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Analysis of adaptation in anteroposterior dynamics of human postural control

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Abstract

To quantify the postural adjustments over time, twelve normal subjects were investigated with posturography during vibration either toward the calf or the paraspinal neck muscles; with eyes open vs. eyes closed. The stimulus response adjustments over time were found to be almost identical for all test conditions, though the responses were smaller during eyes open conditions. To determine the response adjustments and thus the dynamics of postural control, a system identification methodology incorporating both step response and dynamic feedback components was developed. This methodology can be used to quantify both short-term stimulus response and postural adaptive adjustments when evaluating postural control performance. © 1998 Elsevier Science B.V.

Keywords: Postural control; Adaptation; Proprioception; Vision

1. Introduction:

The ability to maintain stability in stance and motion is important in many human activities. Since the human body is not statically stable, maintaining upright posture requires continuous action of tonal adjustments in the antigravity muscles [1]. Because of this, human postural control is very complex and must include functions for detection of movements as well as for evoking and controlling co-ordinate muscular responses. This can be viewed as a dynamic feedback control system, using the information from the visual, vestibular and somatosensory receptors [2,3] to modify the postural responses. A change in gain or intensity from one or more of the sensory systems might affect the postural control performance, evoking and modifying motor output at all levels from the spinal medulla to the cerebral cortex [3–8]. The ability to adjust the motor control mechanism is therefore fundamental in postural control.

The performance of postural control has been analysed in various situations, ranging from experiments reflecting the requirements of daily life to tests of the ability of specific receptor organs to perceive postural information [9–12]. The influence of specific sensory systems on postural control can be analysed by various methods; moving the supporting surface [4,13] or the visual surroundings [14–16], applying galvanic stimulus to the vestibular nerve [17] or by vibratory stimulation applied on the muscles [18–20]. The responses from the postural control system can be analysed from forces actuated by the feet on a force platform, quantified as sway velocity, torque variance or sway path [3,6,21]. EMG recordings of muscular activity or movement analysis systems such as Selspot™ have also been used to analyse postural control performance during motion and stance [3,6,21].

A recent development in posturography is the use of system identification to describe the properties of the postural control system. This identification procedure makes a stimulus response model that describes the relationship between induced disturbances and counter-active postural movements to maintain balance...
[19,22,23]. To assess dynamic feedback control, a stimulus that is well-defined in time and primarily affects the sensory input is needed [24,25]. To fulfill these requirements, vibration toward calf or neck muscles was selected as the stimulus in this study. Vibratory stimulation can be applied in a wide range of amplitudes and frequencies and also according to a pseudorandom binary sequence (PRBS) schedule used in system identification [18,25]. The vibration is known to activate the muscle spindles [20,26] in the same way as passive muscle stretching, causing a reflexive contraction of the muscle [27,28].

The aim of this investigation was to find out if the responses to stimuli adjust according to recognizable patterns in postural control and if these patterns are affected by visual and somatosensory information. For this purpose we developed a method, using system identification, to evaluate responses of perturbed stance containing both feedback and adaptive behaviour.

2. Material and methods

2.1. Subjects

Experiments were performed on ten test subjects and two control subjects (seven men and five women; mean age 34.8 years, range 25–45 years), all with previous experience of the test situation, though not within the preceding 6 months. The subjects had no history of vertigo, central nervous disease, or injury to the lower extremities. At the time of the investigation, no subject was on any form of medication or had consumed alcoholic beverages for at least 24 h.

2.2. Apparatus

Body sway was evoked by applying vibratory stimulation to the gastrocnemius muscles of both legs or to the paraspinal muscles of the neck, care being taken to avoid contact with bone. The vibrators, constructed as cylinders of 0.06 m length and 0.01 m in diameter, were held in place by straps around the calf muscles or by a collar with a strap around the neck. The vibratory effect was 850 mW (1.0 mm amplitude at 60 Hz). Forces and torques actuated by the feet were recorded with 6 d.f. by a force platform developed at the ENT Department [29]. Data was sampled at 10 Hz by a computer (486/25 COMPAQ) equipped with an AD converter (Analog Devices RTI-815).

2.3. Procedure

The subjects were instructed to stand erect but not at attention, their feet at an angle of $\approx 30^\circ$ open to the front. The subjects either focused on a mark on the wall at a distance of $\approx 1.5$ m, or had their eyes closed, as instructed. Spontaneous sway was recorded for 30 s of quiet stance, after which, for the test group, the vibrators were turned on/off according to a computer-controlled PRBS schedule with a pulse time of 0.8–6.4 s. Each test sequence including the 30 s without stimulation lasted 235 s. To verify stimulus-independent variations in measured torque, the two control subjects were not exposed to stimulation.

