tests and gave the impression that the model showed exceptionally good agreement with real conditions. It is important to realise that the agreement depends on the testing conditions, which in this case were ideal for this particular model. The rooms were very small and the external implicit factors could be reduced. In practical applications, the applicability of the model is normally not good. It is important to consider this aspect when performing model evaluations.

4.4.2 Sources of error in predictions
According to Beard (1997) the model error in a computer model consists of the following parts:

- Theoretical and numerical assumptions
- Numerical solution technique
- Software error
- Hardware fault
- Application error

Theoretical and numerical assumptions cause errors due to simplification and approximation of the real world. These assumptions are built into the model itself. An example of this kind of simplification is the two-zone model itself. The two-zone model assumes that the smoke will form a homogeneous upper layer, which is separated from...
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the ambient air by a horizontal interface. These are ideal conditions, which are a simplification of the real condition. If experimental results are compared with model predictions from a zone model a discrepancy will be observed, but how good are the predictions? This depends on the validity of the model for the situation. The assumptions have to be reasonable and the situation within the range of applicability of the model.

Other recognised assumptions are simplifications in the sub-model for plume entrainment, combustion, heat transfer in walls, species concentration etc. (Peacock et al., 1993b; Babrauskas, 1996; Jones et al., 1996; Mowrer and Stroup, 1998). It is obvious that the suitability of the model will depend greatly on the specific situation. This is where the competence of the engineer is important. The effect of the simplifications and assumptions depend on the engineer's knowledge of the limitations of the model. The ability to pick an appropriate model for the problem is crucial.

Another uncertainty often overlooked is the problem of the reproduction of full-scale experiments. Even when two identical experiments are performed, there are always parameters that can not be controlled and it may not even be possible to take them into account in the model. An example of this is the differences between three large-scale bedroom fire tests which were performed by Factory Mutual in USA, between 1973 and 1975. One of the objectives of the tests was to investigate the reproducibility of realistic full-scale room fire tests under well-controlled ambient conditions and furnishing (Croce and Emmons, 1974; Croce, 1975; Alpert et al., 1975). The results were surprising, since the time to flashover in the first and third tests differed significantly from the second, although the experimental setups were identical. The measurements showed that the developing phases of the fires were almost identical once the fire spread started, but the pre-burn time was much longer in test two. The difference was around ten minutes, which is a long time in the early stage of fire development. If the time to flashover had been measured from the time when the fire growth had reached a stable level, for example after 50kW, instead of the time to ignition, the differences would probably have been much smaller, see Figure 23. The major uncertainty in the measurement of time to flashover is likely to originate in the smouldering, i.e., pre-flaming, period, where some of the conditions could not be controlled. This shows that even very well-controlled experiments can be difficult to reproduce, but by paying attention to the measurement of the variable of interest the uncertainty can be reduced.
The uncertainty quantified by comparison between measurements and predictions will therefore not only consist of the model uncertainty. The variability between model predictions and experimental measurements is partly caused by uncertainties in the measurements and the reproducibility of experiments. The major approximation in the statistical method developed to quantify the model uncertainty is that the measurements represent real conditions, i.e. that the uncertainty in the measurements can be neglected compared with the model uncertainty.

The quality of the numerical solution technique and the software error are related to the quality of the computer model and the abilities of the software designer. The software related error differs between computer models and could be minimised by choosing a good model.

Errors due to hardware faults are very rare, but can be caused by errors in the design of the microprocessor or a fault in the manufacturing of the microprocessor (Beard, 1997). A few years ago, an error of this type was observed when Intel released a series of malfunctioning Pentium processors.

Application error is related to the mistakes that can be made by the user. Beard (1997) divides this source of uncertainty into three sub-categories:

- Misunderstanding of the model or its numerical solution procedure
- Misunderstanding of the software design
- A straightforward mistake in inserting the input or reading the output.

Both the skill of the engineer and the user interface of the program will affect this error.

All these sources contribute to form the final model error which is, by definition, the difference between real conditions and the model conditions. In this work no separate analysis of the different sub-categories was performed. The combination of these categories is referred to as the model error.
The uncertainty in prediction models certain to decrease when more knowledge of the different processes is available. Peacock et al. (1998b) suggest that the following issues should be looked into in detail, to improve the smoke transport model CFAST 2.0:

- Numerical solvers
- Entrainment
- User-specified fires
- Leakage area
- Statistical treatment of data
- Experimental measurements

For the engineer it is important that the tools be continuously developed and that research is carried out, but until improvements are made, evaluation of the model error is a necessity.

4.4.3 Sources of error in measurements

The model error is quantified on the basis of Eq. [4.1]. When model predictions are compared with experimental measurements, part of the error is due to measurement error. The measurements are normally approximated to represent real conditions, provided that the measurement errors are negligible compared with the errors in the prediction. This approximation introduces an additional error source, which is discussed by Beard (1992).

- Lack of controlled conditions (e.g. ambient humidity may vary)
- Experimental design (e.g. thermocouple position)
- Direct error in measurement (e.g. error associated with a thermocouple reading)
- Raw data processing algorithm (e.g. assumptions associated with finding an average temperature from a set of raw data).

Beard also recognised the following sources of error in experiments.

- Sensitivity and reproducibility
- Inappropriate interpretation of results from a model

The measurements are approximations of the real world conditions. There are measurement errors present but these errors are assumed to be small compared with the model errors and are therefore overlooked.

4.4.4 Uncertainty in the model error

In fire safety engineering the specified fire scenario is almost never the same. The geometry, fire characteristics and the ventilation conditions change from building to building and from room to room. If the model error and model uncertainty are unique in all the specified scenarios, it is necessary to perform full-scale experiments of all possible types of scenarios to be able to determine the model error and make the adjustment to the predicted temperature. This is, of course, completely unrealistic. The approach taken is therefore to investigate how the model error behaves and to use an approach that makes it possible to apply the results to “real world” situations.
Hanna (1991) has studied uncertainties in dense gas models. He used a simple method to quantify the uncertainties in deterministic models. The objective was to quantify the uncertainties due to model physics error, but it was impossible to separate this source of uncertainty from the uncertainty due to measurement, uncertainty due to data input and output error and also the stochastic variability in the parameter. The situation is similar in smoke transport models and the same general approach can be used to characterise different types of errors introduced in fire modelling.

The total model error can be quantified by comparing experimental measurements with model predictions in a statistical analysis. It is important to be aware of the other components of the quantified uncertainty. When evaluating models and comparing them, the discussion is often focused on the physical part of the model (Babrauskas, 1996). In the statistical analysis performed here it was not possible to separate the different components from each other. Although the discussion often is focused on the physical error, that which is interesting and which represents the total error when the model is applied, is the total model error. One problem is that source of the total model error is correlated to the modeller himself and the complexity of the model. If an analysis of the uncertainty is to be performed and applied in design situations, it is not very practical if the error analysis has to be made for each engineer. At this moment, there is little or no structured information available to quantify and specify the role of data errors in the total uncertainty (Magnusson, 1996). No attempt was made in this work to separate data errors from the other components of the total model uncertainty.

4.5 Include model uncertainty in model output

Depending on the assessment and the level of complexity in the uncertainty analysis required, the uncertainty in the variables can be represented in different ways.

4.5.1 Taking uncertainty in deterministic model into account

The most commonly used fire and smoke transport models in fire safety engineering applications produce deterministic model output (Beard, 1997). In predictions of very well-specified laboratory-scale experiments the effect of uncertainties due to natural variation of input parameters or knowledge uncertainty are not substantial. Researchers develop the most complex models, and many of were originally created to predict the results of small-scale laboratory experiments, where the conditions can easily be controlled. The objectives of these models were often to describe fire phenomena and not necessarily the factors affecting the fire phenomena in practical applications. Therefore, uncertainties are often not dealt within mathematical models.

In practical engineering applications the need to address uncertainty is totally different, from the laboratory-controlled situation. The conditions used as input to the models and the design conditions can not be isolated or kept constant. The uncertainty must be taken into account when decisions are made. If mathematical models are used in applications it is necessary to recognise the additional uncertainties that are introduced when models are used.

In fire safety engineering, the quantification of model uncertainty has not been an issue of great concern. Several scientific papers have been published, but there is no general agreement on how to address the issue in practical design applications or quantitative
Modelling

In a recent investigation by The Swedish Board of Building, Housing and Planning (Boverket, 1997) it was concluded that fire and smoke transport models are commonly used in analytical design methods with no thought of the predictive capability of the model.

In some engineering disciplines models are verified by independent third party organisations (Cole and Wicks, 1995) which provides some kind of quality assurance, approval or certification of the model. Guidelines are also developed to assist the engineer in deriving a conservative model output. In this situation, the uncertainty is dealt with implicitly by the practitioner, but an explicit analysis of uncertainties is the basis for the simplified recommendations. The uncertainty is unknown but if the guidelines are followed the model result will be conservative. There is a lack of guidelines and certification of this type for fire and smoke transport models, and quantitative tools are non-existent. It is therefore very difficult for the practitioner to deal with uncertainties. Most validation of fire models is qualitative and no protocol for quantification has been agreed upon.

To be able to carry out a quantitative uncertainty analysis of the calculations, a quantitative approach to model uncertainty is required. One way of quantifying model uncertainty, even when data are unavailable, is by using expert judgement. Although expert judgement is very subjective, there are well-established methods of using expert judgement as part of the design process (Magnusson and Rantatalo, 1998).

There are techniques to assess lack of knowledge on parameters and relations necessary to model physical or chemical behaviour. These techniques are often referred to as expert judgement techniques and protocols (Cole and Wicks, 1995), and can be used to derive parameter values for use in quantitative risk analysis. Expert judgement often plays a major role in practical applications, but the process is often informal. Attempts to formalise the process of using expert knowledge in fire safety design applications are presented by Magnusson and Rantatalo (1998).

Another way of quantifying model uncertainty is to use statistical methods. An approach to including model uncertainty explicitly in fire safety engineering calculations has been presented by Magnusson et al. (1997). The model uncertainty in the deterministic model output was expressed as a stochastic uncertainty factor, \( U \). By multiplying the uncertainty factor by the model output, a variable \( T_{\text{adj}} \) can be calculated, see Eq. [4.2]. \( T_{\text{adj}} \) is thus the model output with the model uncertainty included.

\[
T_{\text{adj}} = U \cdot T_{\text{predicted}}
\] [4.2]

The uncertainty factor, \( U \), was quantified in an analysis of the model error. A quantitative relationship between model output and model error can be expressed as a functional relation in terms of a constant error, a random error, a proportional factor or a complex function of the model output. If the error can be quantified, it is possible to take the model uncertainty into account and include this uncertainty in the calculations. In Chapter 5 a detailed analysis of the model error in the smoke transport model CFAST 2.0 is presented and a method of quantifying the model uncertainty is described.
Since the model output from most smoke transport models is deterministic (Beard, 1997), the uncertainty must be dealt with explicitly, e.g. as in Eq. [4.2]. A general methodology is illustrated in Figure 24.

The model uncertainty, i.e. the relation between model output, \( f_m() \), and model error, for a deterministic model is known and is expressed as \( y_\sigma \). The model output is adjusted to take the model uncertainty into account. The adjusted value (\( y_{adj} \)) will therefore be probabilistic.

\[
\begin{align*}
\text{Input data} & \quad \begin{cases} 
x_1 \\
x_2 \\
\vdots \\
x_n 
\end{cases} \\
\text{Model} & \Rightarrow f_m(x_1, x_2, \ldots, x_n) \\
\text{Model output} & \Rightarrow y_m \\
\text{Adjustment for model uncertainty} & \Rightarrow \sigma \\
\text{Model uncertainty included in model output} & \Rightarrow f_{adj}(y_m, y_\sigma) \\
\text{PDF} & \Rightarrow y_{adj}
\end{align*}
\]

\textit{Figure 24. Including the model uncertainty in the model output.}

Despite the fact that the condition modelled is deterministic, the uncertainty in the deterministic model prediction causes the adjusted model output to be stochastic. The situation is similar to that presented in Section 2.2.2, where natural variation is neglected and knowledge uncertainty is present. The probability and variation interval for the deterministic reference prediction can be described by a distribution function (\( f_y \)), see Figure 24. The uncertainty due to natural variation in the input data must be taken care of before using the deterministic model. If no separate uncertainty analysis is carried out a conservative input is normally chosen to ensure a conservative model output. If the adjusted model output is used as input in another model or compared with deterministic acceptance criteria, a similar situation occurs, i.e. an uncertain variable must be represented by a single deterministic value. If the whole time period of the fire development is of interest the adjusted model prediction will be expressed as a time-dependent probabilistic model output, see Section 4.1.5.

The effect of the adjustment for such a time period is illustrated in Figure 25. The 2.5% and 97.5% percentile represents the 95% prediction interval for the adjusted temperature. The figure exemplifies how the uncertainty in the model prediction, \( T_p \), is propagated to other variables used in the design applications, \( t_{\text{critical}} \).
4.5.2 Deterministic representation of an uncertain variable

When deterministic values are used, information on uncertainty is lost, although the uncertainty can be dealt with in a "rough" analysis, according to Section 2.3.2.

A design criterion can be expressed as in Eq. [4.3], if high temperature is assumed to be hazardous and $T_{\text{critical}}$ represents the maximum acceptable temperature exposure. $T_{\text{predicted}}$ is the temperature exposure calculated with a smoke transport model, assumed to predict the real conditions accurately. The design is considered acceptable if the design criterion is fulfilled.

$$T_{\text{critical}} \geq T_{\text{predicted}} \quad [4.3]$$

A problem occurs when there is an error in the "deterministic" model output. If there is a model error in the prediction it is not certain that the design solution fulfills the design conditions. If there is quantitative knowledge of the model error, the predicted exposure level can be adjusted and expressed as $T_{\text{adj}}$, according to Figure 24. After adjustment the model prediction is more accurate, but contains uncertainty. Definitive criterions, such as in Eq. [4.3], require a deterministic representative value for the range of possible values that can occur, see Figure 26.
A representative deterministic value is required to make a comparison, according to the simple design verification. Energistyrelsen (1996) suggested that the mean value can be used to represent uncertain variables and the design verification can be performed according to Eq. [4.4].

\[ T_{\text{critical}} \geq T_{\text{adj}} \]  

[4.4]

If the mean of \( T_{\text{adj}} \) is used for verification in the deterministic design criterion, i.e. Eq. [4.4], there is a possibility that the real conditions will not fulfil the design criterion. If the distribution in Figure 26 symbolises the adjusted prediction, Eq. [4.4] would indicate that the criterion was not exceeded if the critical level were 140 °C. It is noted that if the mean of \( T_{\text{adj}} \) is used as a representative value for the distribution, part of the uncertainty interval is greater than \( T_{\text{critical}} \), i.e. there is a probability of failure.

There are several disadvantages of using the mean value as a deterministic comparative value for an uncertain variable. Information about if and how high the probability is that the design criterion is not fulfilled is not presented. If the mean of \( T_{\text{adj}} \) is equal to \( T_{\text{critical}} \) the false impression is given that the design fulfils the design criterion, but there is actually a 50% probability that the criterion is not fulfilled. Another disadvantage is that no difference is observed between two distributions that have the same mean values but different variances, see Figure 27.

An alternative to the mean value is to use a quantile from the distribution. The effect of the uncertainty is then better taken into account. An example is to define a conservative value as one of the bounds of the 95% interval of the variable, i.e. the 2.5% or 97.5% quantile, depending on how hazardous exposure is defined. The design condition will be expressed according to Eq. [4.5].

\[ T_{\text{critical}} \geq T_{97.5\%} \]  

[4.5]

If the design condition is fulfilled there is a 97.5% probability that the temperature will not exceed \( T_{\text{critical}} \). The quantile chosen depends on the sensitivity of the system and the
consequence of a failure (Statens planverk, 1982). In this work the 97.5% quantile represents a conservative value. A conservative value is normally considered good enough and “on the safe side”. Despite the fact that a conservative value is used and $T_{97.5\%} = T_{\text{critical}}$, there is still a 2.5% probability that the system will fail. This is referred to as an accepted risk in technical systems, and is in most systems unavoidable (Grimvall et al., 1998). Zero risk is impossible to achieve. It would be too expensive and often impossible in practice.
5 Quantitative analysis of model error

5.1 Statistical analysis of model error

The engineer must be aware of the predictive capability of the models to derive a confident prediction which can be used in engineering applications. If the approach to the applications is based on analytical methods, then an explicit analysis of the uncertainties must be carried out or conservative values and assumptions used. Appropriate treatment of uncertainty is required to justify the use of analytical methods, see Section 2.3. A major difference between using a prescriptive standard method, i.e. an approved solution, and an analytical approach lies in how the uncertainty is dealt with, see Chapter 3. In approved solutions no specific account must be taken, of uncertainty as this is dealt with implicitly. To assess the uncertainty explicitly and to derive conservative values from uncertain model predictions, as is required in the analytical methods, quantification of the error and uncertainty in the model predictions is necessary.

No general approach has been agreed upon regarding the quantification of model uncertainty or how to take the uncertainty into account. The evaluation guidelines recommend comparison with experiments, but do not describe how to perform the comparison (ISO/PDTR 13387-3, 1998; ASTM, 1996). In this report the following straightforward approach is used.

1. Derive a method to quantify the model error and model uncertainty in smoke transport models by comparison between model predictions and experimental measurements.

2. Create a model to adjust future model predictions for the model error, based on the relationship derived from the comparison.

3. Exemplify the use of the adjustment model in engineering applications

A logical scheme for the adjustment of a deterministic model output is shown in Figure 21, and in Figure 28 a scheme for the model use in an application is exemplified.

![Diagram](image_url)

*Figure 28. Illustration of how model predictions can be corrected for model error in a simple design expression.*
The additional input required, apart from the model output, varies between different applications and can be model output from other sub-models, performance criteria, etc. In this section, statistical models to determine \( f_{adj}(T_p) \) are exemplified. The nomenclature used in this chapter is presented in Appendix B.

The objective of the quantitative analysis is to determine an adjustment model for the model output, \( f_{adj}(T_p) \), to take the model error into account in future model predictions. This will result in a better estimate of the “real” conditions required as input in engineering application, see Eq. [5.1]. The measured temperature is used to represent the real temperature, under the assumption that the error in the measurement is considerably smaller than the error in the model prediction.

\[
T_{real} \approx T_{measured} \approx T_{adjusted} = f(T_{predicted}) = T_{predicted} - e
\]  

[5.1]

The characteristics of the error, \( e \), in a model prediction can be of two different types. If the size of the error in a number of identical predictions varies in a random way, the error is uncertain, see Figure 29.

Figure 29. The interval that represents the estimate of the measured temperature, \( T_m \), from the predicted temperature, \( T_p \).

If a future prediction is to be adjusted based on knowledge of this randomness, it is not possible to tell exactly where the measured value will be if an experiment is performed. Based on the knowledge of the uncertainty in the error in the prediction, an interval can be given for the estimate of the measured temperature. When the error only consists of a random component, it can be expressed as the distribution \( \epsilon_{\text{random}} \) with expectation of zero and the standard deviation \( \sigma_e \). An adjustment of the random error is expressed in Eq. [5.2].

\[
T_{measured} = T_{predicted} - \epsilon_{\text{random}}
\]  

[5.2]
The other characteristic type is referred to as systematic, i.e. a bias. A systematic error is possible to adjust for without uncertainty, in contrary to the random error. The error is characterised in Figure 30.

![Figure 30](image)

**Figure 30.** The error is deterministic and an estimate of the dependent variable $T_{mi}$ can be made without uncertainty.

When the error only consists of a systematic component, it can be described as the distribution $\mathcal{E}_{\text{systematic}}$, with an expectation of $\mu_e$, and no variance, i.e. $(\mu_e, 0)$. The value is described as a distribution without variation, i.e. a constant $\mu_e$. An adjustment of the systematic error is expressed in Eq. [5.3].

$$T_{\text{measured}} = T_{\text{predicted}} - (\mu_e, 0) = T_{\text{predicted}} - \mu_e \quad [5.3]$$

The systematic error represents a bias in the model predictions and is a measure of the accuracy, while the random error represents the uncertainty and is a measure of the variability in the prediction. An error in a model prediction can consists of both a random component and a systematic component, see Figure 31.

![Figure 31](image)

**Figure 31.** The model error is a combination of a random and a systematic error for each model prediction.

The error can be expressed as a stochastic variable, according to Eq. [5.4], where the systematic bias and the variability are represented together.

$$\mathcal{E} = \mathcal{E}(\mathcal{E}_{\text{systematic}}, \mathcal{E}_{\text{random}}) \in (\mu_e, \sigma_e) \quad [5.4]$$
Both types of uncertainty are of interest when making adjustments to the model error. An adjustment must be made to the predicted value to take the error into account, which requires quantitative knowledge of the error. A statistical method that can be used to quantify the error from comparisons of measurements and predictions is presented later in this chapter.

The main difference between the systematic and the random error is that the systematic error in a prediction can be removed or reduced completely, while the random part cannot. For this reason the random part of the error in the model prediction has to be taken into account, to assess the model error correctly. This can be done in different ways according to Section 4.5. The correction of the total error in the original model prediction, $T_p$, is referred to as an adjustment, $T_{adj}$, and can be seen as an estimate of the measured temperature, $T_m$.

This estimate is uncertain due to the quantified uncertainty in the predictions. An example of the result of such an adjustment is represented by the uncertainty interval in Figure 31. The interval represents the modeller’s lack of knowledge of the true error and what the temperature would be if measurements were performed. The model uncertainty is therefore seen as an epistemic uncertainty of type B, see Section 2.1.3.

