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Sandberg, Frida; Stridh, Martin; Sörnmo, Leif

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Frequency Tracking of Atrial Fibrillation using Hidden Markov Models

Frida Nilsson, Martin Stridh, and Leif Sörnmo

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and \( f_i + \Delta f = f_{i+1} \), with a center frequency of \( \tilde{f}_i \),

\[
\tilde{f}_i = f_i + \frac{\Delta f}{2}.
\]  

(1)

A. State transition matrix

The \( P \times P \) state transition matrix \( A \) describes the probabilities of transition between different states. Element \( a_{ij} \) is the probability that state \( x(t+1) = j \) if state \( x(t) = i \). Each row must sum to unity, since the elements of a row correspond to the probabilities of transition to a certain state. The probability of track initialization in state \( j \neq 0 \), \( a_{0j} \), is set equal for each frequency state,

\[
a_{0j} = \frac{u}{P - 1}, \quad j = 1, 2, \ldots, P - 1.
\]  

(2)

Hence, the probability of remaining in the zero-state, \( a_{00} \), i.e., when no signal is present, is

\[
a_{00} = 1 - u.
\]  

(3)

The probability of track termination, \( a_{j0} \), is equal for all frequency states,

\[
a_{j0} = v, \quad j = 1, 2, \ldots, P - 1.
\]  

(4)

The transition probability \( \tilde{a}_{ij} \) between the non-zero states is set to

\[
\tilde{a}_{ij} = \frac{(1 - v)g_{ij}}{\sum_{k=1}^{P} g_{ik}}, \quad i, j = 1, 2, \ldots, P - 1,
\]  

(5)

where

\[
g_{ij} = \frac{1}{d\sqrt{2\pi}} \int_{f_j}^{f_j + \Delta f} \exp \left[ -\frac{(f - \tilde{f}_i)^2}{2d^2} \right] df
\]  

(6)

since the location of the frequency track at the next time step is assumed to be Gaussian with mean \( \tilde{f}_i \) and standard deviation \( d \). This results in an unbalanced \( A \), since the diagonal elements of \( a_{ii} \) depend on state \( i \). Hence, all diagonal elements are set to the smallest diagonal element,

\[
a_{\text{min}} = \min_{1 \leq i \leq P - 1} \tilde{a}_{ii}.
\]  

(7)

Consequently, the other \( P - 1 \) elements of the row needs to be adjusted so that the row will sum to unity. If the elements \( a_{i,i-1} \) and \( a_{i+1,i} \) becomes larger than the diagonal element of the row, \( a_{\text{min}} \), they are set to \( a_{\text{min}} \), and the remaining \( P - 3 \) elements are adjusted so that they sum to unity. This continues until no elements of the row are larger than the diagonal element \( a_{\text{min}} \).

Using the previously described elements, \( a_{ij} \), the design parameters of the state transition matrix \( A \) are the track termination and track initiation probabilities, \( u \) and \( v \), and the degree of frequency changes, modeled by the standard deviation, \( d \).

B. Observation matrix

The \( P \times P \) observation matrix \( B \) describes the probabilities of observing a specific state given the true state. The elements of \( B \), \( b_{ij} \), corresponds to the probability of detection in state \( j \), when the true state is \( i \).

We assume that the signal is a sinusoid with added noise, defined by the following signal model

\[
s(n) = a \sin(2\pi f_0 n) + w(n).
\]  

(8)

The signal amplitude, \( a \), and the frequency, \( f_0 \), is assumed to be constant over the time period where the STFT is calculated. The noise, \( w(n) \), is assumed to be zero-mean, Gaussian with variance \( \sigma^2 \).

The probability density function of the signal magnitude \( R \) at frequency \( f \) is derived in [8]. If a sinusoid with frequency \( f \) is present, it can be expressed as

\[
p_1(R(f)) = \frac{2R(f)N \sigma^2}{\sigma^2} \cdot I_0 \left( \frac{R(f)\sigma N}{\sigma^2} \right) e^{-\frac{N(4R(f)^2 + \sigma^2)}{4\sigma^2}}. \]

(9)

If absent, it can be expressed as

\[
p_2(R(f)) = \frac{2R(f)N \sigma^2}{\sigma^2} \cdot e^{-\frac{N(R(f))^2}{\sigma^2}}, \]

(10)

where \( N \) defines the length of the Fourier transform, and \( I_0 \) is the modified Bessel function.