To minimize the effect of habituation, and to ensure the similarity of the test group with regard to the subject’s experience level before each test, the test order was fixed, starting with neck stimulation known to induce less disturbance than calf stimulation in normal subjects [18].

Four tests were conducted in the following order:
1. Neck vibration—eyes closed

The subjects were allowed to step down from the force platform and relax for 3 min between the tests.

The following parameters were evaluated for adaptive behaviour:
1. Variance of torque with and without prefiltering.
2. Empirical torque variance and torque rate variance.
3. Identification of ARMAX models between stimulus and torque.

2.3.1. Variance of torque with and without prefiltering

Anteroposterior body movements over time were quantified by calculating anteroposterior torque variance. Torque variance was also calculated after spectral separation into variance of low and high frequency content. Fast Fourier transformation (FFT) analysis of posture have shown significant differences mainly above 0.10 Hz between open and closed eyes test conditions [30]. This cut-off level was therefore selected for data analysis. A digital low pass filter of FIR [17] (finite duration impulse response) was used for spectral separation, the filter components being selected to avoid aliasing.

2.3.2. Empirical torque variance and torque rate variance

The measured torque can be separated into two components, static torque caused by body displacement from initial stance, and forces and torques caused by acceleration and retardation of body motions. Both components are considered when evaluating the torque according to the empirical torque variance formula (note the difference compared with the variance formula). The amount of body motion is reflected in the torque rate variance formula, evaluating the torque rate. The respective formulas are
\[ P_c = \frac{1}{T} \int_0^T \tau^2 \, dt \quad \text{and} \quad P_v = \frac{1}{T} \int_0^T \left( \frac{d\tau}{dt} \right)^2 \, dt \]

where \( \tau \) denotes anteroposterior torque around the ankle joint, Nm; \( T \) denotes measurement time, s; \( t \) denotes time, s; \( P_c \) denotes empirical torque variance (Nm\(^2\)); and \( P_v \) denotes torque rate variance (Nm/s\(^2\)).

### 2.3.3. Identification of ARMAX models between stimulus and torque response

Parametric estimation of the transfer function between input stimulus and output anteroposterior torque [19] was done with two parametric model estimation methods.

The first method, denoted ‘Feedback’, is based on the methodology published by Johansson and co-workers [19]. A third-order ARMAX model [24,25,32,33] is assessed to describe the feedback in the postural control system between input vibration stimulus and output torque responses.

The second method, denoted ‘Step and feedback’, uses an approach that combines the above feedback model with a step response model describing the phenomenon of slow changes in posture as a response to the onset of stimuli. The input signal to the step response model is a step at the onset of the stimuli sequence and the measured anteroposterior torque is viewed as the output. A feedback model is then assessed to describe the function between input stimulus, i.e. each vibratory pulse, and the remaining measured torque after removing the estimated step response component. Both the step response and feedback models were parametrically identified with a third-order ARMAX model, where the time delay between input and torque response was optimised by using the Akaike information criterion [25].

In both methods, the three normalised model parameters, i.e. stiffness, swiftness and damping [19] correspond to the parameters of PID (i.e. proportional, integrative and derivative) control used in system and control theory [24,25].

The contribution of the step response component was verified with three methods: group validation, variance ratio and residual analysis [25].

Group validation was done by testing the error mean quotient, i.e. the error mean (the mean remaining torque after removing the estimated step response component) divided by the mean measured torque.

The variance ratio compares the results of different model estimation methods in terms of two quotients. The first quotient is the variance of the model residual divided by the variance of measured torque, \( V(\text{res})/V(y) \). The second is the variance of the error (i.e. variance of the remaining torque after removing the estimated model contributions) divided by the variance of the measured torque, \( V(\text{error})/V(y) \). The model estimation is consistent with the measured values if the criterion \( 0 \leq V(\text{res})/V(y) < V(\text{error})/V(y) \leq 1 \) is satisfied [25].

The residual analysis evaluates, by cross-correlation and auto-correlation, whether the estimated model adequately explains the relationships between input (stimulus) and output (anteroposterior torque) with no input correlated variations remaining [24,25].

