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A Radial Basis Function Method for Approximating the Optimal Event-Based Sampling Policy

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INTRODUCTION

In networked control systems it is desirable to have efficient wireless communication (saving energy and bandwidth) while still ensuring good control performance. By abandoning periodic sampling, communication can be made more efficient by sampling and updating the control signal only "when required" based on the system’s behaviour. This is the concept of event-based control.

In this work we consider the classic LQG problem with an added penalty $\rho$ on the average sampling rate $f$.

AN EQUIVALENT SETUP

The optimal structure has the equivalent representation \[1\]:

\[
\begin{align*}
\text{Sensor \& Event-Generator} & \quad \text{Continuous LTI Plant} & \quad \text{Discrete Controller} \\
\text{Kalman Filter, } \hat{x}(t) & \quad \hat{y}(t) & \quad \hat{u}(t) \\
\text{Simulated LQR, } x_a & \quad \text{Sampling sets } x_a(t_j) = \hat{x}(t_j), \text{ and resets error } \hat{x} = x - x_a \text{ to zero.}
\end{align*}
\]

We search for a threshold in the $\hat{x}$-space, from which we reset $\hat{x}$ to zero and accrue the cost $\rho$. This is an optimal stopping problem.

The optimal policy is given by the relative value function $V(\hat{x})$ of the problem.

The optimal policy can be written as

Optimal Policy:
Sample when $V(\hat{x}) = 0$.

With the optimal controller structure known \[1\], the challenge is to find the optimal sampling policy. We extend the work in \[2\], and make the following contributions:

- Deriving an RBF method to approximate the value function and optimal sampling policy.
- Proving guaranteed existence and uniqueness of optimal RBF weights.
- Numerical validation of the proposed method.

RBF SOLUTION

1. RBF-Approximation:
\[ \hat{V} = \sum_{i=1}^{n} \alpha_i \phi(\hat{x} - \hat{x}_i) \]

2. Rescale PDE & insert $\hat{V}$:
\[
\begin{align*}
\text{min}\{\Gamma \alpha + \beta, -\Phi \alpha\} = 0, \\
\alpha = [\alpha_1, ..., \alpha_j, ..., \alpha_n]^T, \\
\Phi_{ij} = \phi(\hat{x}_i - \hat{x}_j). 
\end{align*}
\]

3. Reformulate as LCP:
\[
\begin{align*}
\Gamma \big( Mz + \beta \big) = 0, \\
z \geq 0, \quad (Mz + \beta) \geq 0 
\end{align*}
\]

4. Solve QP:
\[
\begin{align*}
\text{min } z^T (Mz + \beta), \\
\text{s.t. } z \geq 0, \quad (Mz + \beta) \geq 0 
\end{align*}
\]

5. RBF Weights:
\[ \alpha = -\Phi^{-1} z \]

THE OPTIMAL SAMPLING PROBLEM

Validation

Analytic solution for $A = 0 : V(\hat{x}) = -\frac{1}{4} \max(2\sqrt{\rho - \hat{x}^T P \hat{x}}, 0)^2$