The examples above are based on single-point estimates. To express the error in terms of $\mu_e$ and $\sigma_e$ for each single possible prediction is out of the question. It is also impossible to gain information on the uncertainty from single observations, and it is difficult to obtain data from identical tests. The sources of the model error and uncertainty in the error are discussed in Section 4.4. The effects of the different sources are likely to vary if the conditions assumed for the prediction are changed. This will cause the error to change if a range of predictions is studied. The smaller the range in variation, the smaller the uncertainty interval for the adjusted prediction. A high uncertainty interval will give unrealistic results in practical applications.

However, the professional user of a model is not likely to take model error into account if the adjustment process is complicated, or if a large number of possible predictions call for different adjustment models. There must be a balance between the accuracy of the adjustment and the convenience of use of the adjustment model in practical applications.

The range of application is determined by the degree to which the input parameters are allowed to vary and the type of sub-models involved in the model, i.e. the type of scenario modelled. The error is also likely to vary depending on the range of the model output that is analysed. Key equations such as heat and mass transfer are likely to have a strong influence on the predicted parameters and are a source of error in terms of error in model physics, see Section 4.4.

The approach chosen is therefore to limit the variation in input data to certain scenario configurations and to include the whole range of model output during the time frame of interest. The adjustment model would therefore be applicable for a certain type of situation referred to as scenario configuration, where the variation in geometric data, etc., is limited so that the error in the predictions is of the same magnitude. Examples of possible scenario configurations are: a one-room scenario, a three-room scenario, a one-
room scenario with roof vents, etc. The reason for considering different scenarios is that the impact of different sub-models included in the smoke transport model will be different, and therefore the model error is likely to be different.

From here on term Scenario refers to a number of tests defined as a scenario configuration, described in Chapter 5. The reason that this sub-division of the data is made is to reduce the uncertainty so that the results can be of practical use. The objective of the analysis is to avoid results expressing the predictive capability for scenarios with a large uncertainty interval. Scenario in this context is not to be mistaken for the definition used in risk analysis where the definition is slightly different, i.e. scenario is seen as an event tree and a sub-scenario is a single outcome.

The data used for analysis are taken from the fire development for a certain time period from the time of ignition onwards, for a well-specified situation referred to as a fire test. The model output from most fire models which predict the conditions in the pre-flashover phase of fire development is presented as time-dependent graphs, see Figure 18 and Figure 19. The conditions in a fire test can be both predicted with a smoke transport model and evaluated by measurements. When a comparison is made between experiments and predictions for a complete fire test for a certain condition of interest, e.g. temperature, data are collected at several time points. The measured and predicted value at each time point is called a data point and several data points thus represent a fire test. For each data point \( i \) there will probably be a measurable error, which can be expressed according to Eq. [5.5].

\[
\epsilon_i = T_{pi} - T_{mi}
\]

[5.5]

where \( T_{pi} \) represents a deterministic model prediction and \( T_{mi} \) the corresponding measurement at the same point in time.

The error, as defined in Eq. [5.5], is likely to vary and has been analysed for a range of predictions, by comparing measurements and predictions. The sources of variation are discussed in Section 4.4.4 and depend on how the range of predictions is defined, which was discussed previously in this section.

A single data point will give no information on the variation or uncertainty in the error. An adjustment model based on a single observation will be of no value, if the variation is unknown. A number of observations, i.e. data points, are necessary to estimate the uncertainty. The method of analysing the data is very important if a correct estimate of the uncertainty for a whole scenario is to be made. If the data are not chosen and analysed correctly, the uncertainty in the data will not reflect the uncertainty which is the subject of the analysis. In the following section the uncertainty is discussed.

### 5.2 Uncertainty types in a scenario

The uncertainties in a prediction for a scenario originate from the variables and parameters described in Sections 4.4.2 and 4.4.3 and are divided into two different categories.
Uncertainty in error between tests

Uncertainty of the error within a test

The categories into which the uncertainties are divided depend on how the range of predictions subject to analysis is specified, i.e. how the scenario is specified, and the data available for analysis. If the model prediction is a point-type estimate it is sufficient to use a single category, since it is impossible to obtain more that one data point from each test.

5.2.1 Uncertainty between tests

The first category of uncertainty is a combination of several sources of uncertainty. This category represents the uncertainty in the error between different tests. If two different tests are compared at a certain point in time the size of the errors are likely to be different. The uncertainties arise from differences in test-specific conditions or parameters. These test-specific parameters may be known or unknown. For example, if two tests are run with a small variation in input data, this difference might be the source of a model error. When two “identical” tests are evaluated the predictions from the deterministic model are likely to be the same, although the measurement will probably differ, and therefore the model error. The difference is likely to originate from the uncertainty in reproducibility. The dependence between variation in error and input variables can be determined, but part of the variation is due to parameters which can not be controlled.

If \( i \) different tests are considered, \( i \) independent data points can be derived by selecting one data point from each test or by using an average value for each test. If the error, \( e_i \), is calculated for each data point according to Eq. [5.5] (definition), the differences between the errors will reflect the uncertainty in the error for the scenario defined by the tests, due to variations in the scenario-specific parameters. It is important to notice that the data point from each test has to be of the same magnitude, to separate the uncertainty between the tests from the uncertainty within the test. Uncertainty within a test is discussed further in Section 5.2.2.

The mean and variance for the model error \( e \) for the scenario studied can be estimated by simple statistical methods (Walpole and Myers, 1993), see Eqs [5.6]-[5.8]. The model error is assumed to be independent of all the scenario-specific parameters.

\[
\begin{align*}
\varepsilon & \in (\mu_e, \varepsilon_e) \\
(\mu_e) &= \frac{\sum_{i=1}^{n} e_i}{n} \quad [5.7] \\
(\sigma_e^2) &= \frac{\sum_{i=1}^{n} (e_i - (\mu_e))^2}{n-1} \quad [5.8]
\end{align*}
\]
where

\[(\mu_e)^* \text{ is the estimate of the mean, } \mu_e, \text{ of the model error in the scenario} \]

\[(\sigma_e)^* \text{ is the estimate of the standard deviation, } \sigma_e, \text{ of the model error} \]

\[n \text{ is the number of tests} \]

\[i \text{ is the index number for each test, from 1 to } n \]

\[\varepsilon_i \text{ is the model error from test } i \]

\(\mu_e\) indicates if the prediction is biased or not and is a measure of the accuracy of the model. \(\sigma_e\) is a measure of the variation in the size of the errors in the different tests, i.e. the uncertainty of the model prediction in the scenario studied. In the analysis described above, the observed error, \(\varepsilon\), in each data point in the test originates from the same distribution, \(\varepsilon\). The consequence is that no information on the variation of the error within a test is included in the uncertainty measure.

When the uncertainty between tests is studied specifically it is difficult to find a relation with a specific parameter, since much of the variation originates from uncontrollable variation in test-specific parameters. The variation in this error category is therefore treated as an independent variation, although with sufficient data it would be possible to investigate the dependence between the error and known test-specific parameters.

The use of a measure of \(\sigma_e\) as the total uncertainty in model predictions is appropriate if the error does not vary significantly within the possible range of model output in a test. This is also permissible if the variation in the error within a single test is negligible compared with the variation in the error between tests. In other situations additional analysis is the dependence between \(\sigma_e\) and variables and/or test-specific parameters are required.

Since the estimate is based on a limited number of observations, i.e. data points, there will be additional statistical uncertainty in the estimate. This uncertainty in the estimate of \(\mu_e\) is a function of the number of tests included in the estimate, see Eq. [5.9].

\[
\left(\frac{\sigma_{\mu_e}}{n}\right) = \left(\frac{\sigma_e^2}{n}\right)
\]

The greater the number of tests used, the smaller the uncertainty in the estimate. It is not possible to increase the number of data points by using more than one from each test. The data points within the same test are correlated, i.e. the scenario-specific parameters which are the subject of analysis are the same for all the data points from the same test. No more additional information on the variation between the tests is given by a further data point from the same test. The analysis would not give the variation between tests, since it includes the variation within one of the tests. There is, however, an uncertainty within a test which must be taken into account in the analysis of the total uncertainty for the scenario studied.

### 5.2.2 Uncertainty within a test

The second category of uncertainty is that in the error for the range of model output during a single test. The variation originates from the fact that the predictive capability of the model varies for the range of possible model output, i.e. output variables. The
effects of the approximations and the appropriateness of assumptions are not constant. This uncertainty is referred to as uncertainty within a test.

The results in Figure 32 are used to exemplify different approaches to the analysis and quantification of the error within a test. The fire test used consisted of measurements and predictions in a room next to the fire room, in a three-room configuration. A diffusion flame burner using natural gas, with a heat release rate of 500 kW generated the fire. The model output studied was temperature in the smoke layer. The measured and predicted data in terms of temperature at twelve points in time, \( t_i \), describing the development, form twelve data points \((T_{pi}, T_{mi})\), and are presented in Figure 32. An error \( e_i \) can be identified for each data point \((i)\).

![Figure 32. The measured and predicted temperatures that form the twelve data points used in the statistical analysis.](image)

Figure 32 shows that the size of the error varies during a test and that the variation interval is large. The variation of the error, \( e_i \), in the test can be quantified, without any concern being taken to the relation between the error and the data points, by using a similar approach to that presented in Section 5.2.1. Figure 32 indicates that it may not be appropriate to assume that the error in each prediction can be described by the same random distribution. There seems to be a systematic influence, causing the error to grow with increasing temperature and/or time, which must be evaluated further, otherwise the size of the uncertainty for each prediction will not allow the results to be used in practical applications.

If a systematic functional relation can explain a substantial part of the variation in the error between measurements and predictions within a test, the random part will be small. Different methods of quantifying the error within a test and estimating the random and systematic components in the error in a scenario are presented in Sections 5.3.1-5.3.4. This analysis is based on the data points from the test presented in Figure 32.

### 5.2.3 Uncertainty in scenario

The uncertainty type presented in Section 5.2.1 requires an analysis of how the error varies between tests and the type presented in Section 5.2.2 an analysis within the test. It is important to realise that correlated data within a test will not give any information on the uncertainty between different tests. Neither will data points from many tests at a
specified time give information on how the error varies with the range of model output during a test. Two different groups of uncertainty must be recognised in order to be able to derive the total uncertainty for a prediction in a scenario. The data required to perform an analysis of both types and to be able to separate them consists of a number of tests, each of which contains several data points. The more tests, and the more data points in the test, the better the estimate of the uncertainties, i.e. the uncertainty in the estimate of the uncertainty will be reduced.

There is no general analysis method available which automatically takes all uncertainties into account. Since a compromise must be made between complexity, accuracy and the time required for model development and use there, is no room for features in the analysis that are not necessary. The similarities between different analysis methods lie in the basic statistical tools used, e.g. the way in which the uncertainty in a sample is estimated. The ways in which the tools are combined for each specific situation vary however.

Hanna (1991) presents an analysis of the error in predictions with dense gas dispersion models, where the model error is assumed to be independent of the predicted value. The error in a prediction is described as a random value which is added to the prediction. If the model output is of point type estimate it is not possible to separate the uncertainties and isolate the uncertainty within a test, since only one data point is available from each test. In such a case the uncertainties are combined in the quantification process, but problems associated with correlated data do not occur. If the data point from each test is taken at the reference point, e.g. species concentration at a distance of 500 metres from the source, the analysis of the data points would give an estimate of the uncertainty due to variation in the error between the tests. The analysis would only give information on the variation at that specific distance. Data points could also be at other distances in the tests. Then the analysis would contain information on both the uncertainty between tests and the uncertainty within a test, but it would not be possible to separate them. The uncertainty in the estimates is likely to be high, since only one data point from each test is used. If the variation in the error within a test dominates, this approach is not suitable since no attempt is made to analyse the dependence of the error and the model output.

Ronold and Bjerager (1992) present an approach in which the model uncertainty is represented by an uncertainty factor for predictions of pile capacity in the area of geotechnical engineering. The error is assumed to be dependent on the predicted value. The resulting uncertainty in the analysis presented by Ronold and Bjerager is a combination of the uncertainty in a test that can not be explained by the dependence and the uncertainty due to differences between the tests. Only one data point from each test was used. If more than one data point is used from each test, when several tests are involved in the analysis, it will be difficult to interpret exactly which uncertainty has been measured. To take the effect of the correlation between data points from the same test into account, the uncertainties have to be separated.

In many engineering applications model predictions give point-type estimates. In the fire safety engineering applications exemplified in this report the need for flexibility is greater. That which Hanna's and Ronold and Bjerager’s methods have in common is that only one data point from each test is used since the types of uncertainty are not separated in the analysis. For the applications for which they are developed these
methods are adequate, but they can not be used to analyse the uncertainties present in predictions by smoke transport models, which are subject for analysis in this report.

In fire safety engineering the transient part of fire development is often of interest. The transient part of fire development is called the pre-flashover phase. The time period before flashover occurs is normally between ten and twenty minutes. In structural fire safety engineering, the structure is often designed to withstand fire for periods from half an hour to 3-4 hours. Many factors are constant and the model error is not sensitive to the point in time that is studied, as the conditions have nearly turned into steady-state conditions. It is not then necessary to study the variation within the test since this will provide no additional knowledge on the error.

In the statistical analysis of the uncertainties, there are many advantages in separating the different categories in the output. More information on the total uncertainty can be derived through such an analysis. The main advantage is that the sources of the total uncertainty are clear and the dominating source can be identified. Changing the conditions in the tests involved and/or the part of fire development included, can lead to changes in the relation between the types of uncertainty. This information is of use when scenarios are defined. It can be difficult to predict a suitable range of situations, i.e. variation in input data, and range of the model predictions. If the total uncertainty is found to be unacceptably high for a scenario, one way to reduce it can be to re-define the scenario.

If the types of uncertainties are separated, the functional dependence of the error can be investigated. If part of the variation in the size of the error can be explained by functional dependence between the error and other variables, the variation can be reduced. Another benefit is that if a prediction is similar to a previously performed test, adjustment based on the uncertainty within the test may be possible. This is often not possible in fire safety engineering applications, since a number of test-specific parameters are unknown and uncontrollable. Finally, analysis in which the uncertainty within and between tests is separated is more complex than when they are merged together, but it is likely that the resulting total uncertainty will be smaller (Holmquist, 1998).

Full-scale fire experiments are very expensive which limits the number of tests that can be carried out to characterise a scenario.

5.2.4 Quantitative approach
The objective of this study was not to investigate the effect and importance of each uncertain variable, but rather to develop a methodology with which to derive quantitative measures of the predictive capability of a model so that different models can be compared. The development of the statistical quantitative approach was undertaken with the possibility of deriving data for comparison, the demand for a simple adjustment method in practical applications, requirements of accurate adjustment of the error, the amount of time required for analysis and the development of the statistical model in mind.

This analysis does not comprise a complete variance analysis. The method is based on a straightforward simplistic approach in which the important differences in the data
material used to quantify the error are taken into account. Different methods have been
discovered in the literature, but they have been found to be inadequate, for the purpose
of this study, to develop a simple and accurate expression for the user. The inadequacy
of these methods is not surprising since they have been developed based on
assumptions, approximations and certain types of data for kinds of analysis that differ
from the fire situation. The main idea behind some of the methods is presented in the
following discussion.

To evaluate how much of the total uncertainty originates from the variation within and
between tests, an analysis method that separates the uncertainties is necessary. A simple
analysis is not sufficient, since information on the relation between the uncertainty types
is necessary in order to be able to identify appropriate scenarios.

In the study “Model Uncertainty in Smoke Transport Models” (Lundin, 1997) the first
step was taken towards a method of quantifying the model error during the early stages
of an enclosure fire. The model prediction was made by the computer code CFAST 2.0.
The analysis suggested that a dependence could be used to describe the variation in the
model error within a test.

The statistical method developed together with the Department of Mathematical
Statistics at Lund University is based on an analysis of the functional dependence within
the test and the variation of the error between tests. The uncertainties within and
between the tests are combined to give a total uncertainty. The development of the
method is presented in Section 5.3. The results of the analysis are exemplified with data
points from temperature measurements and predictions from one of the tests presented
in Section 6.

5.3 Development of quantitative methods

When the dependence of the model error and a variable is analysed it is important to
find a variable with strong correlation to the error, and which can explain much of the
variation of the error within a test through a simple relation. As in all projects, time and
money are limited.

One variable that is likely to be a candidate is the model output itself. Time is another.
Eq. [5.10] indicates that both variables affect the error.

\[ \varepsilon = f(T_p, t, x_0, ..., x_z) \]  \[5.10\]

A multi-variable analysis of the dependence is possible and a number of variables can
be included. It is likely that the output will be time-dependent, and therefore the time
will be included in the dependence implicitly, if the relation in Eq. [5.11] is used to
investigate the dependence.

\[ \varepsilon \propto T_p \]  \[5.11\]

Eq. [5.11] suggests that the relation between the error and the predicted value is
proportional.
In Figure 33 and Figure 34 the error is plotted against predicted values of temperature and time.

Both graphs show that a random relation between the error and the variables temperature and time is unlikely. It would be probably be possible to describe the temperature in the examples with a simple functional relation. Since the objective is to create a simplistic model to adjust the model output, it is noted that the relation describing the model output itself seems to be simpler than that describing the time.

Since the temperature predictions are based on differential equations, the temperature at a point in time will effect the prediction, i.e. the error is not likely to be constant. The error is therefore a product of the error in the prediction of the temperature at a point in time and the error in the input from the temperature at the previous point in time. Thus the error propagates and accumulates with time. The error in the model output is therefore likely to be a function of the predicted variable itself. An error in the temperature prediction is likely to have a greater impact the higher the temperature is.

In the analysis of the variation of the error within a single scenario, the functional dependence between $\varepsilon$ and $T_p$ is determined. The data are separated into scenarios as the functional dependence between $\varepsilon$ and $T_p$ is assumed to be different in the different scenarios. Although the differential equations that constitute the model are complex, quantification using the statistical approach is simplified and is not concerned with the “true” relation. The true relation is of concern for the model developer and involves very complex analysis. If a simple relation were to exist, it would certainly have been implemented in the model already.

The statistical approach provides a way of describing the error in a specific situation with limited variation in input. A linear relation is used to describe the dependence, which can be seen as a simple form of response surface, with only two variables. The objective is to describe the effect of the “true” mathematical relation that dominates. The part of the error not explained by the functional relation used in the statistical approach will result in a random error.
Random errors may result from a poor statistical model but can also be the result of variation in errors from other sources, as discussed in Section 4. In this analysis, the variation of the error within a test is assumed to only be dependent on the variable $T_p$, despite the fact of influence on the error from parameters. Since the data within a test are correlated, the effect of test- and scenario-specific parameters can not be observed, since their effect on the error does not vary during a single test. Thereby it is possible to isolate the dependence of the error within a test of correlated values, as a function of $T_p$. The uncertainty for a scenario will be dependent on how the difference between tests effects the dependence between $\varepsilon$ and $T_p$.

In the following sections different types of dependence between the model error and the model output are analysed. Several methods are used to exemplify the importance of the choice of dependence.

### 5.3.1 Model error independent of model prediction

Hanna (1993) presented a simple approach to quantifying the variation in model error in data points from dense gas dispersion models. In the model, it is assumed that the whole error in each data point is random and independent of the predicted value, i.e. each error, $\varepsilon_i$, originates from the same distribution. If the errors in the data points are assumed to originate from the same distribution, then the parameters defining this distribution can be estimated. Hanna used this approach to quantify the variation in errors between tests.

In Figure 35 the errors from the test, $\varepsilon_i$, is plotted to illustrate the variation in size for the range of model output in the test. It is obvious that the error is not constant during the test.

![Figure 35. The spread of the size of the errors, $\varepsilon_i$, i=1..12, in the test used to exemplify the different types of dependence.](image)

One approach to analyse the dependence between $\varepsilon$ and $T_p$ is to assume that the method used by Hanna could also be used to describe the variation within a test. This is exemplified with the errors presented in Figure 35.

The parameters describing the distribution from which each error, $\varepsilon$, in the test originates, are the mean error, $\mu_\varepsilon$, and the standard deviation, $\sigma_\varepsilon$. The arbitrary adjustment of a future model prediction within the test is described by Eqs [5.12] and [5.13].
The number of data points \((i)\) in the test is not sufficient to identify the distribution of the variation of the error. To judge from Figure 35, either a uniform distribution or normal distribution could be used. The effect of the type of distribution chosen to represent the data is minor in relation to the size of the uncertainty itself. The approach presented here in Section 5.3.1 is used to demonstrate that a more rigid analysis must be performed, and this method will not be used in the final model, which is presented in Section 5.4. Therefore, it is assumed that the error originates from a normal distribution. The parameters in the distribution can be quantified using Eqs [5.14]-[5.16].

\[
\begin{align*}
(\mu_e)^\star & = \frac{\sum_{i=1}^{n} (T_{pi} - T_{mi})}{n} \\
(\sigma^2_e) & = \frac{\sum_{i=1}^{n} (T_{pi} - T_{mi} - (\mu_e)^\star)^2}{n - 1} \\
(\sigma^2_{\mu e}) & = \frac{(\sigma^2_e)^\star}{n}
\end{align*}
\]

where
- \(\mu_e\) is the average model error in the test, i.e. a measure of the systematic error
- \(\sigma^2_e\) is the variance of the errors in the test, i.e. a measure of the uncertainty
- \(\sigma^2_{\mu e}\) is the variance of \(\mu_e\), i.e. a measure of the uncertainty in the estimate of \(\mu_e\)
- \(n\) is the number of data points in the test
- \(i\) denotes the number of each data point
- \(T_{pi}\) is the model prediction of the variable in data point \(i\)
- \(T_{mi}\) is the measured value of the variable in data point \(i\)

The results from the analysis of the test are presented in Table 2.