### 2.4. Data analysis

Values were obtained for five periods: the rest period (0–30 s without stimulation), and periods I–IV (30–80, 80–130, 130–180, and 180–230 s, respectively), except for the values obtained from parametric identification, where the entire test time is evaluated for adaptive performance. As vibratory stimulation applied to neck or calf muscles mainly affect muscles predominantly active in the anteroposterior direction [18], only those responses are analysed.

The statistical analysis was performed with SPSS™ software [34] and MATLAB™ version 4.2c analytical software [31]. Levels of significance in values for the three dynamic parameters, stiffness, swiftness and damping, were tested by 1-way ANOVA, as was the significance of differences in values for different test conditions within the same time period. Levels of significance in changes over time in torque and posture were tested by repeated ANOVA. Thus, as the distribution of the values tended to be skewed, logarithmic transformation was done to obtain normal distribution. Normality of distribution was tested with the Kolmogorov–Smirnov and Shapiro–Wilk tests. In all tests \( P < 0.05 \) was considered to be statistically significant.

### 3. Results

The control subjects showed no significant change in any of the examined variables, despite test conditions. The results in the following figures and tables are all values obtained for test subjects exposed to stimulation. Unless otherwise stated, all changes in values are given with reference to values for the respective rest periods.

#### 3.1. Variance of torque with and without prefiltering

(Fig. 1)

During all test conditions, torque variance increased significantly at the onset of stimuli in period I \( (P < 0.01) \). With neck stimulation and eyes closed, torque variance diminished successively from period I to period III \( (P < 0.01) \). With calf stimulation and eyes closed, torque variance decreased significantly in periods II–IV, as compared with period I \( (P < 0.05) \). Torque variance was significantly greater in periods I–III during calf stimulation with eyes closed, than during any other test condition \( (P < 0.05) \).
Fig. 1. Log-transformed median values, quartiles and value ranges throughout the test for torque variance (A), variance of low frequency torque (B) and variance of high frequency torque (C). Asterisks denote significant differences.
Low pass filtered torque increased significantly at the onset of stimuli in period I ($P < 0.005$), except during neck stimulation with eyes closed. With calf stimulation, low pass filtered torque was significantly reduced in periods II and IV, as compared with period I ($P < 0.05$).

Except during neck stimulation with eyes open, high pass filtered torque increased significantly at the onset of stimulation in period I ($P < 0.05$), after which it decreased as the test proceeded. During neck stimulation with eyes closed, torque variance decreased significantly from period I–III ($P < 0.005$). During calf stimulation and eyes closed, high pass filtered torque was significantly reduced in periods II–IV, as compared with period I ($P < 0.05$).

A major difference in high pass filtered torque was found between eyes open and eyes closed conditions. With calf stimulation, the high pass filtered torque was significantly greater with eyes closed compared with eyes open conditions ($P < 0.005$) in periods I–IV. The same pattern was found with neck stimulation, though the difference was significant only for periods I and II ($P < 0.05$).

3.2. Empirical torque variance and torque rate variance (Fig. 2)

During all test conditions, empirical torque variance increased significantly at the onset of stimuli in period I ($P < 0.0005$), remaining significantly greater than during the rest periods throughout the four stimulation periods ($P < 0.01$). There was no significant difference in empirical torque variance between any of the four stimulation periods (I–IV). Moreover, in period I empirical torque variance was significantly greater during calf stimulation with eyes closed than during any other test condition ($P < 0.05$).
Fig. 3. Median values, quartiles and value ranges for swiftness, stiffness and damping parameters. The values in (A) are obtained for the feedback model according to the 'Feedback' method. The values in (B) and (C) are obtained for the step response and feedback models respectively according to the 'Step and feedback' method. Asterisks denote significant differences.
Torque rate variance increased significantly at the onset of stimuli in period I ($P < 0.0005$) during eyes closed conditions and remained greater than during the rest periods throughout the four stimulation periods ($P < 0.005$). During calf stimulation and eyes closed, torque rate variance was significantly lower in periods II–IV than in period I ($P < 0.05$). A similar reduction in torque rate variance occurred with neck stimulation, though the reduction as compared with period I was significant only in period III ($P < 0.05$). Moreover, torque rate variance was significantly greater during eyes closed than during eyes open conditions, both with calf stimulation ($P < 0.0005$) and neck stimulation ($P < 0.05$). Torque rate variance was significantly greater during eyes closed than during any other test conditions in all four periods ($P < 0.005$).

3.3. Identification of ARMAX models between stimulus and torque (Figs. 3, 4 and 5)

3.3.1. ‘Feedback’ method

The swiftness parameter was significantly greater during eyes closed conditions than during eyes open conditions ($P < 0.05$). The stiffness was significantly lower with eyes closed compared with eyes open during calf muscle stimulation ($P < 0.01$). There were no significant differences in the damping parameter between any of the test conditions.

