Mixing Time for the Dead Sea Based on Water and Salt Mass Balances

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Abstract

Water and salt mass balances for the Dead Sea were modeled to consider different possible methods for maintaining its water level and water volume. In the models, precipitation, evaporation, rivers, ground water, input/output from potash companies and salt production, and brine discharge were included. The mixing time in the Dead Sea was modeled by a 1) single-layer (well-mixed) system, and a 2) two-layer (stratified) system. Brine discharge from the desalination plant of the proposed Red Sea-Dead Sea Canal project (RSDSC) was also simulated with the model. In the single-layer approach the water level after 100 years was predicted to change from 411m below mean sea level (bmsl) (1997) to 391 m and 479 m based on a water mass balance including and excluding brine discharge from RSDSC, respectively, and to reach 402 m and 444 m for the two cases based on a salt mass balance. In the two-layer approach the water level after 90 years was predicted to change from 411m bmsl (1997) to 397m and 488m for a water mass balance including and excluding brine discharge from RSDSC, respectively, and to reach 387m and 425m for the two cases using a salt mass balance. The water residence time in a single-layer description increased from 58 to 116 years when excluding brine discharge. In the two-layer approach the exchange or mixing time increased in both layers when adding brine discharge to the system from 1.2 to 1.7 years and 11 to 15.3 years in the upper and lower layer, respectively. The models were also employed to reproduce the historical Dead Sea water level variation. Good agreement was found between the models and historical data using both water and salt mass balances.

Keywords: Dead Sea Water Level; Water-Salt Balance; Read Sea-Dead Sea Canal (RSDSC); Single-Layer and Two-Layer System; Salinity; Residence and Mixing Time; Historical Comparison.

1. Introduction

1.1. General

The Dead Sea (DS) is a salt lake in southwestern Asia. It is bounded on the west side by Israel and the West Bank and on the east side by Jordan, forming a part of the Israeli, Jordanian and Palestinian Authority border (Fig. 1). The DS is fed mainly by the Jordan River, which enters the lake from the north. Several smaller streams also discharge into the lake, primarily from the east. The lake has no outlet, and the inflow of fresh water is transported away by evaporation only,
which is rapid in the hot desert climate. Due to large-scale projects by Israel and Jordan to divert water from the Jordan River for irrigation and other water needs, the surface of the DS has been dropping for at least the past 50 years [1].

The surface of the DS was located 408 m below mean sea level (bmsl) in 1996-1997, being the lowest water surface on earth. In the beginning of the last century, the DS level was about 390 m below mean sea level having a surface area of 950 km$^2$. The surface area of the lake continuous to decrease due to high evaporation and decreasing water inflow, and in 1997 the water level and surface area reached 411 m bmsl and 640 sq km, respectively [2]. The DS is located in the northern part of the Great Rift Valley. On the east side of the lake the Moab plateau rises to about 1,340 m above sea level, and on the west side the Judea plateau rises to approximately half that height. Originally the DS consisted of two basins (a northern and a southern basin) divided by the Lisan Peninsula, which extends from the eastern shore out into the lake. In 1966, the DS covered an area of 940 km$^2$ (76% of the lake was in the northern basin), and its length was 76 km with an average width of 14 km. The total volume of the water in the DS was estimated to be 142 km$^3$ (only 0.5% in the southern basin). The maximum depth was 749 m bmsl, whereas the average depth of the southern basin was only 10 m [2].

The level of the DS has continuously fallen since the early 1930s at an increasing rate (rates of 0.5, 0.7, 0.90, 0.95 m/yr reported in [3,4,5,6], respectively). A summary of estimated rates of evaporation as a function of salinity in the DS is presented in Table 1 based on the compilation by [7] for the period between the late 1950s and 1980. The salinity increased during this period from 225 g kg$^{-1}$ to 279 g kg$^{-1}$.

Table 1
Estimated evaporation rates as a function of surface salinity in the Dead Sea [7]

<table>
<thead>
<tr>
<th>TDS g/kg</th>
<th>Elevation bmsl (m)</th>
<th>Period</th>
<th>Data source</th>
<th>Evaporation rates (m/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>225</td>
<td>393</td>
<td>1942/46</td>
<td>Neumann 1958</td>
<td>1.70–1.75</td>
</tr>
<tr>
<td>240</td>
<td>395</td>
<td>1959/60</td>
<td>Neev and Emery 1967</td>
<td>1.47–1.65</td>
</tr>
<tr>
<td>256–279</td>
<td>401</td>
<td>1979/80</td>
<td>Anati et al. 1987</td>
<td>1.30–1.54</td>
</tr>
</tbody>
</table>
During the last decade, the DS level has dropped by more than 25 meters, and reached a level of about 416 meters (2003) bmsl. Thus, the system is not sustainable at present, and different projects to supply seawater to the DS have been suggested in order to restore the water level. The Red Sea-Dead Sea Canal (RSDSC) was considered as one possible solution with a proposed layout as shown in Fig. 1, which may include a desalination plant. The inflow of seawater (or reject brine after desalination) into the DS will have a major impact on its limnology, geochemistry, and biology.