**Table 2. Estimated parameters describing the model error in the test subject for analysis.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>((\mu_e)^\star)</td>
<td>43°C</td>
</tr>
<tr>
<td>((\sigma^2_e)^\star)</td>
<td>41°C</td>
</tr>
</tbody>
</table>
According to Eq. [5.13], the adjusted value will be a distribution. To visualize the uncertainty in an adjusted value, a prediction interval can be used. The prediction interval is the interval in which the measured value is assumed to be, on the basis of a model prediction. The exact location cannot be determined since there is uncertainty in the model prediction. Eq. [5.17] gives the 95% prediction interval for an adjusted future prediction.

\[
T_{adj} = T_p - \mu_e \pm \lambda_{97.5\%} \cdot \sigma_e \cdot \sqrt{1 + \frac{1}{n}}
\]  

[5.17]

A prediction interval will be used to illustrate the uncertainty in the adjusted prediction for the different quantification methods. A graphical comparison is sufficient to observe the important aspects of the difference, although more sophisticated statistical methods, can be used.

The 95% prediction interval is plotted in Figure 36 and indicates that there is a substantial amount of uncertainty in the prediction, if the assumption made in Hanna’s model is used.

![Figure 36. The 95% prediction interval for adjusted temperatures according to Hanna’s approach.](image)

The variation in the error in the test is high. If the variation is explained as a random and independent error, the result is a large prediction interval, as shown in Figure 36.

If the smoke layer temperature is of interest in a design situation and there is no possibility to treat the uncertainty explicitly, a conservative value must be used. A conservative estimate based on the analysis will be represented by the upper 95% confidence limit in Figure 36. The temperature after 600 seconds can be taken as an example. According to Figure 36, the model prediction is 260°C, the adjusted conservative prediction 311°C and the measured value 186°C. In this specific example, the input in a design equation would be an overprediction of the real conditions by more than 120°C. According to the figure, the upper confidence interval will be very conservative and it is suggested that the statistical analysis method is developed further.

Figure 36 shows that the error varies, but there seems to be some systematic relation between the size of the error and the predicted variable. The error increases at higher
temperatures. The presence of a systematic dependence between $\varepsilon$ and $T_p$ makes Hanna’s model inappropriate for the description of the error within data points from a fire test.

5.3.2 Model error as dependent on predicted value with a proportional factor

If a model error would be simple in nature and easy to describe for a large range of application, the error would have been taken into account in model calibration during the model development phase. There is obviously no such thing as a single correction factor which can be used in all situations, although this is what users of the model want. The quantitative measure of the model error contains uncertainties and can not be represented by a single value, even within a well-defined scenario. In the previous section the assumption that the size of the error was independent of the prediction was found to be false, see Section 5.3.1. The uncertainty interval can be reduced if some of the variation in the error within the test can be explained by something other than random variation.

In Figure 33, the error in a fire test is plotted against the model prediction, i.e. temperature. Corresponding data are shown against time in Figure 34. It is obvious that the relation between both the variables and the error is not random. A strong dependence of both the temperature and time is observed. Both parameters can be subjected to analysis, but for simplistic reasons the model output is used in this analysis.

Ronold and Bjerager (1992) are among those who suggested that deviation between predicted and measured data could be expressed by a proportional model uncertainty factor. That error is thus assumed to be proportional to the predicted value. Ronold and Bjerager used this method in a geotechnical application. They used data from different tests, but the data points were taken from a wide range of model output. Their data therefore contain uncertainty within a test and uncertainty between tests. The uncertainty that can not be explained by their approach will be represented as an uncertainty in the functional relation. This approach is applied to the fire test to determine whether a proportional error is appropriate in quantifying the error.

\[
\varepsilon = \gamma \cdot T_p \quad \text{[5.18]}
\]

\[
T_m = T_{adj} = f_{adj}(T_p) = T_p - \varepsilon \quad \text{[5.19]}
\]

\[
\gamma \in N(\mu_{\gamma}, \sigma_{\gamma}) \quad \text{[5.20]}
\]

\[
f_{adj}(T_p) = (1 - \gamma) \cdot T_p = \delta \cdot T_p \quad \text{[5.21]}
\]

\[
\delta \in N(1 - \mu_{\gamma}, \sigma_{\gamma}) = N(\mu_{\delta}, \sigma_{\delta}) \quad \text{[5.22]}
\]

$\delta$ is referred to as a model uncertainty factor and can be interpreted as the proportional dependence between the measured and predicted values. The factor indicates if the model over- or underpredicts the variable $T_m$. $\delta > 1$ corresponds to an underprediction and $\delta < 1$ to an overprediction.
In the approach exemplified by Ronald and Bjerager (1992) it is assumed that the value of $\delta$ for each future predicted temperature is uncertain and can be described as a distribution, according to Eq. [5.22]. $\delta$ is assumed to be independent of the predicted value, i.e. $\delta_i$ originates from the same distribution. This means that an adjusted value is expressed as a function of the prediction, multiplied by the uncertain value of the factor $\delta$.

Based on the same limited data points used in the previous example, $\delta$ can be quantified. The relation between each individual measured value and predicted value is described by Eq. [5.23].

$$T_{mi} = \delta_i \cdot T_{pi}$$  \[5.23\]

where
- $T_{mi}$ = measured temperature
- $T_{pi}$ = predicted temperature
- $\delta_i$ = the value of the model uncertainty factor for data point $i$

For each data point $i$, a factor $\delta_i$ is calculated. The analysis of $\delta_i$ is then similar to how $\epsilon$ was quantified in Section 5.3.1. $\mu_\beta$ will equal 1.0 if the model is expected to give an unbiased mean estimate of the real value, which indicates the absence of a systematic error.

$$\left(\mu_\delta\right)^* = \frac{\sum_{i=1}^{n} \frac{T_{mi}}{T_{pi}}}{n}$$  \[5.24\]

$$\left(\sigma_\delta^2\right) = \frac{\sum_{i=1}^{n} \left(\frac{T_{mi}}{T_{pi}} - \left(\mu_\delta\right)^*\right)^2}{n-1}$$  \[5.25\]

where
- $\mu_\delta$ = is the mean model uncertainty factor for a single test
- $\sigma_\delta^2$ = is the variance of the uncertainty factors $\beta_i$ within the test
- $n$ = is the number of data points in the test
- $i$ = denotes the number of each data point
- $T_{pi}$ = is the model prediction of the variable in data point $i$
- $T_{mi}$ = is the measure value of the variable in data point $i$

The results of the analysis of the test are presented in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\left(\mu_\delta\right)^*$</td>
<td>0.86</td>
</tr>
<tr>
<td>$\left(\sigma_\delta^2\right)$</td>
<td>0.24</td>
</tr>
</tbody>
</table>
This method expresses the estimate of the measured value based on a future model prediction within the test according to Eq. [5.26].

\[ T_{adj} = \delta \cdot T_p \] [5.26]

The prediction interval for the uncertainty factor and the adjusted value are given by Eqs [5.27] and [5.28], assuming that \( \delta \) is normal distributed.

\[ I_{95\%} = \mu_\delta \pm \lambda_{97.5\%} \cdot \sigma_\delta \cdot \sqrt{1 + \frac{1}{n}} \] [5.27]

\[ T_{adj} = T_p \cdot \left( \mu_\delta \pm \lambda_{97.5\%} \cdot \sigma_\delta \cdot \sqrt{1 + \frac{1}{n}} \right) \] [5.28]

The 95% prediction interval is shown in Figure 37.

Figure 37. Prediction interval for \( T_{adj} \) when the model error is expressed as a model factor \( \delta \), which is a distribution. The same distribution is used for each \( T_{pi} \) to be adjusted.

The approach suggested by Ronold and Bjerager (1992) is not appropriate in describing the model error within a fire test, as it leads to an enormously high uncertainty in high predicted values. One reason for this is that an absolute difference of a few degrees between \( T_m \) and \( T_p \) will cause a considerable effect on the factor \( \delta \) in the early part of the test, while the same absolute difference will only have a small effect on the factor \( \delta \) for high temperatures. A random error caused by other parameters than the one that is analysed can easily cause a variation of a few degrees and have such an effect. If this is the case, the variation in \( \delta \) will differ between the end and beginning of the test, cannot be expressed by the same distribution. \( \delta \) cannot be seen as independent of \( T_p \), and thereby the assumptions for the model is not applicable for the data points analysed. This indicates that the model with a proportional factor quantified according to the method used by Ronold and Bjerager is not suitable to quantify the error of interest in this study. To be able to add a certain weight or impart greater importance to \( \delta \) from high temperatures linear regression is suggested, i.e. the importance of \( \delta \) is determined by the size of the prediction \( T_{pi} \).
5.3.3 Model error based on simple regression analysis

In Sections 5.2.1 and 5.2.2 the variation in the model error within a test is described by methods derived for other types of prediction models and results in very high uncertainty in the error. In the following section a straightforward approach is used, where the functional dependence between the error and the predicted values is addressed with more sophisticated statistical methods than before.

The data points are plotted in Figure 38, where $T_m$ is the dependent variable and $T_p$ is the independent variable. If the functional relation between $T_m$ and $T_p$ can be found it is possible to use this relation to adjust future predicted values and also to calculate the expected model error, since $\varepsilon = T_p - T_m \approx T_p - f(T_p)$

![Figure 38. The data points plotted from the example used in the quantitative analysis.](image)

The accuracy of the estimate $f(T_p)$ is dependent on how good the relation is, i.e. how well the assumed model describes the data in the data points. If there is a large random part in the function $f(T_p)$ the uncertainty of $T_m$ will also be large, as in the earlier approaches.

A simple linear regression model has been used to investigate the functional relation, see the report “Uncertainty in Smoke Transport Models” (Lundin, 1997). The model is summarised below and then complemented with an improvement, which reduces the random part in the quantified relation. The functional relation for a single observations, i.e. data point, is described as

$$T_{mi} = \beta \cdot T_{pi} + \varepsilon_i \quad [5.29]$$

where

- $T_{mi}$ is a measured temperature from data point $i$
- $T_{pi}$ is a predicted temperature from data point $i$
- $\beta$ is a constant regression coefficient
- $\varepsilon_i$ is the residual error for each data point.
All the data points in the test can be described by the following arbitrary functional relation:

$$T_m = \beta \cdot T_p + \varepsilon$$  \[5.30\]

where

- $T_m$ is any measured temperature in the range defined by the test
- $T_p$ is any predicted temperature in the range defined by the test
- $\beta$ is a regression coefficient
- $\varepsilon \in N(0, \sigma_e)$ is the random error independent of the data points.

The factor $\beta$ represents a constant proportional over- or underprediction and $\varepsilon$ an additional error when the bias $\beta$ between $T_m$ and $T_p$ has been taken into account. From here on the systematic error, $\varepsilon_{\text{systematic}}$, is defined as the expectation value of the functional relation between measured and predicted temperature, $E(f(T_p))$. The random error, $\varepsilon_{\text{random}}$, will be referred to as the variance or standard deviation for the functional relation, $V(f(T_p))$. The symbol $\varepsilon$ is used to refer to the residual error.

$\beta$ and $\varepsilon$ are calculated with standard statistical equations (Walpole and Myers, 1993). The nomenclature used is that used in most standard statistical textbooks. It must be noted that the index $i$ is replaced with $j$ in the following equations.

$$T_{mp} = \sum_{j=1}^{n} T_{mj}T_{pj}$$  \[5.31\]

$$T_{pp} = \sum_{j=1}^{n} T_{pj}T_{pj}$$  \[5.32\]

$$T_{mm} = \sum_{j=1}^{n} T_{mj}T_{mj}$$  \[5.33\]

$$T_0 = T_{mm} - \frac{(T_{mp})^2}{T_{pp}}$$  \[5.34\]

$$\beta^* = \frac{T_{mp}}{T_{pp}}$$  \[5.35\]

$$\left(\sigma^2_{\varepsilon}\right) = \frac{T_0}{n-1}$$  \[5.36\]

where

- $n$ is the number of data points in the test.
- $j$ denoted the number of the data point.
Quantitative analysis of model error

\[ T_{mj} = \text{measured temperature in data point } j \]
\[ T_{pj} = \text{predicted temperature in data point } j \]
\[ \beta^* = \text{regression coefficient describing the slope} \]
\[ \sigma_e^* = \text{standard deviation of the residuals} \]

The prediction interval for a future measured temperature (Walpole and Myers, 1993) is given by

\[
I = \beta^* \cdot T_p \pm \lambda_{\alpha/2} \cdot \sigma_e^* \left[ 1 + \frac{T_p^2}{\sum_{j=1}^{n} T_{pj}^2} \right]^{1/2}
\]

The model is applied to the example and the results of the analysis are presented in Table 4. It is important to notice that the standard deviation is related to the random part of the error, \( \varepsilon_{\text{random}} \), and not to the variation in the parameter describing the slope, \( \beta \).

**Table 4. Estimated parameters describing the model error in the test subject to analysis.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \beta )</th>
<th>( \sigma_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.72</td>
<td>18.5 °C</td>
</tr>
</tbody>
</table>

The 95% prediction interval for the adjusted temperature, \( T_{adj} \), is presented in Figure 39. In Figure 40 each data point from the test has been plotted, i.e. predicted values and their corresponding measured values. The dotted line shows the uncertainty interval for the adjusted temperature in the time-temperature graph, transferred from Figure 39.

**Figure 39. Analysis of a linear functional relation between predicted and measured temperatures.**

**Figure 40. Results from the analysis showing the 95% prediction interval, used to estimate measured values based on predicted values.**

The prediction interval shows a reduction in the uncertainty interval compared with earlier approaches. In the test subject to analysis, an adjusted prediction that is conservative is actually closer to the measured temperature than the prediction itself. In a design situation the effect is that the model itself gives too conservative a prediction.
5.3.4 Model error based on regression analysis with intercept
The same concept is used as in the regression model presented above, but the model
does not force the regression line through $x = 0, y = 0$. The model is expanded to
include an intercept, $\alpha$, that can represent any value on the y-axis. The regression model
with intercept is a standard model included in most statistical textbooks.
The parameters $\alpha$, $\beta$, and $\varepsilon$ are calculated with standard statistical equations (Walpole
and Myers, 1993).

\[
T_m = \frac{\sum_{j=1}^{n} T_{mj}}{n} \tag{5.38}
\]

\[
Q_{mp} = \sum_{j=1}^{n} \left( T_{mj} - T_m \right) \left( T_{pj} - T_p \right) \tag{5.39}
\]

\[
Q_{pp} = \sum_{j=1}^{n} \left( T_{pj} - T_p \right)^2 \tag{5.40}
\]

\[
Q_{mm} = \sum_{j=1}^{n} \left( T_{mj} - T_m \right)^2 \tag{5.41}
\]

\[
Q_0 = Q_{mm} - \frac{(Q_{mp})^2}{Q_{pp}} \tag{5.42}
\]

\[
\beta = \frac{Q_{mp}}{Q_{pp}} \tag{5.43}
\]

\[
(\sigma^2) = \frac{Q_0}{(n-2)} \tag{5.44}
\]

\[
\alpha = T_m - \beta \cdot T_p \tag{5.45}
\]

where
$n$ = the number of data points in a single test
$j$ denotes the number of a specific data point
$T_{mj}$ = measured temperature in data point $j$ (a single point on a temperature-time curve)
$T_{pj}$ = predicted temperature in data point $j$ (a single point on a temperature-time curve)
$\beta$ = the slope of the regression line
$\alpha$ = the intercept of the regression line on the y-axis
$(\sigma^2)$ = estimated residual variance of the model errors
A 95% prediction interval for the estimate of $T_m$ based on a future prediction $T_p$ is given in Eq. [5.46] (Walpole and Myers, 1993).

$$I_{95\%} = \alpha^* + \beta^* \cdot T_p \pm 1.96 \cdot \sigma^*_{\epsilon} \sqrt{1 + \frac{1}{n} + \frac{(T_p - \bar{T}_p)^2}{Q_{pp}}}$$  \[5.46\]

The model is applied to the example and the results of the analysis are presented in Table 5.

**Table 5. Estimated parameters describing the model error in the test subject to analysis.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>36 °C</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.55</td>
</tr>
<tr>
<td>$\sigma_{\epsilon}$</td>
<td>8.3 °C</td>
</tr>
</tbody>
</table>

The results are presented graphically in Figure 41 and Figure 42.

**Figure 41.** Measured temperature as a function of predicted temperature.

**Figure 42.** Measured and predicted temperatures and the prediction interval from regression analysis.

Figure 41 presents the data points from the test and the model describing the functional dependence between $T_m$ and $T_p$, together with the 95% prediction interval. The uncertainty in the estimate of $T_m$ is reduced compared with earlier approaches. In Figure 42 the predicted and measured data are plotted against time, together with the 95% prediction interval representing the uncertainty interval for the adjusted prediction. The adjusted prediction is a better estimate of $T_m$ than the original model prediction $T_p$.

The regression parameters are quantified in Eqs [5.38]-[5.45] and describe each single data point according to Eq. [5.47].

$$T_m = \alpha + \beta \cdot T_p + \epsilon$$  \[5.47\]

$$\epsilon \in N(0, \sigma_{\epsilon})$$  \[5.48\]
Since the estimates of the regression parameters are based on a relatively small sample, the estimates contain uncertainty.

\( T_p - E(\alpha + \beta \cdot T_p) \) can be interpreted as the systematic component of the model error, while \( \varepsilon \), together with the uncertainty in the other regression parameters, represents the random component, \( V(\alpha + \beta \cdot T_p) \). The random part is illustrated as the scatter around the regression line in Figure 41, or as the width of the prediction interval in Figure 41 and Figure 42. The systematic part is illustrated as the distance from the predicted value to the middle of the prediction interval in Figure 42.

The relation between the analysed data points is used to create an adjustment model for the error in future predictions for the same range of data points. The adjustment model uses the functional relation quantified in the analysis. The estimate of \( T_m \) from an arbitrary model prediction \( T_p \) within the range of the data used in the analysis, i.e. the test, is presented in Eq [5.49].

\[
T_{adj} = \alpha + \beta \cdot T_p + \varepsilon \quad [5.49]
\]

The upper 95% quantile in the prediction interval can be used to ensure a conservative value of the adjustment prediction:

\[
T_{adj} = \hat{f}(T_p) = \alpha^* + \beta^* \cdot T_p \pm \lambda_{\alpha/2} \cdot \sigma^* \sqrt{1 + \frac{1}{n} + \frac{(T_p - T_p)^2}{Q_{pp}}} \quad [5.50]
\]

\( \lambda_{\alpha/2} \) and \( \lambda_{(1-\alpha/2)} \) are the percentiles corresponding to a prediction interval of \( (1-\alpha) \). Depending on the application, one of these percentiles will be used to define a conservative limit. High temperature is often related to hazard or can be defined as a design condition. If the adjusted model prediction is expressed as a stochastic variable \( T_{adj} \), the upper percentile will serve as a conservative deterministic value. It is important to keep in mind that in some applications the other percentile can be considered conservative, for example when modelling sprinkler activation.

If the regression is improved, the scatter around the regression line will be reduced and the part of the variation that is explained by the model will be increased. The uncertainty in the adjusted value will then be reduced. In Figure 43 the functional relations between the simpler regression, with intercept forced through zero and the model developed in this study are compared. It is obvious that the scatter around the dotted line is smaller than around the solid line. The scatter represents the uncertainty in the functional relation between the variables.
5.4 Quantifying the model error in fire models

In the previous sections different quantitative analysis methods were compared, to establish how the variation within a test could be described. The objective is to find a method in which the random part of the error in each prediction is low. The most suitable method of quantifying the model error in a fire test was concluded to be a regression model with a non-zero intercept, described in Section 5.3.4. The regression model gives the functional relation between \( T_m \) and \( T_p \) in the analysed data. This relation is assumed to be valid for the whole test and therefore any future prediction within the test can be adjusted for the error.

The variation within a single test can thus be analysed. According to Section 5.1 the error is likely to be affected by variation in test-specific parameters. The knowledge of the error and the variation in the error within a single test is of no use in practical applications, where the different types of possible tests are endless. Therefore, tests that are assumed to have similar errors and variations in error are collected into scenarios.

General models are available for the analysis of uncertainty in the model predictions for a scenario (Searle, 1987). The type of model best suited depends on the specific situation and the nature of the uncertainty in the prediction. In this approach the analysis is based on the quantified functional dependence between the error and the predicted parameter. This dependence includes a random component, which is part of the total uncertainty in the prediction for a scenario. To include the effect of uncertainty due to variation in the error between tests, the variation in the functional relation between the tests is used as measure of this uncertainty. No attempt is made to describe the variation between tests with a functional relation. The individual importance of each test is reflected by weights, which takes the width of the range and the number of data points in each test into account. Both types of uncertainties are separated during the analysis in order to be able to quantify the functional dependence within the tests. The uncertainty within the test and the uncertainty between tests are finally merged together and used to derive an adjustment model for the scenario. The method of quantifying the error is

\[ T = \text{Simple regression model with intercept} \]

Figure 43. Systematic difference between simple regression model and regression model with intercept.
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referred to as a weighted regression analysis. The method was developed in co-
operation with the Department of Mathematical Statistics, Lund University, and is
presented below.