3.3.2. ‘Step and feedback’ method

With the step response model, the values obtained for swiftness were significantly greater ($P < 0.05$) during calf stimulation and eyes closed than during any other test condition ($P < 0.005$ vs. tests with eyes open). With calf stimulation, values for the stiffness parameter were significantly lower ($P < 0.01$) during eyes closed than during eyes open. There were no significant differences in damping.

With the feedback model, both swiftness and stiffness parameters differed significantly between eyes open and eyes closed conditions, though not between neck and calf muscle stimulation. Swiftness values were significantly greater during eyes closed than during eyes open conditions ($P < 0.001$), whereas stiffness values were greater during eyes open than during eyes closed conditions ($P < 0.05$). In response to calf muscle stimulation, damping values were significantly greater during eyes open than during eyes closed conditions ($P < 0.05$).

Validation of models identified according to the ‘Feedback’ method showed a manifest offset error mainly caused by changing body displacement as the test proceeded. The behaviour at stimuli onset, with a temporary increase in response followed by a slow decrease, indicates that a time-variant control system is involved (see Fig. 4).

Validation shows a manifest improvement when the model identified according to ‘Step and feedback’ method is used. The improvement is mainly due to the calculation of the body displacement detected by the step response component (see Fig. 5).

3.3.3. Validation by test of error

Validation by test of error (Table 1A) shows a major reduction of the error mean quotient, indicating the step response component to be a major determinant of the measured torque.
3.3.4. Variance ratio comparison

The ‘Feedback’ estimation method was in two cases superior to that with the ‘Step and feedback’ method (Table 1B). In one case, the subject manifested no step response behaviour; and in the other, the subject manifested a change in feedback behaviour as the test proceeded. The ‘Step and feedback’ method was validated by re-estimation of the feedback response component in two intervals of comparable duration.

3.3.5. \( \chi^2 \)-tests of model correlation

Residuals obtained with the ‘Step and feedback’ method were validated with the \( \chi^2 \)-test at a significance level of \( P < 0.05 \). Of 40 tests, auto-correlation was confirmed in eight and crosscorrelation in three.

4. Discussion

The ability to adjust postural control performance is of major importance in daily life but even more so when a lesion or disorder affects postural control [4,5,35]. The importance of this ability was also reflected in this study where normal subjects were able to greatly improve postural control performance, suppressing disturbance or misinformation from one of the somatosensory receptor systems. Evaluation of a patient’s adaptive ability might therefore yield further information and could add another dimension to the evaluation of overall postural control performance. The ability of adaptation to perturbation might be useful for diagnostic purposes and in evaluating treatment efficacy and the progress of patients in rehabilitation programs.

Table 1

(A) Recordings ranked according to the magnitude of the error mean quotient; (B) Variance ratio comparison

<table>
<thead>
<tr>
<th>(A) Error quotient</th>
<th>No. of recordings</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Error mean/measured mean) ( &gt; 1.0 )</td>
<td>3</td>
</tr>
<tr>
<td>1.0 &gt; (Error mean/measured mean) ( \geq 0.1 )</td>
<td>8</td>
</tr>
<tr>
<td>0.1 &gt; (Error mean/measured mean) ( \geq 0.01 )</td>
<td>21</td>
</tr>
<tr>
<td>0.01 &gt; (Error mean/measured mean) ( \geq 0.001 )</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(B) Variance ratio</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( V(error_F)/V(y) &gt; 1 )</td>
<td>4</td>
</tr>
<tr>
<td>( V(error_S)/V(y) &gt; 1 )</td>
<td>0</td>
</tr>
<tr>
<td>( 0 \leq V(res_F)/V(y) &lt; V(error_F)/V(y) )</td>
<td>40</td>
</tr>
<tr>
<td>( 0 \leq V(res_S)/V(y) &lt; V(error_S)/V(y) )</td>
<td>40</td>
</tr>
<tr>
<td>( V(error_F)/V(y) &gt; V(error_S)/V(y) )</td>
<td>38</td>
</tr>
<tr>
<td>( V(error_S)/V(y) &gt; V(error_F)/V(y) )</td>
<td>2</td>
</tr>
</tbody>
</table>

The variance ratios from the ‘Feedback’ method are suffixed with F and the ratios from the ‘Step and feedback’ method are suffixed with S.