The elements of the observation matrix \( B \) are derived from \( p_1 \) and \( p_2 \). If no signal is present, the probability of no detection (zero-state), \( b_{00} \), and the probability of detection in state \( i \), \( b_{0i} \), is

\[
b_{00} = \prod_{i=1}^{P-1} \int_{D}^{\infty} p_2(r)dr = \left[ 1 - e^{-\frac{D^2N}{\sigma^2}} \right]^{P-1},
\]  

(11)

\[
b_{0i} = \frac{1 - b_{00}}{P - 1},
\]  

(12)

where \( D \) is the detection threshold.

If a signal with frequency between \( f_m \) and \( f_m + \Delta f \) is present, the probabilities of detection in state \( m \), \( b_{mm} \), no detection (zero-state), \( b_{0m} \), and detection in state \( i \neq m \), \( b_{mi} \) are given by

\[
b_{mm} = \int_{D}^{\infty} p_1(r) \cdot \left[ 1 - e^{-\frac{D^2N}{\sigma^2}} \right]^{P-2} dr,
\]  

(13)

\[
b_{0m} = \left[ 1 - e^{-\frac{D^2N}{\sigma^2}} \right]^{P-2} \int_{0}^{D} p_1(r)dr,
\]  

(14)

\[
b_{mi} = \frac{1 - b_{0m} - b_{mm}}{P - 2},
\]  

(15)

respectively.

The design parameters of \( B \) is the SNR, given by the parameters \( \alpha \) and \( \sigma^2 \), and the detection threshold, \( D \).

C. Optimal detection threshold

An optimal detection threshold, \( D_{\text{opt}} \), can be obtained if the following error cost function criterion is used,

\[
C_e = \alpha P(R_1 > D \cap \cdots \cap R_P > D | i = 0) + \beta P(R_1 < D \cup \cdots \cup R_P < D | i \neq 0)
\]  

(16)
where the sum of $\alpha$ and $\beta$ equals 1. Setting the derivative of $C_e$ to zero, one obtains an expression of the probability of a magnitude equal to the detection threshold, $D$, when a signal is present:

$$p_1(D) = \frac{2DN(P-1)\alpha\mu_0 - \frac{\beta^2\sigma^2}{\sigma^2}}{\sigma^2\beta(1-\mu_0)} e^{-\frac{\beta^2\sigma^2}{\sigma^2}} \left(1 - \frac{\beta(1-\mu_0)(P-2)b_{00}}{\alpha\mu_0(P-1)b_{00}}\right),$$

(17)

where $\mu_0$ is the long term probability of the zero-state.

By setting $\alpha = (1-\mu_0)$ and $\beta = \mu_0$, false dismissals are emphasized if the probability of being in the zero-state is small, and vice versa. This gives

$$p_1(D) = \frac{2DN(P-1)}{\sigma^2} e^{-\frac{\beta^2\sigma^2}{\sigma^2}} \left(1 - \frac{(P-2)b_{00}}{(P-1)b_{00}}\right).$$

(18)

The optimal threshold, $D_{\text{opt}}$, is then

$$D_{\text{opt}} = \arg \max_D p_1(D).$$

(19)

Since $b_{00}$ depends on $D$, $D_{\text{opt}}$ has to be calculated iteratively. By using the cost function optimality criterion, the only design parameters of $B$ are $a$ and $\sigma^2$.

### III. EVALUATION AND RESULTS

#### A. Signals and parameter values

The HMM method was evaluated using simulated AF signals. To investigate the robustness of the method, real noise was added. The AF signals are simulated as a sinusoid, with varying amplitude, fundamental frequency and its $M$ harmonics [4]:

$$s(n) = -\sum_{i=0}^{M-1} a_i(n) \sin(i\theta(n)),$$

(20)

where $a_0$ and $a_i$, $i = 1, \ldots, M$, are the amplitudes of the fundamental and the harmonics, respectively, and $\theta(n)$ is the phase.