5.4.1 Model error based on multi-test regression model with intercept
The regression model presented below is based on the same type of regression analysis
as described in Section 5.3, but information from all tests in a scenario is used. The
analysis is more complicated because the uncertainty in the model error for a scenario is
calculated as a weighted uncertainty from the uncertainty within each test and the
uncertainty between the tests in the scenario.

In presenting the development of the model, temperature predictions from CFAST 2.0
are used as model output subject to analysis. Predictions of smoke layer height are also
analysed in this study, with the same method and the results are presented in Chapter 7.
The data used in the analysis of the smoke layer height are also taken from the database
presented in Chapter 6.

Let \((T_{pij}, T_{mi}), \ j = 1, 2, \ldots, n_i\) be \(n_i\) ‘predictions’ and ‘measurements’ within a test \(i\) (for
example \(T_p\) and \(T_m\) in temperature-time curves) and \(i = 1, 2, \ldots, L\) be \(L\) different tests in
a scenario. In the earlier examples, the number of tests within the scenario has been one,
i.e. \(i = 1\) has been used. Observe that the index \(i\) is introduced again, but with a different
definition than in earlier sections.

\(\beta^*\) is a weighted average of \(\beta^*_i\) from each test, and \(\beta_{all}^*\) is derived from all the data
points when uncertainties are not separated.

\[
T_{m_i} = \frac{\sum_{j=1}^{n_i} T_{m_{ij}}}{n_i} \quad [5.51]
\]

\[
T_m = \frac{\sum_{i=1}^{p} \sum_{j=1}^{n_i} T_{m_{ij}}}{\sum_{i=1}^{p} n_i} \quad [5.52]
\]

\[
\beta^*_i = \frac{Q_{mp}^{(i)}}{Q_{pp}^{(i)}} \quad [5.53]
\]

\[
\beta_{all}^* = \frac{\sum_{i=1}^{p} \sum_{j=1}^{n_i} (T_{m_{ij}} - T_m)(T_{p_{ij}} - T_p)}{\sum_{i=1}^{p} \sum_{j=1}^{n_i} (T_{p_{ij}} - T_p)^2} \quad [5.54]
\]
Information about the uncertainty in the model predictions is obtained by studying the uncertainty in each individual test and from uncertainty analysis of all the data points together. The importance of these components is determined by a weighting factor $c_i$.

$$\beta^* = \sum_{i=1}^{p} c_i \cdot \beta_i + \left(1 - \sum_{i=1}^{p} c_i\right) \beta_{all} \quad [5.55]$$

The weights $c_i$ reflect the importance of the different tests by taking the number of data points within the test and the length of the measuring interval into account.

By determining the functional dependence between the error and the predicted value for a scenario and including the range of temperature during fire development, it is possible to adjust any model prediction within this scenario. The factors $c_i$ can be calculated using an algorithm written by Björn Holmquist at the Department of Mathematical Statistics, Lund University, presented as a Matlab file in Appendix A.

With the weighted average $\beta^*$, an estimate of the combined intercept can be calculated by using all the data points. Information about the uncertainty within tests in proportion to the uncertainty between tests is included in $\beta^*$, but could be derived from a similar expression as Eq. [5.56].

$$\alpha^* = T_m - \beta^* \cdot T_p \quad [5.56]$$

The systematic error for the scenario can now be expressed with the regression line according to Eq. [5.57]. The equation express the mean bias between the prediction and the measurement, for the range of model output defined in the scenario.

$$\epsilon_{systematic} = T_p - E(\alpha + \beta \cdot T_p) \quad [5.57]$$

In this statistical method it is assumed that the major sources of uncertainty in the functional relation between $T_m$ and $T_p$ can be related to the uncertainty within each test, i.e. the residual variance $\sigma_e^2$, and the uncertainty between the intercepts for the tests, $\sigma_a^2$. The residual variance is assumed to be the same in the different tests and the uncertainty in $\beta^*$ is neglected and assumed to be the same for all tests. In a future prediction, $\sigma_a^2$ represents the uncertainty of where the regression line would lie, i.e. which test the prediction belongs to, and $\sigma_e^2$ the uncertainty in how far from the regression line the data point is likely to be, i.e. the uncertainty within a test. Since the regression model is used to adjust the future prediction, account is automatically taken of the dependent part of the error, represented by $\epsilon_{systematic}$.

The separation of total random uncertainty into the components $\sigma_e$ and $\sigma_a$ makes it possible to investigate which of the two different types of uncertainties dominates. Proposals for definitions of scenarios, where the total uncertainty is reduced, require this information. The alternative, i.e. merging all the data from the different tests together and performing a single regression analysis, is not sufficient. The final expression for the adjustment model for a scenario with multiple tests is:
The expression is stochastic, since \(\alpha^*\) and \(\varepsilon\) contain uncertainties. The uncertainty is calculated with the Matlab file presented in Appendix A. The adjusted temperature contains uncertainty and is expressed as a distribution. In a stochastic analysis the expression in Eq. [5.58] can be used directly, but in many engineering application a deterministic conservative value is of more use.

A conservative value can be derived from a prediction, with the adjustment model presented in Eq. [5.59]. The distribution representing the model uncertainty is assumed to be normal distributed and a conservative value is derived from one of the bounds in the two-sided prediction interval. Which bound is used depends on if a high or a low value is considered hazardous, see Section 4.5.2. The 95% bounds for a single prediction \(T_p\) are given by:

\[
I_{T_{adj}} = \alpha^* + \beta^* \cdot T_p \pm \lambda_{97.5\%} \sqrt{\left(\sigma^a_n\right)^2 + \left(\sigma^\varepsilon_n\right)^2}
\]

[5.59]

The deterministic adjustment model, which gives a conservative estimate if high temperature is regarded as hazardous, is:

\[
T_{adj} = f (T_p) = \alpha^* + \beta^* \cdot T_p + \lambda_{97.5\%} \sqrt{\left(\sigma^a_n\right)^2 + \left(\sigma^\varepsilon_n\right)^2}
\]

[5.60]

When the parameters are quantified the adjustment function for a scenario can be expressed as:

\[
T_{adj} = f(T_p) = \left(\alpha^* + \lambda_{97.5\%} \sqrt{\left(\sigma^a_n\right)^2 + \left(\sigma^\varepsilon_n\right)^2}\right) + \beta^* \cdot T_p = U_{adj} + \beta^* \cdot T_p
\]

[5.61]

The 95% percentile for a normal distributed variable is 1.96 \(\approx 2\). For simplistic reasons the input needed for the conservative adjustment model can be reduced to two parameters, \(U_{adj}\) and \(\beta^*\), once the statistical analysis for the scenario is performed, see Eq. [5.61].

An alternative approach to the weighting procedure in Eq. [5.55] is a re-sampling method, see Section 5.5.2. The benefit of a re-sampling method is that better knowledge is gained of the uncertainties associated of the estimates of parameters describing distributions and also the shape of the distributions. The estimates of \(\alpha^*\) and \(\beta^*\) are based on a relatively small number of tests for each scenario. The limitations of the method presented above are presented in Section 5.5.

To exemplify this method a number of scenarios are evaluated quantitatively. The scenarios are presented in Chapter 6. The use of different scenarios makes it possible to evaluate the effects of the differences between the configurations on the model error. The results of the quantification examples are presented in Chapter 7.
5.5 Limitations of the quantitative analysis

All models, including statistical analysis models, are based on assumptions, approximations and simplifications. This Section presents a brief overview of the general limitations which must be considered when a model is used and simplifications in employed in the statistical model.

5.5.1 Limitations in the use of quantitative analysis

When adjusting model predictions it is important to be aware of the limitations in the adjustment model $f_{adj}(T_p)$. The adjustment model is based on a quantitative analysis of the model error compared with experimental measurements and predictions.

This section is entitled “Limitations of the quantitative analysis”, but a more appropriate title would have been “The importance of model limitations”. All models have limitations due to the assumptions, approximations and simplifications on which they are based. If these are not taken into consideration the model may be inappropriate and give poor predictions of the real conditions. If the effect of using the model outside its range of applicability is not analysed, it will be impossible to get an idea of the uncertainties involved. The uncertainties in the model prediction are often as useful to the modeller as the prediction itself (Britter, 1993).

For empirical models, the range of the experimental interval used to develop the model is an important limitation. This interval may be defined as a range of values within which predictions can be made, for example temperatures between 20 and 800°C, for a smoke transport model. It can also be defined as limitations in the situation modelled in terms of limits on the input data. If the fire model is of the two-zone type, it is, by definition, inappropriate to use this model after flashover has occurred, since the assumption of two-layer conditions in the room is incorrect. The definition of flashover in terms of measurable physical properties is not exactly defined (Peacock et al., 1998a), but is often defined in engineering applications as the condition when the smoke temperature at ceiling height reaches 600°C (Hägglund et al., 1974). As a fire develops towards flashover, the temperature is somewhere between 500 and 800°C. The two-layer assumption is not strictly valid when flashover is about to occur. It is therefore reasonable to assume that the model predictions will be less accurate just before flashover than in the temperature interval of 150 - 300°C, even if predictions are possible up to 800°C.

If validation shows that a model is in good agreement with experimental results, it may still only be used for the type of experimental scenario and range of model output used in the validation. It is not possible to expand the applicability of the model without making additional comparisons. If the model is used outside the validated range, there will be lack of knowledge about the quality in the predictions. A simple example is shown in Figure 44 and Figure 45.
In Figure 44 the relation between a dependent variable Y and an independent regressor X is shown by black filled squares. The dotted line is an estimate of the linear relation between the variables based on a quantitative analysis of the plotted data. The linear relation can be used as a model to estimate Y for a given value of X. The model is based on several observations or measurements of Y and X in the interval 50 < Y < 125 and 25 < X < 140, i.e. the black squares. In this interval a linear relation seems to be a good estimate of the real relation between the variables.

The linear model can be used outside the interval for which the model was developed, see Figure 45. Whether or not the model will make accurate predictions of Y, depends on the relation between Y and X in that interval. If the linear relation is valid, measurements will lie on the line, as do the crosses in Figure 45. If however, the assumption of a linear relation is incorrect, data may deviate from the regression as illustrated by the circles in Figure 45. The use of a model outside the interval for which it has been designed is therefore not recommended until the model has been compared with experiments in the new interval.

Models are often based on the results from small-scale experiments in limited situations. In practice there is a great need to expand the applicability of models. An example in fire modelling is the plume model, used in the smoke transport model CFAST 2.0. The plume model was empirically derived for small fires and is implemented in the computer code normally used to model larger fires. Comparison with experiment show that the relation seems to be valid for a much greater range of heat release rates than the original empirical plume model was developed for. Thus, the model was found to be valid over a greater range than that for which it was developed.

### 5.5.2 Assumptions and limitations in the statistical method

When the errors in the data points are compared to identify the systematic and random error in a test, the comparison is often based on a limited number of data points. The number of data points used in the analysis is referred to as the sample size. An example of sample size is the number of different tests in a scenario, when parameters such as...
\( \mu, \sigma \), or \( \beta \) are estimated in the weighted regression analysis. It can also be the number of data points in a test when \( \alpha, \beta \) and \( \sigma_e \) are estimated for a single test.

The statistical estimates of parameters describing distributions are based on the assumptions that the mean and variance of a sample reflect the same parameters in the distribution, from which the sample is assumed to originate. A sample consists of \( n \) observation and \( n \) is normally referred to as sample size. If \( n \) observations are randomly drawn from a distribution \( p \) times, the mean and variance of each sample observations will vary. Therefore, there is an uncertainty associated with the estimates of the parameters in the distribution based on a sample. This uncertainty is referred to as statistical uncertainty or the uncertainty in the estimate and depends on the sample size. If the sample size is increased, the accuracy of the estimate will increase.

The statistical method presented in Section 5.4.1 is limited to take the uncertainty in the estimate of the mean value into account. An example is given in Eqs [5.62] and [5.63], where the mean value and variation of a sample is estimated.

\[
(x)^\prime = \frac{\sum_{i=1}^{n} (x_i)}{n} \tag{5.62}
\]

\[
\sum_{i=1}^{n} \left( x_i - (x)^\prime \right)^2 \left( \frac{1}{n-1} \right) \tag{5.63}
\]

The uncertainty in the estimate of the mean value of \( x \) is presented in Eq. [5.64].

\[
(\sigma_x)^\prime = \left( \frac{\sigma_x}{n} \right) \tag{5.64}
\]

Since there is an uncertainty in the estimate of the mean of \( x \), the parameter can be described as a distribution, according to Eq. [5.65].

\[
\bar{x} \in N((\mu_x)^\prime, (\sigma_x)^\prime) \tag{5.65}
\]

\[
\mu_x = E(x) \tag{5.66}
\]

The distribution is according to the central limit theorem approximately normal distributed for large sample sizes. If the shape of the distribution and the parameters describing the distribution are known, the uncertainty can be taken into account. The uncertainty in the estimate of the mean error is, for example, used when the uncertainty represented by the prediction interval is calculated.

If the knowledge of the shape of the distribution is limited, and the parameters describing the distribution are uncertain, the prediction interval will also be uncertain.
This uncertainty is often neglected since it is small in relation to the uncertainty that originates from the variance of the sample itself, but for small sample sizes the effect may be greater.

Another aspect is that the estimate of the variance of the sample, $\sigma_x$, is based on the same sample that is used to estimate the mean, $\mu_x$, and the estimate thereby contains uncertainty. The parameter $\sigma_x$ effects the size of the prediction interval used in the adjustment model and uncertainty in this variable is also likely to have an effect.

This leads to a rather abstract and complicated statistical discussion. Although a rigid analysis has been conducted there are still limitations and simplifications in the model. Below follow some short comments on the major assumptions.

In the analysis presented in Section 5.4.1, the uncertainties related to the observed variation of the model error within and between the tests in a scenario constitutes the main part of the total uncertainty. This gives rise to the following assumption.

- The parameter $\beta$ is assumed to be the same for all tests within the same scenario. The uncertainty in this parameter is neglected.

- The statistical uncertainty, $\sigma_x/n$, in the estimate of the intercept, $\alpha$, for the whole scenario is neglected, although $\sigma_x$ is taken into account since it represents one of the dominating uncertainties.

- There is a lack of analysis of the uncertainty of the parameters describing the uncertainty.

- Only limited data are available for use in the analysis. More data would reduce the statistical uncertainty.

These assumptions are believed to affect the results, but the magnitude is small. One indication is that the variation in the data is well described by the results of the analysis. Further analysis of the neglected uncertainties could provide information on the uncertainty in the prediction interval for the adjusted prediction. No new dominating components would be introduced by a more sophisticated analysis. The effect would be a more qualified and accurate estimate. The practical affect is assumed to be negligible, since the important variation of the error is taken into account, i.e. the functional dependence $\varepsilon = f(T_p)$, $\sigma_x$ and $\sigma_\alpha$. The effect of neglecting an important parameter, such as the functional dependence between the model error and the prediction of the model analysed, is described in Section 5.3. The effect of the neglected uncertainties will decrease if the number of tests and data points are increased.

If the statistical analysis is to be improved more advanced statistical methods will be required. The approach to the model error assessment does not need to be changed, but the estimates of the parameters discussed above can be improved. One such tool is the re-sampling technique.

If the test consists of $n$ data points, a new sample is produced with $n-1$ of these data-points randomly taken from the original sample. The mean value is then calculated for
the new sample according to Eq. [5.62]. The procedure is repeated, for example 10,000 times. The re-sampling will thus give 10,000 mean values, \( \mu_x \). This sampled data are then subjected to analysis according to the standard equations based on a large sample size. The mean and variance of \( \mu_x \) can be estimated with Eqs [5.64] and [5.66], based on the 10,000 \( \mu_x \) simulations and a histogram can be used to indicate the shape of the distribution of \( \mu_x \). This should be compared with the method based on Eqs [5.62]-[5.66] and the assumption of a normal distribution. The same approach can be used for any parameter in the analysis, for example \( \sigma_x \).

The lack of large sample sizes to ensure accurate predictions by statistical models makes re-sampling an alternative. Re-sampling is a numerical sampling method which is more effective than the analytical methods for small sample sizes, if the analytical results rely on large sample sizes. A simple example can be the mean value of a sample, presented in Eq. [5.62], and the uncertainty in this value estimated with Eq. [5.64]. These parameters can also be estimated by re-sampling.

There are different ways of performing re-sampling. One way is described above, where one of the data points is excluded in each calculation. Another method is to draw \( n \) data points from the original sample with \( n \) samples, but to use a replacement technique. Efron and Tibshirani (1993) present a good introduction to different re-sampling techniques, e.g. Bootstrap and Jackknife.

In the statistical method presented in this dissertation the main sources of uncertainty are to be found in parameters that can not be estimated by the bootstrap method, and the uncertainty in these parameters is neglected. In many applications, it is not necessary to know the exact shape of the distribution and many approximations, such as the central limit theorem give reasonable results. The re-sampling technique also requires substantially more programming to incorporate it into the problem analysed. The increased complexity of the re-sampling is concluded not to be worth while. The re-sampling technique does not lead to a method in which the types of uncertainty do not need to be separated. Neither can information on the uncertainty between tests be gained from a single test. Re-sampling is a tool that is used to gain better information on the shape of the distribution under investigation. The separation of uncertainties is still necessary and the analysis will be based on the same data points.

In this type of statistical analysis, the sample size is the number of full-scale tests that is included in the scenario. It is often difficult to obtain reliable full-scale data since it is expensive and time-consuming to execute experiments. Experimental data are increasingly being regarded as commercial assets and are not published. This tendency has unfortunately also been observed in research organisations and international committees. It is therefore difficult to obtain a large sample size, i.e. many tests, to use in the analysis of the model error in prediction of full-scale conditions.

The re-sampling method is useful when there is a lack of data, since the lack of data results in a high statistical uncertainty. The action necessary to reduce the uncertainty, which is the objective of the analysis, is to use a more effective approach to the quantification. Re-sampling may be a very useful tool in the quantification of model error, but is not used in the statistical analysis in this dissertation.
6 Scenario configurations

The experimental data used for the statistical analysis consists of measurements of temperature and the height of the interface from full-scale tests. The measurements are published in a number of scientific journals and these are summarised in a database which have been presented in previously published reports (Bragason, 1994; Lundin, 1997). The model predictions of the same tests by CFAST 2.0 (Peacock et al., 1993a), were also included in the database. Additional simulations have been carried out with the models FAST 3.1 (Peacock et al., 1997), FASTLite 1.0 (Portier et al., 1996) and FPETool 3.2 (Deal, 1995) for the single-scenario analysis. The input for these models was almost identical to the input data used for CFAST 2.0 and is therefore not discussed in further detail. The measured data and predicted results in the database are divided into scenarios referred to as Scenarios A-E. The input data for the predictions together with the corresponding temperature vs. time, T(t), and interface height vs. time graphs, z(t), are presented by Bragason (1994).

6.1 Description of the scenarios

In the following sections, briefs descriptions are presented of the scenarios analysed with the statistical model presented in Chapter 5. The scenarios have been re-named from the original database, since the nomenclature used there is seen to be confusing. For a more detailed description of the tests included in the scenarios, readers are referred to the original references, which are presented in the description of each scenario below.

6.1.1 Scenario A – Single enclosure

Scenario A involves smoke filling in a single room, with very limited openings. The floor area was 5.6 x 5.6 m² and the height 6.1 m. The only connection with the surroundings was an opening with 0.25 m high and 0.35 m wide, situated at the bottom of the room. The construction material was concrete, see Figure 46 and Figure 47.

Data are available from five tests for scenario A. The major difference between the tests is the rate of heat release. The fire was a typical pool fire with kerosene as fuel. The fire

Figure 46. Plan view of test room in scenario A (Hägglund et al., 1985).

Figure 47. Side view of test room in scenario A (Hägglund et al., 1985).
development was very rapid and the fire reached steady-state conditions after 30-60 seconds. The maximum rate of heat release could be changed by varying the size of the pool. The maximum rate of heat release varied between 30 kW to 390 kW.

Measurements and predictions of both temperature and interface height are available in the database. This scenario is referred to as scenario V1 in the database constructed by Bragason (1994). The original measured data is presented by Hägglund et al. (1985).

6.1.2 Scenario B – Two rooms connected by a doorway
Scenario B involves the spread of smoke from a small room to a large room, the two rooms being connected by a doorway. The smaller room was 3 x 4 m² in area and 2.6 m high. The adjoining room had a floor area of 5.6 x 5.6 m² and a ceiling height of 6.1 m. The rooms were connected by a doorway with a height of 2 m and width of 1 m. From the larger room, there was a 0.25 m high and 0.80 m wide opening to the outside at the bottom of the room. The rooms were made of concrete, see Figure 48.

Data are available for two tests in scenario B. The difference between the tests was the rate of heat release. The fire was a kerosene pool fire, with a very rapid development. The fire reached steady-state conditions after 90 seconds. The maximum rate of heat release was varied by varying the pool area. The maximum rate of heat release varied between 330 kW and 670 kW.