Postural control performance was found to be dependent on the duration of stimuli exposure and on the availability of visual input. The adaptive improvement of postural performance was seen in a progressive reduction of the body sway in response to stimuli and in the changes of posture. The adjustment pattern of declining responses to stimuli as the test proceeded reaching a steady state after about 40–50 s (see Fig. 1) was found independently of test condition. The onset of stimuli also induced an immediate change of posture. The majority of the subjects deviated forward during neck vibration and backward during calf vibration. The posture returned thereafter progressively to the initial position or to a slightly forward deviation compared with initial position.

There were significant differences in the responses evoked during eyes open and eyes closed test conditions, with respect to the values obtained for the parameters of the identified model, high frequency content of the torque and torque rate variance. If visual input was available, a major reduction of measured torque was found at frequencies above 0.1 Hz. This implies that postural control is less dependent on fast corrective movements to maintain stability if visual information is available. Although visual information is suggested to be mainly in use when detecting movements within the 0.01–0.1 Hz range [10], its major effect on postural control performance in terms of reduction of measured torque, is seen above 0.1 Hz. Hence, non-linear relationships may exist between the spectral content of visual perception and its effect on postural control responses.

Adaptation of postural control, engaging muscles remote from the disturbed proprioceptive muscle receptors and joints, has been suggested to be dependent on both vestibular and cervical receptors contributing correct sensory information about the movements of the body [36]. However, the present findings that similar responses may be evoked by disturbing calf and neck muscles independently of visual information suggests that the controlling and adaptive behaviour should be considered as a more generalised response pattern to repeated perturbations. The nature of postural control adjustments, as reflected in the results of this study, indicates adaptation to contain two separate processes. One process may be seen in the progressive reduction of body sway in response to stimuli. The present results suggest postural displacement comprises an additional slower simultaneous adaptive process, as reflected in the decline of the step response over time and in the interrelation of low frequency content of the torque and empirical torque variance. The decline of the step response may be considered to represent long-term compensatory behaviour.

Attempts to quantify adaptive changes of postural control are beset by a number of methodological prob-
lems. Methods of time-series analysis are based on the assumption of time-invariant statistical properties and of reproducibility of the results [25]. But by definition adaptation does affect reproducibility. Each time the subject is exposed to a stimulus, the response pattern may tend to be affected by experience gained from previous exposure [4,5,14,37]. The use of unpredictable pseudorandom stimuli might reduce the effect of anticipation, but the sensations of exposure are the same.

Although the estimation method requires a time-invariant relationship between input and output, time-variant adaptive changes can be described by using a step signal reflecting a component of the adaptive process. But a time-invariant feedback model can not entirely describe the initial adaptive changes in responses to separate vibratory pulses. This weakness is reflected in the failure of residual analysis to consistently manifest white-noise properties. Satisfactory feedback models, both before and after removal of the estimated step response component, could only be verified by residual analysis during the periods where the stimulus response relationship is in steady-state.

When studying adaptive processes, the disturbance of postural control must be of such magnitude that an adjustment of control performance is induced [3,19]. However, the necessary magnitude of disturbance might be highly individual and dependent on the postural control ability of the exposed subject. Thus, posturographic measurements from weak patients may be highly susceptible to adaptation, even at low magnitude disturbance. Moreover, measured responses might also be modified by fatigue when the subject is exposed to high magnitude disturbance.

5. Conclusion

Postural control manifested the same stimulus-response pattern almost independent of test conditions during exposure to high intensity stimulation. The response amplitude was generally smaller during open eyes conditions, since the availability of visual input was found to significantly reduce torque responses above 0.1 Hz. By using a system identification approach we could develop an improved methodology, including step response and dynamic feedback components, to describe the dynamics of postural control.

Appendix A. Modelling of the postural control system

The following assumptions are made in order to formalise and simplify analysis (Fig. 6).

Assumption 1: The body is stiff and has a mass $m$ (kg).

Assumption 2: The body centre of mass is located at distance $l$ (m) from the platform surface.

Assumption 3: There is a dynamical equilibrium between the torque of the foot and the forces acting on the ‘pendulum’.

Assumption 4: Assume that there is a stabilising ankle torque $T_{bal}(t)$.

Assumption 5: Assume that there is a disturbance torque $T_d(t)$ from the environment.