The time varying amplitude $a_i(n)$ of the $i$ : th harmonic is given by

$$a_i(n) = e^{-\gamma_i(a + \Delta a \sin(2\pi F_a F_s n))},$$

(21)

where the amplitude $\Delta a$ is the modulation peak amplitude and $F_a$ is the modulation frequency. The exponential decay of the amplitudes of the harmonics is denoted $\gamma$.

The fundamental AF frequency varies around $F_0$ with maximum frequency deviation of $\Delta F$ and a modulation frequency $F_m$. Hence, the phase is given by

$$\theta(n) = 2\pi F_0 F_s n + \frac{\Delta F}{F_m} \sin(2\pi F_m F_s n).$$

(22)

The signal amplitude is set to $a = 100$ and the modulation amplitude and frequency is set to $\Delta a = 30$ and $F_m = 0.08$, respectively. AF signals with four different frequency trends were created: constant frequency, varying frequency, gradually decreasing frequency and stepwise decreasing frequency. The frequency trends of the simulated signals are shown in Fig. 1.

![Fig. 1. Frequency trends of the simulated AF signal with (a) constant frequency, (b) varying frequency, (c) gradually changing frequency and (d) stepwise changing frequency.](image)

![Fig. 2. Noise obtained from ECG signal.](image)

Noise, present in ECG signals, was added to the simulated AF signals. In order to acquire the noise signal, QRST complexes was removed from ECG signals with normal sinus rhythm, using average beat subtraction. Figure 2 shows the noise signal.

The SNR is defined by

$$\text{SNR} = 20 \log \frac{V_s}{\sigma_v}$$

(23)

where $V_s$ is the peak-to-peak amplitude of the simulated AF signal, and $\sigma_v$ is the standard deviation of the noise. Test signals with SNR between 0 dB and 10 dB was created. Figure 3 shows a simulated AF signal with added noise at 5 dB SNR.

The frequencies between 3 and 12 Hz are divided into segments of $\Delta f = 0.1$ Hz, so that state 1 contains frequencies between 3 and 3.1 Hz, and so on until state $P$ with frequencies between 11.9 and 12 Hz. This gives a total of $P = 91$ states, including the zero-state.

Since absence of fibrillation is assumed to be unlikely, the track initiation probability is set to $u = 0.98$ and the track termination probability is set to $v = 0.01$. The standard deviation is set to $d = 0.5$.  

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The signal and noise amplitudes of the observation matrix $B$ are set to $a = 0.1$ and $\sigma^2 = 0.1$ respectively. Note that the HMM is not matched to the simulation model.

B. Results

An example of HMM frequency tracking is presented in Fig. 4. The HMM improves the frequency tracking performance at SNRs ranging from 9 down to 0 dB. For signals with very high SNR the spectral peaks can be clearly distinguished. Hence, the observed states equal the true states, and there is no need to employ the HMM. For signals with very low SNR, where the observed frequencies are so corrupted by noise that no observed state is equal to the true state, the frequency tracking cannot be improved by the HMM.

The average RMS error of the estimated frequencies, obtained with and without the HMM, is presented in Fig. 5. For example, the average RMS error drops from 1.2 to 0.2 Hz, when the HMM is applied at 5 dB SNR. When the optimal state, given by the Viterbi track, is the zero-state, the error is undefined. A high zero-state occupancy percentage tend to give a lower average RMS error. Therefore it is important to compare not only the average RMS error, but also the zero-state occupancy percentage.

The results suggests that the assumption of white noise in the HMM is not crucial, since the performance for colored noise is good.

IV. CONCLUSIONS

Frequency tracking with HMM was evaluated using simulated AF signals with varying amplitude and frequency embedded in noise obtained from ECG recordings, showing that HMM improves the frequency tracking substantially.

REFERENCES