Measurements and predictions of both temperature and interface height are available in the database. This scenario is referred to as scenario V2 in the database constructed by Bragason (1994). The original measured data is presented by Hägglund (1992).
6.1.3 Scenario C – Three rooms including a corridor
This scenario consists of two rooms connected by a corridor, i.e. the third room. The corridor had an area of 2.4 \( \times \) 12.2 m\(^2\) and the rooms 2.3 \( \times \) 2.3 m\(^2\) and 2.3 \( \times \) 0.9 m\(^2\). The height in the corridor was 2.4 m and in the rooms 2.2 and 2.4 m. The experimental layout is presented in Figure 49. A detailed description of the geometry is presented by Bragason (1994).

\[\text{Figure 49. Experimental layout for scenario C, i.e. three rooms including a corridor (Peacock et al., 1993b).}\]

During the experiments measurements were conducted at three different locations in the three-room configuration. Therefore the scenario was divided into three separate scenarios denoted scenario C, room 1, scenario C, room 2 and scenario C, room 3. A number of different parameters were changed during the experiments, e.g. the ventilation conditions and rate of heat release. Six tests are included in the database, but during the experiments room 3 was excluded in some tests and less data is available for that scenario. The fire was located in room 1 for all tests and is created with a diffusion flame burner using natural gas. Fires with heat release rates of 100 – 500 kW were used.

Scenarios C, room 1-3 is referred to as scenarios V3, room 1-3 in the database constructed by Bragason (1994). The original measured data were presented by Peacock et al. (1991).
6.1.4 Scenario D – Large-scale spaces

Scenario D involves the spread of smoke in large spaces. The floor area measured 720 m$^2$ and the height was 26.3 m. A plan and section of the room are presented in Figure 50.

![Figure 50: Plan and section of the room where the experiments were conducted (Yamana and Tanaka, 1985).](image)

Four tests were conducted in which the ventilation conditions were changed. The following conditions were evaluated: no smoke ventilation, natural ventilation and mechanical venting. The fire was assumed to be same in all tests and consisted of a methanol pool fire. The total rate of heat release was measured to be 1300 kW.

Measurements and predictions of interface height are available in the database. This scenario is referred to as scenario V4 in the database compiled by Bragason (1994). The original measured data were presented by Yamana and Tanaka (1985).
6.1.5 **Scenario E – Single room connected to a corridor**

Scenario E involves the spread of smoke from a small room to a corridor, which are connected by a doorway. The room in which the fire originated measures 4.2 x 3.3 m² and was 2.4 m high. The adjoining corridor had a floor area of 19 x 2.4 m² and the same ceiling height as the room. The doorway between the room and the corridor was 2 m high and 1 m wide. There was a 0.94 m high and 0.15 m wide opening to the outside at one end of the corridor. The construction was made of concrete, see Figure 51.

![Figure 51. Sketch of the geometry in scenario E.](image)

Data from only two tests are available for scenario E. The difference between the tests was the rate of heat release. The fuel was methane and the rate of heat release varied between 100 kW and 225 kW.

Measurements and predictions of interface height are given in the database. This scenario is referred to as scenario V8 in the database constructed by Bragason (1994). The original measured data were presented by Rockett et al. (1989).

**6.2 Modification of data**

In the report “Uncertainty in Smoke Transport Models” (Lundin, 1997) it was shown that quantification of the functional relation between the error and interface height predictions could be approached in the same way as for temperature predictions, i.e. with a linear functional relation between $\varepsilon$ and the predicted value. For this reason it was suggested that the input data in the analysis ought to be modified due to obvious errors in the predictions by the two-zone model in the beginning of a tests, to better be represented by a linear relation.

The error is partly due to the fact that the two-zone models neglect the transportation time for smoke from the base of the fire to the ceiling, and the smoke layer is assumed to form directly. If the dependence of the error and the predicted interface height are described by a linear relation, the impact of the assumption can be observed in the first part of the test, i.e. for high interface heights.

To reduce the effect of this assumption the range of data analysed in the tests was restricted. Data from the very first part of the tests was excluded. Only predictions from 70% of the ceiling height and below were used in the analysis. The effect of this
restriction of the data is illustrated in Figure 52, where the regression analysis for the original and modified data is presented. It has been observed that the effect is greatest in scenarios with high ceiling heights. The correction is therefore not necessary in scenario E, for example.

Since the interface descends during the fire the data point in the top-right corner of the figure above represents the first data point in the test, the data point to the left of that point will be the second etc. If the difference between the first and second data points is studied it can be seen that the predicted height has decreased while the measured height is unchanged. This difference between predictions and measurements is due to the fact that the smoke transport time from the fire to the ceiling is neglected in the model. If the data points where the smoke filling process is not indicated by the measurements are excluded from the analysis, the linear model describes the data much better, according to the solid line in Figure 52.

The data presented in this chapter is used to exemplify the statistical analysis model presented in Chapter 5. It was outside the scope of this project to perform full-scale tests, to derive parameters for scenarios of direct practical application. Data from several experiments have been published, the information included is often insufficient to be able to make model predictions of the experiments. There also seems to be a feeling that experimental results are competitive resources and thus regarded as confidential major organisations and companies. Hopefully, this trend will change and the attitude towards co-operation between organisations and companies will improve.
7 Results

The results of the quantitative analysis of the error in smoke transport model predictions are presented in this chapter. The analysis is based on the statistical method presented in Chapter 5, which is based on measurements and predictions described in Chapter 6. Two following two different types of quantitative analysis have been conducted:

- multi-scenario analysis of predictions from a single smoke transport model,
- single-scenario analysis of predictions from four different smoke transport models.

How can the knowledge of the quantitative model error in model predictions, derived in this study, be used? The applications for smoke transport models stretch from verification of a design criterion in a sub-scenario to a complete risk analysis for a building. All engineering applications have in common that when quantitative tools are introduced instead of prescriptive regulations, the uncertainty must be assessed. The assessment can vary widely; from the evaluation of predictive capability to an uncertainty analysis. In the following sections examples are given of how the results from the statistical model analysis presented in this dissertation can be used. The main objective of this study was to develop a methodology quantifying the model error and model uncertainty in the model output from smoke transport models, and to derive a method for the use of this information in engineering applications. The results are based on limited data, so the results presented in this section serve only as examples of how the methodology can be used. The following results are presented and discussed in five sections.

- Results of the analysis of model output in terms of temperature and interface height predictions for a whole scenario.
- How to take the model error into account in future predictions, based on the quantitative analysis.
- How to evaluate the definition of scenarios, i.e. the difference between the tests included.
- Qualitative evaluation of the predictive capability.
- Quantitative evaluation of the predicted capability.
- Comparison of results with those from other statistical analysis methods, applied to the same data.
- Comparison of the predictive capability of different smoke transport models.
- Concept of uncertainty analysis with uncertain model output.
- The use in fire safety engineering applications.
7.1 Analysis of model output from smoke transport models

The error in a model prediction is assumed to consist of a bias, i.e. a systematic part, and a random part referred to as the uncertainty or variability in the model prediction. The bias has been found to be dependent on the size of the predicted value, while the uncertain part is assumed to be independent of the predicted variable. The parameters in the linear (functional) relation between the prediction and the size of the error ($\varepsilon$) is determined in the quantitative analysis. The random part of the error is divided into uncertainty within a single test and the uncertainty between different tests in the same scenario. The quantitative information on the model error forms the basis in the adjustment model that is used to take the model error in the original model prediction into account.

The computer program Matlab (1994) was used to perform the analysis with the statistical model presented in Chapter 5. The input-file for Matlab is presented in Appendix 1. The model output analysed in this report consists of the temperature in the smoke layer and the smoke-free height from floor level, i.e. the interface height. The simulation results from Matlab are presented in Appendix C and D and are summarised in this section. The measured and predicted data from scenario 3, presented in Chapter 6, are used to exemplify the analysis results in greater detail.

7.1.1 Model error in temperature predictions

Example of the output from Matlab calculations are presented in Figure 53 and Figure 54. The scenario used in the analysis consisted of three rooms, including a corridor, and is described in detail in Chapter 6. The model output analysed is the temperature predictions in the room next to the fire room. The factors varied between the single tests in the scenario were the heat release rate and the ventilation to the third room.

![Figure 53. Data used for statistical analysis of the model error for scenario C.](image)

![Figure 54. The 95% prediction interval for estimate of measured temperature for scenario C.](image)

In Figure 53 the data points from the different tests are presented together with the regression line. Scenario C, room 2, in Figure 53 consists of six different tests and the tests can be distinguished by the different symbols in the figure.
The regression line represents the linear relation between the predicted and measured temperature, where the y-values measure the predicted temperature ($T_p$), represented on the x-axis, adjusted for the mean bias between measured and predicted temperature. To estimate $T_{m}$ based on a prediction $T_p$ the error must be taken into account. Adjustment of the systematic bias in the prediction is not sufficient. The random part of the error must also be taken into account, to derive an appropriate value. The random error can be visualised as the scatter from the individual data points around the regression line in Figure 53.

The uncertainty in the estimate of the measured temperature is represented by a 95% prediction interval. The prediction interval expresses the uncertainty in $T_{adj}$, which is the estimate of $T_m$ based on the model prediction ($T_p$). The uncertainty originates from uncertainty within tests, $\sigma_e^*$, and from differences between the tests $\sigma_{a}^*$. A prediction interval should not be mistaken for a confidence interval. A prediction interval describes the uncertainty in a single estimate of the variable represented on the y-axis, based on a single value from the x-axis. An example is the uncertainty interval for $T_{adj}$ based on a model prediction $T_{p}$, in Figure 54. A confidence interval describes the uncertainty in the regression line (Walpole and Myers, 1993). Both uncertainties originate from the variation in the sample used in the regression analysis.

The result of the statistical analysis of the data in a scenario is the quantified functional relation between the predicted and measured data, including the uncertainty in this relation. The result is incorporated into an adjustment model, see Eq. [7.1], which adjusts a model prediction to take account of the model error. The quantitative knowledge of the model error is only valid for the type of scenario for which the functional relation has been derived.

The concept for the adjustment model is the same for all the scenarios analysed in this work, but the model parameters $a^*$, $b^*$ and $e^*$ are specific for each scenario. To simplify the notations sub-sequentially $\alpha$, $\beta$, $\epsilon$, etc. will be used instead of their corresponding notations for the estimated values. Eq [7.1] express the distribution of the uncertainty interval presented in Figure 54, as a function of the predicted variable.

$$T_{adj} = \alpha + \beta \cdot T_p + \epsilon$$ [7.1]

$T_p$ can be any model prediction within the range limited by the definition of the scenario. $\alpha$ and $\epsilon$ are stochastic variables expressed as $N(\mu_a, \sigma_a)$ and $N(0, \sigma_\epsilon)$ and $\beta$ is a constant. The last three parameters are dependent on the scenario. The parameters for the adjustment model for the different scenarios analysed are presented in Table 6. A similar summary is presented for the adjustment of interface height predictions in Table 9. The temperature was predicted by the smoke transport model CFAST 2.0.
Table 6. Parameters for the adjustment model for temperature predictions for CFAST 2.0.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\mu_a$ [°C]</th>
<th>$\beta$</th>
<th>$\sigma_a$ [°C]</th>
<th>$\sigma_c$ [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.1</td>
<td>0.59</td>
<td>3.2</td>
<td>3.6</td>
</tr>
<tr>
<td>B</td>
<td>-1.8</td>
<td>0.86</td>
<td>1.8</td>
<td>5.9</td>
</tr>
<tr>
<td>C, room 1</td>
<td>158</td>
<td>0.45</td>
<td>72</td>
<td>17</td>
</tr>
<tr>
<td>C, room 2</td>
<td>36</td>
<td>0.55</td>
<td>5.7</td>
<td>5.8</td>
</tr>
<tr>
<td>C, room 3</td>
<td>12</td>
<td>0.57</td>
<td>0.33</td>
<td>1.7</td>
</tr>
</tbody>
</table>

The results presented in Table 6 show that the regression parameters differ between the scenarios. This indicates that the model error is sensitive to the type of scenario layout, as suspected. There is also an obvious difference between the types of uncertainty that dominate in the different scenarios, both in size and importance. The parameters $\mu_a$ and $\beta$ give an indication that there is a significant systematic error for all the scenarios. The predictive capability of the model can be seen as questionable if no account is taken to the error in the predictions.

In scenario C, room 3, the uncertainties are very small compared with scenario C, room 2. The uncertainty due to variation between tests is high for scenario C, room 2. This indicates that the tests for that particular scenario ought to be chosen in another way or another model should be used to quantify the error and variation in the error, to reduce the uncertainty.

If the statistical model is compared with that used by Lundin (1997), the parameter $\beta$ can not be interpreted in the same way. In the improved statistical model used in this work $\beta$ is a proportional coefficient representing part of the bias, i.e. the systematic error. In the earlier model, where the regression line was forced through zero, the parameter $\beta$ represented the total systematic error which could be interpreted as the mean over- or underprediction.

Model predictions adjusted according to the adjustment model developed in this study can be presented in two ways:

- $T_{\text{adj}}$ expressed as a stochastic variable,
- $T_{\text{adj}}$ expressed as a conservative value.

The model output and the uncertainty in the output can be merged together to give a stochastic variable according to Eq. [7.1] and can be used in a probabilistic uncertainty analysis where the model parameters are treated as stochastic variables. The adjusted model prediction is then described as an uncertain variable similar to the model described in Section 4.1.4. If model predictions from a whole fire test are adjusted, the output will be a time-dependent stochastic variable, according to Section 4.1.5. By treating the model output as a stochastic variable, the uncertainty can be included in the output and it is possible to visualise the uncertainty in the output.
It is suitable to use a conservative value if the model prediction from the smoke transport model is to be expressed as a deterministic value, e.g. if the prediction is used as input in a deterministic design equation or in a comparison with deterministic performance criteria. Eq. [7.2] can be used to derive a conservative estimate, if high temperature is assumed to be hazardous. The conservative value is defined as the upper percentile in the 95% prediction interval.

\[ T_{adj} = f(T_p) = \mu + \beta \cdot T_p + 1.96\sqrt{\sigma_a^2 + \sigma_e^2} \]  

[7.2]

If a low temperature represents a hazard, for example in sprinkler activation calculations, the lower quantile can be used instead, see Eq. [7.3].

\[ T_{adj} = f(T_p) = \mu + \beta \cdot T_p - 1.96\sqrt{\sigma_a^2 + \sigma_e^2} \]  

[7.3]

Eqs [7.2] and [7.3] represent the upper and lower percentiles in the prediction interval presented in Figure 54.

If no consideration needs to be taken of the relation between the two main types of uncertainties separately in the analysis, a simplified form of Eq. [7.2] can be used, see Eq. [7.4]. This is convenient in many fire safety engineering applications, where models are used. The only real need to separate the uncertainties is when the model and the scenario are evaluated made. When a proper scenario has been found and the parameters determined by the statistical analysis, the parameters can be presented according to Table 7 to make it easier for the user.

\[ T_{adj} = f(T_p) = \beta \cdot T_p + (\mu + 1.96\sqrt{\sigma_a^2 + \sigma_e^2}) = \beta \cdot T_p + U_{adj} \]  

[7.4]

Table 7 simplifies the use of the results from the analysis, since the amount of information is reduced. The simplification is traded against the possibility of changing the percentile of the prediction interval used and of separating random and systematic errors.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\beta$</th>
<th>$U_{adj}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.59</td>
<td>11</td>
</tr>
<tr>
<td>B</td>
<td>0.86</td>
<td>10</td>
</tr>
<tr>
<td>C, room 1</td>
<td>0.45</td>
<td>303</td>
</tr>
<tr>
<td>C, room 2</td>
<td>0.55</td>
<td>52</td>
</tr>
<tr>
<td>C, room 3</td>
<td>0.57</td>
<td>15</td>
</tr>
</tbody>
</table>

An alternative to using Eq. [7.4] to adjust a model prediction is to use a diagram similar to Figure 54, for the scenario. The diagrams for the scenarios in Table 7 are presented in Appendix C.

When the model output is represented by a conservative value, the uncertainty in the result can not explicitly be analysed or presented. A conservative value is a value that is...
considered to be “on the safe side”. Which way is the most appropriate to take the uncertainty into account depends on the situation in which the model output is used and the requirements on the quantitative assessment, see Section 2.3.

It can be seen that there is great uncertainty in the model predictions for scenario C, room 1. When the data were analysed it was discovered that one of the tests differed substantially from others in this scenario.

In Table 8 the results for scenario C, room 1, are compared with the results obtained when the deviating data is removed. A change in uncertainty is noted but it must also be recognised that the range of predictions which can be adjusted has been reduced. The uncertainty is still high, but the example shows the impact of a re-definition of the tests included in the scenario. The reason for the increment in $\sigma_e$ is the increased statistical uncertainty in the estimate, due to a reduced number of data points, i.e. sample size.

Table 8. Comparison of parameters for the adjustment model when the scenario is changed.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\mu_a$ [$^\circ$C]</th>
<th>$\beta$</th>
<th>$\sigma_a$ [$^\circ$C]</th>
<th>$\sigma_e$ [$^\circ$C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C, room 1</td>
<td>158</td>
<td>0.45</td>
<td>72</td>
<td>17</td>
</tr>
<tr>
<td>C, room 1 (one test removed)</td>
<td>130</td>
<td>0.45</td>
<td>25</td>
<td>19</td>
</tr>
</tbody>
</table>

7.1.2 Model error in interface height predictions

The results of the simulations of the interface height predictions are presented in a similar way to those of the temperature predictions in the section above. The analysis of the prediction of smoke-free height, i.e. interface height, in scenario A is exemplified in Figure 55 and Figure 56.

![Figure 55. Data used for statistical analysis of the model error for scenario A.](image1)

![Figure 56. The 95% prediction interval for the measured interface height for scenario A.](image2)

The same type of adjustment made for the model error in the temperature predictions is made for predictions of the interface height, see Eq. [7.5].

$$z_{adj} = \alpha + \beta \cdot z_p + \varepsilon$$

[7.5]
A low smoke-free height is considered hazardous in most applications, so the lower quantile of the 95% prediction interval is used, see Eq. [7.6].

\[
z_{adj} = f(z_p) = \mu_a + \beta \cdot z_p - 1.96 \sqrt{s^2_a + s^2_e}
\]  

[7.6]

The parameters for the adjustment model for the scenarios analysed are presented in Table 9.

**Table 9. Parameters for the adjustment model for interface height predictions by CFAST 2.0.**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>(\mu_a) [m]</th>
<th>(\beta)</th>
<th>(\sigma_a) [m]</th>
<th>(\sigma_e) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.57</td>
<td>1.7</td>
<td>0.12</td>
<td>0.25</td>
</tr>
<tr>
<td>B</td>
<td>-2.1</td>
<td>3.4</td>
<td>0.94</td>
<td>0.28</td>
</tr>
<tr>
<td>D</td>
<td>-5.7</td>
<td>2.6</td>
<td>3.0</td>
<td>0.46</td>
</tr>
<tr>
<td>E</td>
<td>-1.0</td>
<td>1.9</td>
<td>0.2</td>
<td>0.036</td>
</tr>
</tbody>
</table>

The conclusions which can be drawn from the results presented in Table 9 are similar to the results from the analysis of error in the temperature predictions. The regression parameters differ between the different scenarios. There is also an obvious difference between the type of uncertainty dominating in the different scenarios, both in size and importance.

A simplified expression, similar to Eq. [7.4], can be derived for the adjustment of the interface height if low interface height is considered hazardous, see Eq. [7.7].

\[
z_{adj} = f(z_p) = \beta \cdot z_p + (\mu_a - 1.96 \sqrt{s^2_a + s^2_e}) = \beta \cdot z_p + U_{adj}
\]  

[7.7]

The parameters are presented in Table 10.

**Table 10. Scenario-specific parameters for the simplified adjustment model in Eq.[7.7].**

<table>
<thead>
<tr>
<th>Scenario</th>
<th>(\beta)</th>
<th>(U_{adj}) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.7</td>
<td>-1.1</td>
</tr>
<tr>
<td>B</td>
<td>3.4</td>
<td>-4.0</td>
</tr>
<tr>
<td>D</td>
<td>2.6</td>
<td>-11.6</td>
</tr>
<tr>
<td>E</td>
<td>1.9</td>
<td>-1.4</td>
</tr>
</tbody>
</table>

An alternative to using Eq. [7.7] to adjust a model prediction is to use a diagram similar to Figure 54, for the scenario. The diagrams for the scenarios in Table 10 are presented in Appendix C.

According to the analysis results presented in Table 7 and Table 10 considerable corrections must be made to the predicted values to obtain conservative values that can be used in practical calculations. The corrections are of the magnitude where the physical background for the prediction almost disappears. There is an obvious need for continuous model development.
7.2 Qualitative evaluation of analysis results

Most of the analysis of the predictive capability of smoke transport models carried out so far has been of a qualitative nature. Statements like “two-zone models will conservatively estimate the fire environment” (Luo et al., 1997), i.e. overpredicting the temperature in the smoke layer, are commonly used. In many applications it is sufficient to know that the model prediction is on the safe side. Whether an over- or underprediction is on the safe side or not very much depends on the situation being analysed.

In this section the quantitative analysis results are used to derive a qualitative measure of the predictive capability of the smoke transport model CFAST 2.0, and the results are compared with previously presented evaluation results. A qualitative assessment of model error and model uncertainty is less time-consuming than a complete quantitative analysis. Whether or not a quantitative approach is needed depends on the type of application, see Chapter 2.

The solid lines in Figure 57 and Figure 58 represent the unbiased model prediction for a whole test, i.e. the predicted values equal the measured values exactly for the whole range of model predictions. If a model has such accuracy, adjustments of the predictions are of no relevance. The analysis of the model error in Chapter 5 showed that this is not the case for smoke transport models. The adjusted temperature is therefore likely to deviate from the line.

The dotted lines represent the 95% prediction interval for the adjusted value, for a specific scenario. One of the intervals is normally considered conservative, depending on how the hazard and the design criteria are defined. In the following section the adjusted variable is treated deterministically and referred to as the upper 95% percentile for temperature prediction and the lower percentile for interface height prediction to ensure a conservative estimate of the conditions, see Chapter 4.