The tangential torque equilibrium for a standing person subject to gravitation $g$, and with a body moment of inertia $J$, is then

$$J \frac{d^2 \theta}{dt^2} = mg l \sin \theta(t) + T_{bal}(t) + T_d(t), \quad J = ml^2$$

(1)

Assumption 6: Assume that $T_{bal}(t)$ stabilises the posture with PID-control (proportional, integrative, derivative) with the components P, I, D determined by coefficients $\kappa$, $\eta$ and $\rho$. The parameter $\kappa$ may be interpreted as a spring constant, and $\eta$ might be compared with a viscous damping as obtained with a dashpot. The parameter $\rho$ may be interpreted as a constant for slow reset action in the control system.

Assumption 7: The vibration stimulus $v$ introduces an erroneous input into the stabilising system, causing misperception of the position $\theta$ (stretch) and angular velocity $d\theta/dt$ (rate) so that the P, I, D actions of feedback system are modified to
The sway velocity $v$ is assumed to disturb both stretch and rate perception but at different proportions $b_1$ and $b_2$, respectively. The torque equilibrium of Eq. (1) and assumptions 6 and 7 give the two equations

$$J \frac{d^2 \theta}{dt^2} = mg l \sin \theta(t) + T_{\text{bal}}(t) + T_d(t)$$

$$T_{\text{bal}}(t) = -mg l \sin \theta(t) - \kappa J \theta(t) - \eta J \frac{d \theta}{dt} - \rho J \int_0^t \theta(t) \, dt + (b_1 + b_2)v(t)$$

Elimination of $T_{\text{bal}}$ gives

$$J \frac{d^2 \theta}{dt^2} = -\eta J \frac{d \theta}{dt} - \kappa J \theta(t) - \rho J \int_0^t \theta(t) \, dt + T_d(t)$$

$$+ (b_1 + b_2)v(t)$$

There are three states that affect motion, namely angular velocity $d\theta/dt$, angular position $\theta$, and the bias compensation. A Laplace transform [25] and algebraic simplification gives the transfer function

$$\theta(s) = \frac{1}{J} \left( \frac{b_1 + b_2}{s^3 + \eta s^2 + \kappa s + \rho} \right) V(s) + \frac{1}{J s^3 + \eta s^2 + \kappa s + \rho} T_d(s)$$

A transfer function from vibration stimulus $V$ and disturbance $T_d$ to the torque $T_{\text{bal}}$ is found via Eq. (6) for linearised motion around the equilibrium $\theta = 0$ where $\sin \theta \approx \theta$ and

$$T_{\text{bal}}(s) \approx (J s^2 - mg l) \theta(s) - T_d(s)$$

$$= \frac{(b_1 + b_2)(s^3 - g)}{s^3 + \eta s^2 + \kappa s + \rho} V(s)$$

$$- \frac{\eta s^2 + (\kappa + g)}{s^3 + \eta s^2 + \kappa s + \rho} T_d(s)$$

Normalisation of the transfer function, Eq. (6), with respect to frequency gives for the stimulus dependence

$$T_{\text{bal}}(s) = \frac{(b_1 + b_2)}{s} \left( \frac{s}{\omega_0} \right)^3 - \frac{g}{\omega_0} \left( \frac{s}{\omega_0} \right)^2 \frac{s}{\omega_0} V(s)$$

A more functional characterisation of the motion based on the transfer function properties may be formulated using the concepts:

Swiftness: $\omega_0 = \sqrt[3]{\rho}$

Stiffness: $\kappa/\omega_0^2$

Damping: $\eta/\omega_0$

The swiftness parameter is a bandwidth (rad/s) and provides information about the highest angular frequency of the disturbance for which the postural control system gives adequate correction. The stiffness and damping are dimensionless stability parameters. A high value of stiffness means rapid response to disturbance, i.e. rapid compensation for small deviations from equilibrium. A high value of damping means good damping of sway velocity.

**ARMAX model estimation of the transfer function**

An ARMAX model estimate the discrete time transfer function between input $u(t)$ and output responses $y(t)$ considering a noise factor $e(t)$.

$$y(t) = \frac{B(q)}{A(q)} u(t) + \frac{C(q)}{A(q)} e(t)$$

For a third order model, the estimated ARMAX pole polynomial is

$$A(q) = q^3 + a_1 q^2 + a_2 q + a_3$$

The parameter values are converted from discrete to continuous-time by inverse sampling with

$$1 - \frac{1}{h} \log \Phi$$

with a sampling interval $h$ and a matrix $\Phi$. Computation of the continuous-time characteristic polynomial gives the physiological feedback parameters used in Eq. (6).

$$A(s) = s^3 + \eta s^2 + \kappa s + \rho$$

**References**