If an adjusted prediction is under the solid line, the model overpredicts the real world conditions, and the model prediction has been reduced to compensate for the model error. If the original model output is an overprediction and used without taking the error into account the prediction in this case is on the safe side. A comparison of the
conservative adjusted value and the solid line is possible to make such a quantitative statement.

Figure 57 shows that model tends to overpredict the temperature for predictions over 100°C, i.e. the model predictions are higher than the corresponding adjusted value. Below 100°C the model underpredicts the conditions. This indicates that whether the model over- or underpredicts the conditions during a test can vary. It is therefore concluded that a general qualitative statement as to whether the model over- or underpredicts the conditions would not be appropriate for the whole fire test.

Figure 58 shows that the same conclusion is valid for interface height predictions from scenario E. For high interface height predictions, the model seems to underpredict the height of the smoke layer. For lower interface heights the predictions seems to be overestimated. The conditions are often considered to be hazardous when the smoke layer descends, i.e. the interface height is low. If the model overpredicts the interface height, critical conditions will occur earlier than the model prediction indicates. If the model predictions are not adjusted they can be unconservative, i.e. not on the safe side.

According to Appendix C CFAST 2.0 seems to underpredict the temperature at the beginning of fire development and overpredict the temperature for high temperatures in scenarios C. In the other scenarios, it seems to be only in the very first data point where the predicted temperature is lower than the measured. The interface height predictions at the beginning of fire development are underpredictions, while the model seems to overpredict the interface height at the end of the fire scenario. The same change in over- and underprediction is valid for all the interface height predictions. The point at which the predictive characteristics of the model changes varies between the different scenarios. Using Eqs [7.4] and [7.7] a quantitative measure of the over- or underprediction can be derived.

As far as temperature predictions are concerned the quantitative analysis gives the same results as earlier published qualitative analysis results, i.e. when the temperature is high the model overpredicts the temperature.

### 7.3 Quantitative evaluation of analysis results

In Chapter 5 it was shown that the size of the uncertainty interval for the adjusted prediction differs, depending on the how the functional relation between the predicted value and the model error is determined. The functional relation makes it possible to express the model error in quantitative terms and to take the error into account when the predictions are adjusted.

Even if qualitative measures of smoke transport models have dominated traditionally, the demand is growing for quantitative tools to assess the uncertainty. Calculation results are being used more often for actual design and not only to check if conditions are fulfilled or not. The need for quantitative assessment of the uncertainty is more and more obvious.

The adjustment model in this study is based on information on the systematic and random errors in the prediction of a specific scenario. How the adjustment model is
expressed depends on what assumptions are made about the relation between measured and predicted values when the statistical analysis model is derived. Since there is no standard covering the quantification of model uncertainty in smoke transport models, a number of different methods are likely to developed.

The parameters $\alpha$ and $\beta$ expressing the functional relation, see Eq. [7.1], are a weak base for comparison of different statistical models. It is difficult to get an idea of how the impact on the adjusted value depends on the parameters, and the way in which the functional relation is expressed might differ. An example is the statistical model used in this report and the model used in “Uncertainty in Smoke Transport Models” (Lundin, 1997), see Section 7.1.1.

A simple result to interpret and use as a measure of the predictive capability or effect of the adjustment in a comparison would be ideal. For a specific predicted value in a scenario a measure that can be used is quantified according to Eq. [7.8].

$$\frac{T_p - T_{adj}}{T_p}$$  

[7.8]

One way to approach the examples above is to analyse and compare the ratio for two adjustments made by different statistical models or different smoke transport models. A positive value of the ratio in Eq. [7.8] can be interpreted as how many percent the model overpredicts a conservative value, and a negative value how many percent the model underpredicts a conservative value.

This type of comparison is not suitable for drawing general conclusions of the predictive capability of the model or the effect of the adjustment model, since the ratio will vary within the range of the model output in a test. The ratio in Eq. [7.8] will vary during a test and the result: “the overprediction by the model within a test varies of between 10-20%” would be difficult to interpret and not of much use. What value would then be appropriate? It is impossible to describe the impact of the adjustments with a single measure for a whole test. In many situations simplicity is gained at the expense of accuracy and information. Earlier in this dissertation it has been concluded that the predictive capability varies during a test, and that the whole range of a test must be included for the adjustment model to be used in practice.

The idea of using multiple tests and the whole range of tests when defining a scenario was to create an adjustment model that is applicable to a large range of predictions. Studying how the quotient in Eq. [7.8] varies during a fire test is therefore not recommended, since this is not a useful way to measure the effect of the adjustment. This comparison is likely to be more confusing than clarifying.

The difference between statistical models can be observed as a difference in the width of the uncertainty interval and the position of the adjusted conservative value. Since the whole test is of interest it is suitable to use diagrams to illustrate the results from the adjustment models, instead of the uncertainty intervals at some selected points in the range of model output in a test.
7.3.1 Comparison of statistical models

One way to visualise the difference between the two statistical models discussed in this dissertation is to plot the uncertainty interval for the adjusted predictions, for the range of model output defined for a scenario. The scenario used to exemplify the difference is C, room 2, see Figure 59. The figure clearly shows that there is a difference in uncertainty in the adjusted value from the two different statistical models. There is a much wider uncertainty interval when the model based on a simpler statistical analysis is used to adjust the model prediction.

Another way of demonstrating the difference in the results is to plot the original model prediction in a time-temperature graph, together with the adjusted predictions from both statistical models. In Figure 60, predictions from a test from scenario C, room 2 and the uncertainty interval for the value adjusted with the statistical model described in Chapter 5 and the model presented by Lundin (1997) are shown. The comparison clearly shows the same result as in Figure 59. If a conservative value is derived from each statistical model the difference is of the order of 100°C for adjustments of a prediction of 250°C.

The difference between the statistical models results in differences in the accuracy of the adjustments. The main difference between the statistical model presented in this dissertation and the model presented in Lundin (1997) is in how the functional relation within a single test is quantified. In the earlier model the regression line was forced through zero. If the model error is analysed from the physics perspective, additional conditions such as a fixed intercept is a natural part. The model error is likely to be very small at the initial temperature (t = 0). The analysis undertaken in this report is based on a statistical analysis, where the objective is to describe the data in the scenario as accurately as possible. No particular account is taken of a single data point, and therefore the regression line is not forced through zero, or through the initial condition, T(t = 0) and the adjustment of each prediction will be uncertain, even if a certain prediction is known to be exact. As a result of the improved, but more complex statistical method, the random part of the error has been decreased and a larger part of the error can be explained by a systematic error, which can be decreased.
7.3.2 Comparison between different smoke transport models

The model CFAST 2.0 is more or less a standard tool used for fire modelling. Although there are many other smoke transport models available, CFAST 2.0 is used and well-known by practising engineers all over the world. CFAST 2.0 is commonly used to predict conditions in fire development and has often been subject of analysis and comparison with other models (Hostikka et al., 1998; Mowrer and Stroup, 1998). The model was developed by the National Institute of Standards and Technology (NIST), USA, during many years. Continuous improvement and development has been undertaken. The model is now available in updated versions, where the algorithms have been improved and new features added. The model has also been released with a simpler user interface and is called FASTLite 1.0. The most updated release is FAST 3.1, which is a part of the Hazard I tool package. In addition to these three models, FPETool 3.2 is also included in the analysis. FPETool 3.2 is also developed by NIST and uses some of the same algorithms as the other models.

To illustrate how the model improvements have affected the model error a multi-model analysis was carried out of a single scenario to exemplify the impact of the development. The bases for the simulation models are the same, originating from CFAST 2.0, but have certain differences. It is dangerous to develop new models from the same original model. The same errors might be included in the models and differences are not addressed and discussed, since they are not identified in model comparison exercises. The user community can also be mistaken in thinking that uncertainty analysis is not necessary since the model is well-known and widely accepted and therefore use the results uncritically.

Several different smoke transport models have been used to simulate the same test and the data are presented together with experimental measurements in Figure 61. It can be seen that there is a wide difference between the different models. To illustrate how a comparison can be made, all four models have been used to simulate all the tests in scenario A. The statistical analysis method was applied to the data and the results presented in Table 11.

**Table 11. Results from statistical analysis of 4 different smoke transport models.**

<table>
<thead>
<tr>
<th>Model</th>
<th>$\mu_a$ [°C]</th>
<th>$\beta$</th>
<th>$\sigma_a$ [°C]</th>
<th>$\sigma_x$ [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFAST 2.0</td>
<td>2.1</td>
<td>0.59</td>
<td>3.16</td>
<td>3.6</td>
</tr>
<tr>
<td>FAST 3.1 (Q_{rad} = 15%)</td>
<td>4.6</td>
<td>0.51</td>
<td>2.24</td>
<td>2.07</td>
</tr>
<tr>
<td>FAST 3.1 (Q_{rad} = 35%)</td>
<td>2.3</td>
<td>0.66</td>
<td>2.21</td>
<td>2.05</td>
</tr>
<tr>
<td>FASTLite 1.0</td>
<td>0.75</td>
<td>0.74</td>
<td>1.97</td>
<td>2.43</td>
</tr>
<tr>
<td>FPETool 3.2</td>
<td>-3.5</td>
<td>1.0</td>
<td>4.01</td>
<td>4.75</td>
</tr>
</tbody>
</table>

Any uncertainty in the input data is totally removed since the same input data were used for all the models. In FAST 3.1 it is possible to specify a value for the radiative fraction of the heat release and two different values were used. In all the other models the default value is 15% and can not be changed. The effect of the possibility of specifying this parameter can be seen in the parameters expressing the systematic error, but very little difference in the parameters representing the random part. A further discussion of the effect of the radiative fraction is found in Chapter 8.
Figure 61 shows that the model predictions differ between the smoke transport models. The graphical comparison shows that all models overpredict the temperature in the test used to exemplify the results, and that the magnitude differs. That indicates that the model error differs between the models.

![Figure 61. Experimental measurements and model predictions from several smoke transport models for test T2 in scenario A.](image1)

![Figure 62. The conservative adjustments of predictions when high temperature is hazardous and the original measurements from the test.](image2)

Using the results in Table 11, the conservative adjusted temperatures were calculated for all the models. In Figure 62, these are plotted together with the measurements from the tests that had been modelled. Since the models overpredict the temperature, the adjusted values are lower than the original predictions. It can be seen that the adjusted values also differ substantially between the models, which indicates that the random part of the error is different.

A limitation in this comparison is that the range of the series is not very large, but considerable differences in the temperature predicted for use in design calculations were observed. It is noted that the predictions of FPETool 3.2 differ from those of the other models. At the end of the investigated interval the agreement with measured values seems very good. The statistical analysis shows a high level of uncertainty in the error. It is suspected that a different functional relation between the error and the predicted value from the one used in the statistical model could reduce the uncertainty.

One of the models has the emission rate from the flame as an input parameter, i.e. the part of the rate of heat release that is transferred from the flame as radiation. In the other models this is a constant value that can not be changed. Two different rates were used. The default value of 15% in the other models was used, as well as 35% which is considered more appropriate by Afgan and Beer (1974). The simulation results show that this is an important parameter and a possible explanation of the overprediction of the models.

The scenario chosen was a one-room configuration so that all the models could be used. Unfortunately, the maximum measured temperature in this scenario is below 60°C, which is insufficient in many practical applications. The comparison should therefore to be seen as an exemplification of the possibility of comparing different models and to illustrate the differences between their predictive capability.
By comparing the results for FAST 3.1, the effect of changing the radiative fraction can be evaluated. As can be seen in Figure 59, the overprediction is higher when the radiative fraction is set to 15%, which is seen as an underestimation of the radiative fraction by Afgan and Beer (1974). If 35% is used instead, which is a more realistic value, the overprediction decreases. Since the default value for the other models is 15% this assumption can explain the overprediction to a certain extent. A small difference between the parameters in the adjustment model is noticed, for the predictions with 15% and 35% radiative fraction. The parameter $\mu_{ab}$, i.e. the intercept on the y-axis, differs by only a few degrees and the parameters expressing the uncertainty are also of the same magnitude. The parameter $\beta$, which reduces the predictions if the parameter is <1 differs and is, as expected, higher for the higher predictions, i.e. a radiative fraction of 15%. The adjusted conservative values for different radiative fractions differ very little according to Figure 60. The adjustment model takes the effect of the error due to the underestimated radiative fraction into account. Since the adjusted predictions are similar but the original predictions differ, the conclusion is drawn that the effect on the error due to difference in the radiative fraction is almost completely explained by a difference in the systematic error, in the example analysed.

### 7.4 Uncertainty analysis

An area of application in which knowledge of model uncertainty is a necessity is uncertainty analysis. Most engineering calculations are subject to various sources of uncertainty, see Chapter 2. The objective of uncertainty analysis may vary. To be able to calculate the total uncertainty in the final result is obviously one common goal of uncertainty analysis.

To be able to evaluate the impact of the uncertainty that is present in the model predictions after adjustment on the total uncertainty a quantitative analysis must be made. Expert judgement or engineering judgement alone is not sufficient. The relation between uncertainty in the input parameters, i.e. parametric uncertainty due to natural variation, and model uncertainty, i.e. uncertainty due to lack of knowledge, can be analysed if the uncertainties are propagated through the calculations. In the statistical analysis in this report emphasis has been placed on the model uncertainty. The uncertainty in the input parameters has not been considered in the analysis. In practical applications all the types of uncertainty presented in Chapter 2 must be considered to achieve an adequate analytical design result.

In this report a method of adjusting for the model error and taking the model uncertainty into account in a quantitative manner has been presented. By using Eq. [7.1] the adjusted variable can be expressed as a stochastic variable and information on the uncertainty is therefore included. The adjusted model output can then be used as input in another design application, which might be subject of uncertainty analysis. The method described above is appropriate as long as the model uncertainty in the model output is of interest. The problem becomes more complex if an uncertainty analysis of the model output is performed and the parameter uncertainty is to be included. The most commonly used smoke transport models are all deterministic. It is thus difficult to propagate the uncertainty through the models, according to the methods presented in Section 4.1.1-4.1.3.
To illustrate the effect of several uncertain input parameters in a deterministic model, a large number of simulations would be required to cover the range of all possible variable combinations. To combine this with the uncertainty in the model would be a very time consuming exercise. There are alternative approaches to propagating the uncertainty in the input parameters and the uncertainty in the model through the calculation. A more efficient way is to use a response surface to describe the smoke transport by a series of simple linear models, i.e. a meta-model or response surface (Iman and Helton, 1988). The response surface methodology has been applied for CFAST 2.0 calculations on numerous occasions (Magnusson et al., 1997; Frantzich, 1998). In these studies the time before critical conditions are reached, but it is possible to derive response models for other forms of output. The present work focuses on model uncertainty, and a detailed discussion of uncertainty analysis methods is outside the scope of this project. Magnusson (1997) presents an overview of available methods and the overview below is limited to exemplifying the use of the analysis results from this. The presentation of the response surface method is only schematic.

The purpose of the response surface methodology is to convert a complex model of the type presented in Chapter 4 into a simpler analytical model. The predictions of the smoke transport model, within a certain range, can then be expressed as an analytical function with a reduced number of input variables. The effect of uncertainty in these variables on the model output can then be analysed with standardised methods. For a well-defined range of input data, \( x_1 - x_n \), an approximate response surface would give the same model output as CFAST 2.0, see Eq. \([7.9]\).

\[
f_{\text{CFAST}2.0}(x_1, x_2, \ldots, x_n) = f_r(x_1, x_2, \ldots, x_n)
\]  \([7.9]\)

When some of the input data is uncertain, they can be expressed as stochastic variables, which can be modelled in the analytical expression expressed by Eq. \([7.9]\). This can be done with simple spreadsheets, using Monte-Carlo simulations. There are also analytical ways of propagating the uncertainty in the analytical expressions. Such methods are used in the \(\beta\)-index method (Thoft-Christensen and Baker, 1982), and computer programs based on this method (Strurel, 1995). The prediction can then be seen as the type of model described in Section 4.1.4 and exemplified in Eq. \([7.10]\).

\[
f_r(X_1, X_2, \ldots, X_n), X_i \in (\mu_i, \sigma_i)
\]  \([7.10]\)

If the model error is quantified according to the approach presented in this dissertation, the uncertainty in the model output can be expressed according to Eq. \([7.11]\). Both the parametric uncertainty and the model uncertainty are then taken into account.

\[
f_{\text{adj}}(T_p) = f_{\text{adj}}(f_r(X_1, X_2, \ldots, X_n)) = \alpha + \beta \cdot f_r(X_1, X_2, \ldots, X_n) + \varepsilon
\]  \([7.11]\)
7.5 Applications to engineering

The examples in the previous sections have mainly focused on the use of the results in the analysis and comparison of uncertainty. This section is concerned with the use of the results when the model is used in engineering applications. When model output from a smoke transport model is a part of a quantitative assessment, the error in the prediction has to be taken into account. How the adjustment is made depends on whether the model output is to be treated probabilistically or deterministically. If the analysis is probabilistic, the uncertainty can be represented by a stochastic variable, see Eq. [7.1]. The most commonly used form of the model output is deterministic. By taking the uncertainty in the model output into account a conservative value can be derived from the distribution representing the uncertainty, see Eq. [7.4]. An alternative to using Eq. [7.4] is to use the graphs presented in Appendix C, as is exemplified in Figure 54.

In many applications, the model output is compared with a critical level, e.g. analysis of critical exposure of components, property, life safety, fire reconstruction etc. A design criterion can be whether or not a fix level or limit is exceeded or the time before the limit is exceeded. In the following simplified example the design or analysis problem is: “If a fire occurs, will the temperature exceed the critical temperature?”. The verification is based on an analytical prediction of the conditions with a smoke transport model. To get as close as possible to reality, the model prediction used as a basis for the verification is adjusted for model error. The adjustment is based on analysis results of a scenario similar to that for which the prediction is made. A simple form of verification is presented in Eq. [7.12].

To simplify the verification of the conditions predicted by the smoke transport model, an adjustment can be made to the level representing the design criterion. The adjustment to the prediction is explicitly presented in Eq. [7.13]. The critical temperature can be adjusted according to Eq. [7.14] with an algebraic exercise. This approach makes it possible to check the model output directly against an adjusted critical level, where the model error for the current scenario is taken into account. If numerous calculations are performed for the same scenario and checked against the same criterion, this alternative adjustment method can save time.

\[
T_{critical} \geq T_{adj} \quad [7.12]
\]

\[
T_{critical} \geq \beta^* \cdot T_p + U_{adj} \quad [7.13]
\]

\[
\frac{T_{critical} - U_{adj}}{\beta^*} \geq T_p \quad [7.14]
\]

The effect of the model error on the time before critical conditions are reached can also be derived with the two approaches described above. The model output is often presented as a function of time. If the output is presented graphically, it may be more efficient to adjust the critical level according to the scenario instead of adjusting the model output. This adjustment is made to simplify and speed up the verification in a scenario where many computer iterations are necessary. It is important to realise that the
adjustment of the design criterion is only valid for the model for which the parameters $\beta$ and $U_{\text{adj}}$ are derived. The verification is also only valid for the specific scenarios, since the parameters are dependent on the scenario. The verification according to Eq. [7.14] will be conservative.

To illustrate the effect of the error in the predictions and to illustrate the two different methods of taking the error into account, the examples presented in Figure 63 and Figure 64 are used. In the example presented in Figure 63 and Figure 64 a test is taken from scenario C, room 2. The critical level used as criterion in the example is $200^\circ\text{C}$ in the smoke layer. In Figure 63 the predicted temperature is marked with filled squares and the dash-dotted line represents a critical temperature level, $T_{\text{crit}}$. Since the model overpredicts the temperature, a critical time, $t_{\text{crit}}$, derived without adjustment for model error, would be an underprediction of the real time to reach critical conditions. In Figure 63 the dashed line represents the prediction after conservative adjustment. If the adjusted values are used to derive the time to reach critical conditions, a longer time would result. Since the adjusted temperature predictions are conservative and high temperature is considered hazardous, the derived time will also be conservative.

If multiple computer simulations are to be performed, it can be time saving to adjust the critical level instead of the prediction according to Eq. [7.14]. The result is presented in Figure 64 and, not surprisingly, both methods have the same effect on the time to reach critical conditions. In the test illustrated in the figures above, neglecting the model error in the model prediction has a large impact on the time elapsed before critical conditions are reached. According to the figures the effect will increase with time.
7.5.1 A fictive example

To investigate the effect of an adjustment in an engineering application an adjustment of a prediction in a fictive example is illustrated below. The room subject to analysis is similar to the room in scenario A, which is a single room with floor area of 5.6 x 5.6 m² and height of 6.1 m. All surrounding materials are made of concrete. The rate of heat release is assumed to have a linear growth period of 40 seconds to a maximum steady-state level of 600 kW. The only opening from the room to the outside is a small opening with a width of 0.35 m and height of 0.25 m located at floor level.

A comparison is made between the predicted and adjusted conditions, in terms of temperature and smoke layer height. The comparison is made at the point in time when the smoke layer height specified in the Swedish building code (Boverket, 1998) as the design criterion for life safety is exceeded. This point in time is normally referred to as the time to reach critical conditions. The height of the smoke layer of floor level corresponding to the design criterion is calculated with Eq. [7.15].

\[
z_{crit} = 1.6 + 0.1 \cdot H = 1.6 + 0.62 = 2.22 m
\]  

According to the predictions of the smoke transport model CFAST 2.0 the smoke layer descends to a level of 2.2 m after 25 seconds. The temperature at this time is 58 °C. Conservative adjustments are calculated, based on the predictions and a comparison is presented in Table 12. The simulation results from CFAST 2.0 are presented in Appendix E.

Table 12. Results of comparison between predicted and adjusted simulation results.

<table>
<thead>
<tr>
<th></th>
<th>Model predictions</th>
<th>Adjusted model predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(z) at (t = 25) s</td>
<td>2.22 m</td>
<td>2.64 m</td>
</tr>
<tr>
<td>(T) at (t = 25) s</td>
<td>58 °C</td>
<td>45 °C</td>
</tr>
</tbody>
</table>

The model prediction used in the example is at the beginning of a test. The results in the table show that the model prediction is somewhat higher than the conservative adjusted value and that the smoke layer height is underpredicted.
An illustration of the difference in smoke layer heights is presented in Figure 65.

![Diagram showing difference in smoke layer heights]

Figure 65. Comparison between predicted and adjusted smoke layer height, when predicted height equals the design criterion.

If the data in Appendix C are used, the time to critical conditions for the adjusted predictions can be derived. Even if the thickness of the smoke layer differs by over half a metre, the critical conditions will be reached only 5 seconds. The critical level arises in a period when the descent of the smoke layer is very rapid, and therefore the effect of the model error is small for this specific scenario and this particular model. The effect of adjustment of the model prediction on the design will not be significant in this case. The situation may be totally different if a prediction is made when the rate of descent of the smoke layer has decreased and conditions are close to steady state. Another observation which can be made from the example is that the predictions of the smoke transport model are obviously close to the conservative adjusted value. The difference is only a few degrees or decimetres. In Figure 57 and Figure 58 it is shown that the conservative adjusted value and the model predictions are of the same magnitude at the beginning of a scenario. The effect would be greater if, for example, a higher temperature were to be investigated or if the temperature were closer to the steady-state temperature, i.e. not at the beginning of the test. A high overprediction of temperature can give a false indication of flashover or other damage.

The cables to safety components in nuclear power plans are often assumed to malfunction when the temperature exceeds 200°C (Gilles, 1998). If a risk analysis of the safety systems is made, the risk due to fire is likely to be part of the analysis. If the conditions are predicted with a smoke transport model without taking the model error into account the results might be very misleading. A prediction of 200°C with the model would indicate failure, but with adjustments according to scenario A, the temperature in a real fire would not be over 129°C, see Eq. [7.16].

\[
T_{adj} = 11 + 0.59 \cdot T_p \approx 11 + 0.59 \cdot 200 = 129°C
\]  

[7.16]

The model overpredicts the conditions by 55% and the difference is too high to be neglected in practical applications. In a design situation “a little bit extra safety” might not be much to argue about, but in an analysis situations it can be very misleading when
different alternatives are compared. The difference in damage or risk for component failure is most likely to be noticeable if the temperature differs this much.

Hostikka et al. (1998) performed an extensive code comparing exercise in the “Round Robin for Code Assessment, Scenario B” and concluded that an overall deviation in model prediction varied from ±20% to a factor 2. It must be noted that in this uncertainty interval a big portion comes from uncertainty associated with the modeller, and not only the model. The results found in this work therefore seem to agree with earlier work.
8 Discussion

8.1 The design process

The degree of fire protection prescribed in a design solution derived with a prescriptive method can sometimes seem very conservative, compared with the fire protection indicated by an analytical method. It is often argued that prescriptive solutions are “over-designed” and include unnecessary fire protection. One way for fire safety engineers to market an engineering approach to building contractors is the possibility of saving money by showing that fire protection requirements can be reduced.

When performance criteria are interpreted from design objectives, consideration must be given to the questions: “What happens if the models used or assumptions made are wrong?” “What if there is a failure in the analytical design?” and “What will the consequences be?”. It is likely that the severity of a design failure will vary depending on the system in question, i.e. the importance of safety systems differs. The fact that there is a probability of failure in the design process, due to many different reasons, can call for redundancy of certain sensitive protection systems, which is higher than a strictly technical analysis of the situation would suggest. In such an analysis the consideration of uncertainties is limited to the uncertainty in input data and in mathematical prediction models. Errors and failures due to human error are seldom included in quantitative design criteria. Redundancy in protection systems is one way to deal with the possibility of human error, model error or other types of failures that can occur in the design process. When a certain fire protection system is designed it can be difficult to understand the need for or justify “extra” protection. The extra protection in prescriptive solutions can depend on lack of knowledge, or requirement that the building perform despite non-perfect engineering assessment (USNRC, 1998), but it can also be an implicit consideration of the importance of the system in the total fire protection of the building. When analytical design solutions are compared with prescriptive solutions, the comparison can easily be misleading, due to the fact that insufficient consideration has been given to uncertainties.

When quantitative tools are introduced into the design and optimisation of fire protection in buildings it is important to be aware of their uncertainties. It must also be recognised that a practical design of fire protection must include the possibility of failure occurring during the design process. Hostikka et al. (1998) concluded that: “the user is the most critical link in the chain of using computer fire simulation models for fire safety engineering”.

The use of calculations and models as a design tool implies a responsibility to take the total uncertainty in the assessment into account explicitly, even if it is not always possible to assess these uncertainties with a quantitative approach. In the area of fire safety engineering there is a lack of tools to help the engineer fulfil this responsibility. The need for tools varies with the method and the situation at hand. The results obtained in this study shed some light on the uncertainties associated with smoke transport models. It is important to recognise that the error and uncertainty associated with the prediction model is just one link in the chain of the total uncertainty. There are very few guidelines on how to deal with model error and model uncertainty in a quantitative way.
One reason is that the knowledge in this area is limited. Discussions on uncertainties in the calculation methods and calculation tools frighten the practitioner, since it is likely that the credibility of the tools will be reduced. It will also be difficult to argue for more time to be able to include uncertainty analysis, since time costs money. There is continuous pressure on the engineer and it is possible that this will lead to short-cuts that will affect the uncertainty in the design solution and also knowledge of the uncertainties.

Without knowledge on the quality of the tool, it is difficult to make a decision as to which prediction model should be use. Answers to questions like, “How accurate is the model?” and “What are the limitations?” must be answered, in order to make a good choice.

In this study, a statistical model was developed, able to give quantitative information on the predictive capability of smoke transport models. The predictive capability is determined by studying the error in the model predictions and uses the error as a measure of the predictive capability. Based on the results of the statistical method, an adjustment model is derived to take the model error into account by adjusting the original model predictions.

8.2 Comments on the results

8.2.1 Qualitative assessment
A qualitative assessment of predictive capability can be useful, if there is no need to adjust the prediction for the model error. Qualitative information can be used to judge whether the predictions are on the safe side or not. It is, however, important to be aware of the limited applicability of qualitative assessment of uncertainty. No correction or adjustment for the systematic part of the model error can be made, and no comparison or analysis of the uncertainty in the original or adjusted prediction can be done if qualitative information is used.

Analysis of both temperature and interface height predictions have been carried out for scenarios A and B. The results indicate that the temperature is overpredicted and the height to the smoke layer from floor level is underpredicted. There errors result in conservative predictions of both these variables in most fire safety engineering applications, but whether or not a prediction is conservative depends on the definition of the problem being analysed.

In the statistical analysis of the data for scenario C, it was shown that the model predictions changed from being unconservative to conservative after a short period of time. Changes like this make it inconvenient to use qualitative assessment of the predictive capability for the whole range of prediction covered by a scenario. In a general approach to the predictive capability, which is the objective of this study, a quantitative assessment is suggested. If adjustments are made to the predictions according to the method presented here, conservative values will be derived.
8.2.2 Improvements to the adjustment model

In the analysis of the predictive capability of models for a number of scenarios a general conclusion is that improvements to the statistical model have led to an increase in the systematic part of the error and decrease in the random part. The accuracy of the adjustment is highly dependent on the relation between these two types of error. An accurate adjustment results in a small uncertainty interval for the adjusted value. Figure 66 illustrates the adjustment of a prediction with high systematic error and low random error. In Figure 67 the opposite situation is illustrated, where the random error is large and the systematic error small.

In the examples above, a single predicted value is studied. It is assumed that the original model output is an overprediction, i.e. higher than the real value. If the systematic part of the error is larger than the random part, the adjustment will result in a lower value than the original prediction, according to Figure 66.

The improved statistical analysis is more complex and demanding to carry out, but the analysis does not have to be performed repeatedly when suitable scenarios have been defined and an appropriate database derived. The increased complexity in the statistical analysis model therefore seems to be justified as the uncertainty in the adjustments is reduced and the use of the adjustment model simplified.

8.2.3 Analysis and evaluation of scenarios

The results show that the predictive capability varies between scenarios. The suspicion that the predictive capability varied for different types of scenarios has thus been proven, and the division of the data in the database into scenarios makes sense. The need to define scenarios is confirmed since a general approach would result in a great variation of the error. The statistical methodology can effectively be used to evaluate the appropriateness of the prediction model for a certain scenario. Alternative definitions of scenarios can also easily be evaluated. The different scenarios considered in this study does not nearly cover the range of scenarios that is needed in practical applications. To derive suitable types of scenario definitions for practical applications an analysis of the situations normally modelled is required. The range of each scenario must be limited by a maximum tolerable uncertainty in the adjustments, which is dependent on the tests used to represent the scenario in the analysis.

If a scenario has a high degree of uncertainty in the adjustments, it can be due to high random variation of the error within the tests or differences between the different tests.
The variation is strongly linked to the differences between the tests, i.e. the effect of test-specific parameters, and the range of predictions described by the tests.

It is difficult, in advance to determine the optimal balance between accuracy and the range of applicability of a scenario, i.e. how wide a range of different situations can be adjusted with the scenario-specific adjustment model. With the statistical model it is possible to evaluate the effect of the re-definition of a scenario. Other actions taken to widen the range of applicability of the adjustment model or increase the accuracy of the adjustments can also be evaluated. The following are suggestions that can be evaluated in a sensitivity analysis with the statistical model, if a definition or re-definition of a scenario is considered.

- Re-define the scenario by excluding tests with substantially different test-specific parameters. This action will decrease the range of applicability, since the variation of the test-specific parameters is decreased, and the range of values that can be adjusted by analysis of the particular scenario will thus be changed. This is exemplified in Table 8, Section 7.1.1, and is appropriate if the parameter $\sigma_a$ in the adjustment model is large.

- Add tests to the scenario which have similar test-specific parameters to the tests included in the original definition. If the same definition of the scenario is used and more tests are included in the type of scenario, the uncertainty in the estimates in the statistical model will be reduced.

- Reduce the range of the model output that is included in the tests. In this work the whole range of tests was used. It is noted that several of the tests have a relatively short range, measuring for example from room temperature to one or two hundred degrees. If additional tests are used with a wider range of output they can be included in the scenario directly and the range of predictions covered by the scenario increased. Alternatively only a certain range of the predictions from the tests are used so that the range covered by the scenario is indifferent. The predictions outside the range can be used to define a new scenario and analysed separately. The model FPETool 3.2 is observed to have much better predictive capability for higher values in the range of output investigated in scenario A, see Figure 61. To reduce the uncertainty, a re-definition of the range of predictions can be made. This is suitable if the parameter $\sigma_z$ is large.

- Develop the model expressing the functional relation between the error and the predicted value in a more accurate way. In the development described in this thesis the relation between the error and the predicted value is assumed to be linear. This seems to comply fairly well with the predictions of CFAST 2.0 for most scenarios, but improvements can be made and the assumption might not be appropriate for all smoke transport models. This is suitable if the parameter $\sigma_e$ is large.

- Increase the systematic part of the error by determining the variation between tests with a functional relation. The model error, $\varepsilon$, can be expressed as a function not only of the predicted value, but also of other parameters characteristic for a test, for example, rate of heat release. Such a study would require a greatly extended experimental database. This action will reduce the parameter $\sigma_{ae}$ in the adjustment model.
Reduce the uncertainty in measurements. Even if this uncertainty is not subject to analysis, it is included in the analysis results. A reduction in this source would reduce the final uncertainty.

In the results presented in Chapter 7 it was noted that the uncertainty for scenario C, room 1, is an order of magnitude greater than the other scenarios. This difference may be due to many factors. After examining the analysis results, it was found most likely that the high uncertainty is due to high variation of the test-specific parameters in the tests which defines the scenario. When the data are scrutinised it is noted that there is in particular one test in C, room 1, that deviates from the rest and causes much of the uncertainty. That test is characterised as a fast growing fire with higher temperatures than the rest of the tests. When the data from this particular test were removed from the data defining the scenario before the analysis was made, it is shown that the uncertainty is affected. In this particular case there were a substantial decrease in the random part of the error while the systematic error was almost unaffected.

8.2.4 Comparison of prediction models
All the two-zone models included in the single-scenario analysis tended to overpredict the temperature. It was also concluded that CFAST 2.0 underpredicts the interface height. Although no detailed analysis of assumptions and errors in the components and sub-models used in the smoke transport models was performed, an observation was made during the modelling which warrants comment.

The proportion of the heat release that is emitted as radiation, i.e. the radiative fraction, can be varied in FAST 3.1. A low radiative fraction means that a higher proportion of the energy released by the fire will heat up the smoke gases. All the models, except FAST 3.1, have a default value of 15%, which corresponds to a methanol fire. 15% is a low rate and is valid for flames with a low soot content. A common combustible material in sofas and other furniture where the fire is likely to originate in a building is polyurethane. Flames from polyurethane are very sooty and have a much higher radiative fraction than the default value in the models. The venting conditions and the type of fuel effect the radiative fraction considerably (Tewarson, 1995). In the comparison of the effect of different radiative fractions described in Chapter 7, the fuel used in the experiment was heptane. According to Afgan and Beer (1974) the radiation emission is around 35% for heptane. The tests where a radiative fraction of 35% was used should thus give better results, which was proven correct. The effect of the change in this parameter was considerable. It is a parameter that is sensitive to the model output and can not be changed in most models, which partly explains the general overprediction. The possibility of altering the value in FAST 3.1 is seen as a good model improvement.

The two-zone model is based on the conservation equations for energy, mass and momentum. If the predicted smoke temperature is higher than the real temperature, the density of the smoke layer will also be affected. Higher temperature will result in lower density. If the density is lower for the same amount of smoke the smoke takes up more space than smoke with a higher density. Assuming that the model for mass transport into the smoke layer is reasonably accurate, the interface will be lower in the predicted situation to compensate for the lower density. The error in the temperature will affect many sub-models in the smoke transport model. This has not been studied in detail.
Comparison between results from CFD models and CFAST 2.0 shows good agreement in mass flow calculations, which indicates that the mass balance is reasonably accurate (Holmstedt, 1998) for CFAST 2.0. The comparison with the CFD model is only an indicator and one should be careful to verify models by using other models.

The results indicate that the temperature predictions by CFAST 2.0 overestimate the actual temperature. If the temperature is overpredicted the interface height is underpredicted according to the conservation equations, which has also been observed in the analysis. Several qualitative evaluations of smoke transport models in limited ranges of model output have indicated that the temperatures is overpredicted, e.g. Luo et al. (1997). The effect of the radiative fraction, which is known not to be suitable for the type of fuel that would normally be present in a building fire, will be an overprediction of the temperature. The fact that the interface height is underpredicted is therefore not surprising and is consistent with the theory on which the models are based.

The results of the statistical analysis make it possible to analyse improvements to the models or the development of new features, i.e. the possibility of varying the radiative fraction. The statistical model is a tool with which an advanced sensitivity analysis can be performed, where the effect is obtained for the whole range of values. The fact that there is a difference in the predictive capability of the models indicates that although they originate from the same model, same development and progress have been made. The need for analysis of other models and model output is obvious, since smoke transport models predict many more variables. There are no specific guidelines on how to perform a quantitative analysis of smoke transport models, but the general principles presented in this dissertation can be followed. The lack of clear guidelines on how to quantify the predictive capability makes it difficult for the parties responsible for the decisions based on model predictions, to take the model error into account. Efforts being made through international collaboration like The International Council for Building Research Studies and Documentation – work group 14 (CIB/W14) and The Society of Fire Protection Engineers (SFPE) to develop such guidelines. Third party organisations must be able to assess the uncertainty in model predictions to be able to draw up design guidelines where conservative design is ensured. Both consulting firms and building authorities are using model predictions to evaluate safety and are also in need of more clear guidance.

**8.3 Approximations and limitations**

In the analysis, measured values are used to represent real values, which introduces error and uncertainty in the measurements as part of the quantified uncertainty. This is discussed in Chapter 4. Another source of error and uncertainty is the fact that all conditions and variables can not be controlled from test, to test or taken into account at all. Since a scenario consists of several tests, this uncertainty has an effect on the analysis results.

Part of the data used in the analysis consists of model predictions. In a recent code assessment exercise one of the objectives was to compare simulations performed by different modellers using the same model. Modelling was carried out by professional engineers, and the differences in the result were significant (Hostikka et al., 1998). In this work efforts have been made to reduce the error associated with the modeller by
using simple scenarios and a competent and well-educated engineer to perform the modelling exercises. However, it is recognised that the error and possible variation due to the modeller can not be totally excluded. It is concluded that in practical applications the error and uncertainty due to the prediction model used, is only part of the problem. There are several other sources of error which must also be dealt with, e.g. the uncertainty associated to the modeller and the input data. To be able to compare the uncertainties and evaluate them a common format is needed. The quantitative uncertainty analysis is one tool that can be used and the model uncertainty can, by using the method presented in this dissertation, be expressed in terms which can be assessed in such an analysis (IEC, 1995).

Prediction models will always be developed and improved, but it is not obvious that model development is the most efficient way to deal with model error in practical applications. Even if the model is improved there will still be errors and uncertainty in the output, which have to be taken into account in applications. The improvement of models will lead to higher accuracy but can also result in a more complex model. Even if the development of computational fluid dynamic (CFD) models is rapid and more models are becoming available commercially, the need for simpler tools will always exist. The engineer who performs the model simulation in a design project will probably have to consider factors such as simulation time, quality of results, available computational power, price of model, etc., when choosing which model to use. There is no point in selecting a model that is more complicated or complex than necessary. Zone models are still useful in many situations, even if they are not as accurate as CFD models. Running CFD models requires more knowledge in thermo- and fluid dynamics than that required to operate zone models. It is also more complicated and time-consuming to analyse the results from complex models, since they tend to include much more information. The two-zone model will certainly continue to be an engineering tool commonly used in fire safety engineering applications. Knowledge about the error and limitations in the prediction tools and knowledge about how to use the tools appropriately is necessary if the analytical approach is to be seen as an alternative approach to a prescriptive one. The analysis results show that there is a need for the continuous development of engineering models, but also that there are tools available to assess the uncertainty in the models.

To reduce the uncertainty in interface height predictions the range of data in the tests was restricted, as described in Chapter 5. Although no in-depth analysis of the analytical expressions in the prediction models was performed it is concluded that the predictions at the beginning of the test were not suitable for modelling with the assumptions used in the two-zone models analysed. The measured and predicted value in each data point is taken at the same time point. An alternative may be to adjust the timescale for the predicted results, before the data is divided into data points. A method that can be used if the time–temperature graphs have a similar shape, but are displaced on the timescale, is to define each data point as \( (T_{p_1}(t_1), T_{m_1}(t_1+\Delta t)) \). The parameter \( \Delta t \) is used to calibrate the curves before the analysis is performed, and is an alternative to the reduction of data from interface height predictions used in this work.

Unstable conditions at the beginning of measurements can be the reason why the predictions of smoke temperature change from underpredictions to overpredictions at the beginning of the tests in scenario C, room 1-3. The timing is important and the
accuracy in the data at this particular stage can be low, depending on the original purpose of the experiments.

The analysis was based on limited data and the conditions were relatively simple. A wider range of data should be analysed to provide a more extensive evaluation of the predictive capability of models. The scenarios in this report should be seen as examples and have been used to develop the statistical analysis model. Situations similar to those in which the models are used in practice should be analysed in more detail in order to derive appropriate adjustment factors. Many of the examples presented in the results, Chapter 7, are based on data that have been included in the analysis. The examples are used to illustrate how the results can be used for various purposes. For purposes such as the detailed validation of the statistical model and the adjustment model, tests that have not been used in the analysis must be used.

8.4 Future research

The use of quantitative predictions in fire safety engineering applications is increasing and thus the need for tools to assess the uncertainty introduced into the result due to modelling. The results presented in this dissertation constitute a step forward, but further work must be carried out in order to ensure that proper tools are developed to deal with model uncertainty in engineering applications.

Today, design guidelines do not give explicit recommendations on how to address model uncertainty in quantitative terms. If the results of quantitative engineering analysis are to be of use for decision-makers, appropriate assessment of uncertainties is needed. Since the level of complexity of engineering analysis varies, suitable tools must be devised to assess uncertainty for the different levels.

There is a great need to develop tools to deal with the uncertainty present in design applications when analytical methods are used. Such tools could be new standard methods, simplified verification methods, design values, engineering codes of practice or design calculation methods. Simplified tools would include the treatment of uncertainty, and provide conservative solutions. The need for uncertainty assessment by the engineer would be decreased considerably and thus by the time required to perform the engineering analysis.

Examples of actions which should be undertaken to enable uncertainty to be taken into account in calculations and to simplify the assessment of uncertainty are: derivation of conservative design values, conservative adjustment for model error, improvement of design equations etc. This can not be done without the analysis of the total uncertainty in the design results.

There is a definite need for international co-operation in this area. Engineers world-wide face the same problems and it would be inefficient for every company to perform its own evaluation of the models. There are different approaches to ensure that model predictions are carried out with sufficient accuracy. The first would be for regulators to evaluate and approve which models are acceptable. This is an old-fashioned way to tackle the problem. A more modern methodology would be for third-party organisations or governmental institutes to evaluate models and produce correction factors, etc. The
methodology presented in this report can provide one way of approaching the problem from that direction. Since the analytical method introduces uncertainty associated with the decision of the individual engineer, there is a need for national and international professional societies to address the issue explicitly and harmonise the required assessment of uncertainties.

There are many hand calculation and computer models available. In fire engineering guidelines there are no specific recommendations for certain models, which is good from the view point of flexibility, but there is a need for quality or accuracy approval of the models (Energistyrelsen, 1996; IEC, 1995). In several types of applications in other engineering disciplines, calculation tools must be approved and certified by a third-party organisation or even a regulatory committee, e.g. the off-shore industry. In fire safety engineering the whole responsibility is placed on the engineer. The following suggestions are made for further research which may lead to improvements in the quantitative evaluation of models.

- Construction of a more extensive database, to include scenarios similar to situations in which predictive tools are used in real world applications.
- Analysis of the predictive capability in scenarios that are commonly the subject of analysis in practical applications, with a clear description of the treatment and use of “adjustable” parameters. Such an analysis will make it possible to take the model error into account in future predictions.
- Detailed comparison of the uncertainty in different smoke transport models in order to reduce the impact of model selection on the final results in engineering assessments.
- The formulation of guidelines on how to take the uncertainty into account in fire safety design and fire risk analysis.
- A study of the important parameters that affect the model error to derive the appropriate scenarios and to determine the dependence of important parameters and the model error. This can effectively be carried out in co-operation with model developers.
9 Conclusions

The are numerous situations in which the model error and model uncertainty must be assessed. The results of this study can be used as a tool to evaluate and assess the uncertainty in the model output from smoke transport models. The results consist of the development of a statistical model to analyse the model output from smoke transport models, a multi-scenario analysis of the smoke transport model CFAST 2.0, and a single-scenario analysis of four different smoke transport models. The following final conclusions have been drawn, from the results and discussion chapters of this dissertation.

The results can be seen as examples of what can be achieved when the statistical method is used on appropriate data to assess uncertainty in fire safety engineering applications. The results of the statistical analysis provide quantitative information on the predictive capability of smoke transport models.

The quantitative output of the statistical analysis of a scenario constitute parameters for the adjustment model, resulting in an equation that can be used to make conservative adjustments of model predictions, within a specific scenario. The adjustments for the possible predictions within a scenario can also be presented in a graph which can be used derive conservative values, see Appendix C. The conservative values are then used in deterministic analysis or design applications. The adjustments can also be expressed as a distribution, which contains information on the uncertainty, and which can be used in probabilistic analysis, e.g. uncertainty analysis. A general adjustment factor can not be derived, due to the variation of the predictive capability in the different types of scenarios.

The improved statistical model describes the systematic error in the model predictions more suitably, in the sense of that the random error can therefore be reduced. Since it is possible to compensate for the systematic error, the reduced random error will result in a reduced uncertainty in the adjusted prediction; i.e. the model uncertainty will be reduced.

To derive parameters for the adjustment model so that the adjustments are applicable in practical engineering applications, an analysis must be conducted on measured and predicted data from full-scale tests.

It must be recognised that many of the models available are originally research tools, which can not be used uncritically. The uncertainty must be assessed bearing in mind the application for which the model is to be used, and not only the experimentally controlled conditions under which the model was developed.

The analysis of the model CFAST 2.0 showed that its predictive capability was questionable and that consideration must be taken of the model error. The error varies depending on the scenario. The temperature seems to be overpredicted by the model and the interface height underpredicted. In one of the scenarios the overprediction was not observed until the predicted temperature exceeded 100°C. It has not been investigated whether this deviation was due to poor calibration of time measurements between measurements and predictions, or is a characteristic of the model.
Although the models overpredict the smoke temperature, the general conclusion that the model output is conservative cannot be drawn. Whether or not the results are conservative depends on how the hazard being analysed is specified.

The limitation of the radiative fraction is cited as one of several reasons for this overprediction. This limitation is present in all the models except for FAST 3.2, where the parameter can be varied.

The comparison of smoke transport models showed that their predictive capabilities varied. If the model error is not taken into account, the choice of model will thus have considerable effect on safety and decision-making based on the model predictions.

The results show that the predictive capability of smoke transport models can be questioned but also that the error in the prediction can be reduced. The “corrected” predictions are shown to be better estimates of the real temperature than the original model predictions.

The uncertainty in model predictions must be viewed in relation to other uncertainties. The design fire, the design scenario and the modeller himself is likely to effect the end result. The different kinds of uncertainty in design calculation effect each other, so the efforts of assessment of uncertainties must be undertaken with an open perspective to avoid sub-optimisation. The quantification of model uncertainty as presented in this study is a good base for such a comparison.

Errors in model predictions require adjustment of the model output, which may lead to the loss of the physical basis of the prediction. This will effect general confidence in analytical methods and cause a step back in the development of fire safety engineering tools.

Quantitative assessment of model uncertainty is necessary to derive tools to assess uncertainty in practical applications, for example design values, design equations, design based on risk etc.
Acknowledgements

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References


References


Palisade Corporation. @RISK, User’s Guide. 1996.


Appendix A, Matlab file for the statistical model
Matlab file for the statistical model

This appendix presents the statistical model used to analyse the model error in the smoke transport models which have been analysed. The model is written as an m-file, which can be run with the computer program Matlab (1994).

**modeluncert.m**

Before the m-file modeluncert.m is run, the measured and predicted values must be defined as vectors. Let \((T_{p_{ij}}, T_{m_{ij}}), j = 1, 2, \ldots, n,\) be \(n\) ‘predictions’ and ‘measurements’ within a test \(i\) (for example \(T_p\) and \(T_m\) in temperature-time curves) and \(i = 1, 2, \ldots, L\) be \(L\) different tests in a scenario. In the earlier examples, the number of tests for each scenario was one, i.e. \(i = 1\). The number of tests included in the analysis is declared in the variable (q). The general definition of the vectors used in the m-file is then:

\[
\begin{align*}
p_i &= [T_{pi1} \ T_{pi2} \ .. \ T_{pin}] \\
m_i &= [T_{mi1} \ T_{mi2} \ .. \ T_{min}]
\end{align*}
\]

For a scenario which is the subject of analysis the vectors are defined as:

\[
\begin{align*}
p_1 &= [T_{p11} \ T_{p12} \ .. \ T_{p1n}] \\
m_1 &= [T_{m11} \ T_{m12} \ .. \ T_{m1n}]
\end{align*}
\]

\[
\begin{align*}
p_2 &= [T_{p21} \ T_{p22} \ .. \ T_{p2n}] \\
p_2 &= [T_{m21} \ T_{m22} \ .. \ T_{m2n}]
\end{align*}
\]

\[\ldots\]

\[
\begin{align*}
p_L &= [T_{pi1} \ T_{pi2} \ .. \ T_{pin}] \\
m_L &= [T_{mi1} \ T_{mi2} \ .. \ T_{min}]
\end{align*}
\]

\[
q = [L]
\]

An example of a definition of measured and predicted values for a scenario which consist of two tests are presented below:

\[
\begin{align*}
p1 &= [26 \ 46 \ 64 \ 76 \ 85 \ 91] \\
m1 &= [16 \ 23 \ 33 \ 40 \ 50 \ 57]
\end{align*}
\]

\[
\begin{align*}
p2 &= [29 \ 49 \ 67 \ 79 \ 88 \ 94] \\
m2 &= [18 \ 24 \ 34 \ 42 \ 53 \ 60]
\end{align*}
\]

\[
q = [2]
\]

In this appendix the numerical algorithm used to calculate the regression parameters is presented, but no routines for graphical plotting are included. References are given to equations in the main text.
Model Uncertainty in Fire Safety Engineering

% File start
for y=1:q,
    if y==1, x1=p1', y1=m1'; end;
    % The measured and predicted values
    if y==2, x2=p2', y2=m2'; end;
    % are defined in internal variables.
    if y==3, x3=p3', y3=m3'; end;
    % In this example a maximum of five
    if y==4, x4=p4', y4=m4'; end;
    % tests is used in the scenario.
    if y==5, x5=p5', y5=m5'; end;
end %y

sa2=2; ss2=2; Qyy=[]; Qxx=[]; Qxy=[];
% Initial definition of variables
mx=[]; my=[]; b=[]; yy=[]; xx=[]; n=[];
% used in the algorithm.

hold on
for i=1:q
    if i==1, y=y1; x=x1; end
    if i==2, y=y2; x=x2; end
    if i==3, y=y3; x=x3; end
    if i==4, y=y4; x=x4; end
    if i==5, y=y5; x=x5; end
    if i==6, y=y6; x=x6; end

    Qyyi=sum((y-mean(y)).^2);
    % Eqs [5.51] – [5.52]
    Qxxi=sum((x-mean(x)).^2);
    Qxyi=sum((y-mean(y)).*(x-mean(x)));
    mx=[mx, mean(x)];
    my=[my, mean(y)];
    Qyy=[Qyy, Qyyi];
    Qxx=[Qxx, Qxxi];
    Qxy=[Qxy, Qxyi];
    b=[b, Qxyi/Qxxi];

    % Eq. [5.53]
    yy=[yy; y];
    xx=[xx; x];
    n=[n, length(x)];
end %i

% Eq. [5.54]

hatbeta=sum((yy-mean(yy)).*(xx-mean(xx)))/sum((xx-mean(xx)).^2);
Q000=sum((yy-mean(yy)).^2)-sum((yy-mean(yy)).*(xx-mean(xx)))^2;
Q000=Q000/sum((xx-mean(xx)).^2);

c0=0.1;
% Definition of starting value for c0.
% Iterative calculation of Eq. [5.55].
for iter=1:10
    ci=c0*Qxx/(c0*sum(Qxx)+(1-c0)*sum((xx-mean(xx)).^2));
    betastar=ci.*b+(1-sum(ci))*hatbeta;
    hatai=my-betastar*mx;
    alphastar=mean(yy)-betastar*mean(xx);
    starai=c0*hatai+(1-c0)*alphastar;
    SUM1=sum(n.*(starai-alphastar).^2)/sum(n);
    Q00=Q000/sum((xx-mean(xx)).^2);
end %iter

for i=1:q
    if i==1, y=y1; x=x1; end
    if i==2, y=y2; x=x2; end
    if i==3, y=y3; x=x3; end
    if i==4, y=y4; x=x4; end
    if i==5, y=y5; x=x5; end
    if i==6, y=y6; x=x6; end
    Q00=Q00+sum((y-alphastar-betastar*x).^2);
QPE = QPE + sum((y - hatai(i) - betastar * x).^2);
end %i

QLOF = Q00 - QPE;
ss2 = (QPE + (1 - c0)^2 * QLOF) / sum(n);
disp(sprintf('Q00=' num2str(Q00) ' QLOF=' num2str(QLOF) ' QPE=' num2str(QPE)))
sa2 = c0^2 * QLOF / sum(n);
c00 = 3/4 + sqrt(9/16 - Q00 / (2 * QLOF));
if Q00 / (2 * QLOF) > 9/16, c00 = 3/4; end
c0 = sa2 / (sa2 + ss2);
dd = [ss2='num2str(ss2) ' sa2='num2str(sa2)];
dd = [dd, 'c0=' num2str(sa2 / (sa2 + ss2)) ' c00=' num2str(c00)];
disp(dd);
c0 = c00;
end %iter

hold on
LW = 3;
N = sum(n);
xx0 = [1:0.1:6];

% Print out important parameters.

dd = ['alphastar=' num2str(alphastar) ' betastar=' num2str(betastar)];
dd = [dd, ' ss2=' num2str(ss2) ' sa2=' num2str(sa2)];
disp(dd);
Appendix B, Nomenclature
### Nomenclature

In this appendix the nomenclature used in the development of the statistical analysis presented in Chapter 5 is given.

#### General statistical nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>quantile or percentile</td>
</tr>
<tr>
<td>( \mu_x )</td>
<td>mean value of ( x )</td>
</tr>
<tr>
<td>( \sigma_x )</td>
<td>standard deviation of ( x )</td>
</tr>
<tr>
<td>( f(x) )</td>
<td>a function of variable ( x )</td>
</tr>
<tr>
<td>( E(x) )</td>
<td>expectation value of variable ( x )</td>
</tr>
<tr>
<td>( I_{95%} )</td>
<td>95% prediction interval</td>
</tr>
<tr>
<td>( V(x) )</td>
<td>variance of variable ( x )</td>
</tr>
<tr>
<td>( x )</td>
<td>stochastic variable ( x )</td>
</tr>
<tr>
<td>( x_i )</td>
<td>a specific value of the variable</td>
</tr>
<tr>
<td>( (x)^* )</td>
<td>estimate of variable ( x )</td>
</tr>
<tr>
<td>( z_{95%} )</td>
<td>95% percentile</td>
</tr>
</tbody>
</table>

#### Model-specific nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>regression coefficient</td>
</tr>
<tr>
<td>( \beta )</td>
<td>regression coefficient</td>
</tr>
<tr>
<td>( \delta )</td>
<td>model uncertainty factor</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>model error</td>
</tr>
<tr>
<td>( \varepsilon_{\text{random}} )</td>
<td>random error</td>
</tr>
<tr>
<td>( \varepsilon_{\text{systematic}} )</td>
<td>systematic error</td>
</tr>
<tr>
<td>( \mu_\alpha )</td>
<td>mean value of ( \alpha )</td>
</tr>
<tr>
<td>( \sigma_\alpha )</td>
<td>standard deviation of ( \alpha )</td>
</tr>
<tr>
<td>( \sigma_\varepsilon )</td>
<td>standard deviation of ( \varepsilon )</td>
</tr>
<tr>
<td>( f_{\text{adj}}() )</td>
<td>adjustment model</td>
</tr>
<tr>
<td>( t )</td>
<td>time</td>
</tr>
<tr>
<td>( T )</td>
<td>temperature</td>
</tr>
<tr>
<td>( T_{\text{adj}} )</td>
<td>adjusted temperature</td>
</tr>
<tr>
<td>( T_m, T_{\text{measured}} )</td>
<td>measured temperature</td>
</tr>
<tr>
<td>( T_p, T_{\text{predicted}} )</td>
<td>predicted temperature</td>
</tr>
<tr>
<td>( T_{\text{real}} )</td>
<td>“real” temperature</td>
</tr>
<tr>
<td>( z )</td>
<td>interface height, smoke layer height</td>
</tr>
</tbody>
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Appendix C, Multi-scenario analysis
Multi-scenario analysis
This appendix contains the results from the quantitative analysis of temperature and interface height predictions by CFAST 2.0 for the scenarios presented in Chapter 6.

Temperature
Scenario A, Single enclosure

![Graph showing measured temperature plotted against predicted temperature, with regression line.]

Figure C1. Measured temperature plotted against predicted temperature, together with the regression line.

Figure C2. Mean estimate and 95% prediction interval for the adjusted predicted temperature.

Parameters calculated with the statistical model:

\[ \mu = 2.1 ^\circ C \quad \beta = 0.59 \quad \sigma_\alpha = 3.2 ^\circ C \quad \sigma_e = 3.6 ^\circ C \]

Temperature
Scenario B. Two rooms connected by doorway.

![Graph showing measured temperature plotted against predicted temperature, with regression line.]

Figure C3. Measured temperature plotted against predicted temperature, together with the regression line.

Figure C4. Mean estimate and 95% prediction interval for the adjusted predicted temperature.

Parameters calculated with the statistical model:

\[ \mu = -1.8 ^\circ C \quad \beta = 0.86 \quad \sigma_\alpha = 1.8 ^\circ C \quad \sigma_e = 5.9 ^\circ C \]
Temperature
Scenario C, room 1, Three rooms including a corridor

Parameters calculated with the statistical model:
\[ \mu_a = 158 \, ^\circ C \quad \beta = 0.45 \quad \sigma_a = 72 \, ^\circ C \quad \sigma_e = 17 \, ^\circ C \]

Temperature
Scenario C, room 2, Three rooms including a corridor

Parameters calculated with the statistical model:
\[ \mu_a = 36 \, ^\circ C \quad \beta = 0.55 \quad \sigma_a = 5.7 \, ^\circ C \quad \sigma_e = 5.8 \, ^\circ C \]
Temperature
Scenario C, room 3, Three rooms including a corridor

Parameters calculated with the statistical model:
\[ \mu_\alpha = 12 \, ^\circ\text{C} \quad \beta = 0.57 \quad \sigma_\alpha = 0.33 \, ^\circ\text{C} \quad \sigma_\varepsilon = 1.7 \, ^\circ\text{C} \]
Interface height
Scenario A, Single enclosure

![Graph showing measured interface height plotted against predicted interface with regression line.](image1)

Figure C11. Measured interface height plotted against predicted interface. The regression line is also shown.

Parameters calculated with the statistical model:

\[ \mu_\alpha = -0.57 \text{ m} \quad \beta = 1.7 \quad \sigma_\alpha = 0.12 \text{ m} \quad \sigma_e = 0.25 \text{ m} \]

Interface height
Scenario B, Two rooms connected by doorway

![Graph showing measured interface height plotted against predicted interface with regression line.](image2)

Figure C13. Measured interface height plotted against predicted interface height. The regression line is also shown.

Parameters calculated with the statistical model:

\[ \mu_\alpha = -2.1 \text{ m} \quad \beta = 3.4 \quad \sigma_\alpha = 0.94 \text{ m} \quad \sigma_e = 0.28 \text{ m} \]
Interface height
Scenario D, Large-scale spaces

Figure C15. Measured interface height plotted against predicted interface height. The regression line is also shown.

Figure C16. Mean estimate and 95% prediction interval for the adjusted model prediction of the interface height.

Parameters calculated with the statistical model:
\[ \mu_a = -5.7 \text{ m} \quad \beta = 2.6 \quad \sigma_a = 3.0 \text{ m} \quad \sigma_e = 0.46 \text{ m} \]

Interface height
Scenario E, Single-room connected to a corridor

Figure C17. Measured interface height plotted against predicted interface height. The regression line is also shown.

Figure C18. Mean estimate and 95% prediction interval for the adjusted model prediction of the interface height.

Parameters calculated with the statistical model:
\[ \mu_a = -1.0 \text{ m} \quad \beta = 1.9 \quad \sigma_a = 0.2 \text{ m} \quad \sigma_e = 0.036 \text{ m} \]
Appendix D, Single-scenario analysis
**Single-scenario analysis**

This appendix contains the results from the quantitative analysis of the model error in predictions by CFAST 2.0, FAST 3.1, FASTLite 1.0 and FPETool 3.2 of scenario A.

**Temperature**

**CFAST 2.0**

![Figure D1. Measured temperature plotted against predicted temperature. The regression line is also shown.](image1)

![Figure D2. Mean estimate and 95% prediction interval for the adjusted predicted temperature.](image2)

Parameters calculated with the statistical model:

\[
\mu_a = 2.1 \, ^\circ\text{C} \quad \beta = 0.59 \quad \sigma_a = 3.16 \, ^\circ\text{C} \quad \sigma_e = 3.6 \, ^\circ\text{C}
\]

**Temperature**

**FAST 3.1 Qrad = 15%**

![Figure D3. Measured temperature plotted against predicted temperature. The regression line is also shown.](image3)

![Figure D4. Mean estimate and 95% prediction interval for the adjusted predicted temperature.](image4)

Parameters calculated with the statistical model:

\[
\mu_a = 4.6 \, ^\circ\text{C} \quad \beta = 0.51 \quad \sigma_a = 2.24 \, ^\circ\text{C} \quad \sigma_e = 2.07 \, ^\circ\text{C}
\]
Temperature
FAST 3.1 Qrad = 35%

Parameters calculated with the statistical model:

\[ \mu_\alpha = 2.3 \, ^\circ\text{C} \quad \beta = 0.66 \quad \sigma_\alpha = 2.21 \, ^\circ\text{C} \quad \sigma_\epsilon = 2.05 \, ^\circ\text{C} \]

Temperature
FASTLite 1.0

Parameters calculated with the statistical model:

\[ \mu_\alpha = 0.75 \, ^\circ\text{C} \quad \beta = 0.74 \quad \sigma_\alpha = 1.97 \, ^\circ\text{C} \quad \sigma_\epsilon = 2.43 \, ^\circ\text{C} \]
Temperature
FPETool 3.2

Figure D9. Measured temperature plotted against predicted temperature. The regression line is also shown.

Figure D10. Mean estimate and 95% prediction interval for the adjusted predicted temperature.

Parameters calculated with the statistical model:

$\mu_{\alpha} = -3.5 \, ^{\circ}\text{C} \quad \beta = 1.0 \quad \sigma_{\alpha} = 4.01 \, ^{\circ}\text{C} \quad \sigma_{e} = 4.75 \, ^{\circ}\text{C}$
Appendix E, Description of fictive example
Description of fictive example
This appendix presents the model input and model output for the simulation of the example presented in Chapter 7.

Model input:
One-room configuration: width = 5.62 m
length = 5.62 m
height = 6.15 m

Openings: width = 0.35 m
soffit = 0.3 m
sill = 0 m (floor level)

Heat release: steady-state of 560 kW after 40 seconds (linear growth)

Construction material: concrete

Model output:

Figure E1. Temperature prediction for a fictive example by CFAST 2.0.

Figure E2. Interface height prediction for a fictive example by CFAST 2.0.