Inflow of seawater (density $\approx 1.03$ g/ml) to the highly saline DS (density $\approx 1.24$ g/ml) can be approximated to the inflow of freshwater. The impact of such an inflow can be estimated by comparison with the freshwater inflow during the rainy winter of 1991/2, when the lake level rose by 2 meters and the surface brines were diluted by up to 30% [8].

1.2. Chemistry of the Dead Sea

When the DS is exposed to large runoff, the salt content can drop from its usual 35% salinity to 30% or lower [9]. Under normal conditions, the DS salinity is about nine times greater than the average ocean salinity. Depending on the season, the uppermost 35 m of the DS has a salinity
that ranges between 30% and 40% with a temperature between 19°C and 37°C. Below a transition zone, the bottom layer of the DS consistently has a temperature of about 22°C with complete saturation of halite sodium chloride (NaCl). Because the near-bottom water is saturated, salt precipitates out of solution onto the sea floor.

The DS is presently also saturated or oversaturated with respect to aragonite (CaCO₃) and anhydrite (CaSO₄) [10]. Kinetic factors make gypsum (CaSO₄·2H₂O), the hydrated form of anhydrite, the actual calcium sulfate mineral that precipitates from the DS brine. Any mixing between the calcium-rich DS brine and a sulfate-rich seawater will result in gypsum precipitation (CaSO₄·2H₂O) [11]. The DS salts are commercially important and their production enhances evaporation of water, speeding up the reduction in lake water level. Wisniak [12] presented a description of the properties of the DS together with a chemical analysis of the processes utilized to exploit it commercially. Several numerical models have been developed to determine the water balance [4,6]. In the DS, horizontal variations in temperature and salinity may occur [13,14], but they are typically neglected in the modeling. Due to the peculiar chemical composition of DS water, the accepted definition of “salinity” is not useful for the water in this lake and it has been replaced by an equivalent salinity [15], referred to as “quasisalinity”. The carnallite (KCl. MgCl₂.6H₂O) is next to crystallize, however, the latter is only expected to happen when the brine reaches a specific gravity of 1.3 g/cm³ [3].

Table 2 presents a comparison of chemical analyses from the DS, the Jordan River, and the Mediterranean Sea based on more than 40 years of data. Opinions vary which salt content the DS has, where Gavrieli et. Al [11] reported a density and salinity for the DS of about 1.237 g/ml and 342.4 g/l, respectively. Vengosh and Rosenthal [16] have reported that the DS salinity is about 332.06 g/l. Mixing of the DS water due to waves and wind is slow compared to other water bodies [17]. The DS may be considered as a stratified water body based on 44 available data sets on potential temperature, quasi-salinity, and potential density. The data on these parameters indicate that the top-layer depth of the DS is about 10% of the maximum depth (see Figure 2); if an average between June 1998 and December 2007 at the Ein-Gedi 320 station is considered [18].
### Table 2
Chemical analysis of water from the Dead Sea, Jordan River, and the Mediterranean Sea [11, 16, 19, 20, 21, 22, 23]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(g/l)(^{19,20})</td>
<td>(g/l)(^{22})</td>
<td>(g/l)(^{16})</td>
<td>(g/l)(^{21})</td>
</tr>
<tr>
<td>Cl</td>
<td>180.8</td>
<td>208.0</td>
<td>216.0</td>
<td>219.25</td>
</tr>
<tr>
<td>Mg</td>
<td>34.50</td>
<td>41.96</td>
<td>42.5</td>
<td>42.43</td>
</tr>
<tr>
<td>Na</td>
<td>33.50</td>
<td>34.94</td>
<td>34.3</td>
<td>39.70</td>
</tr>
<tr>
<td>Ca</td>
<td>13.00</td>
<td>15.80</td>
<td>17.1</td>
<td>17.18</td>
</tr>
<tr>
<td>K</td>
<td>6.30</td>
<td>7.56</td>
<td>6.65</td>
<td>7.59</td>
</tr>
<tr>
<td>Br</td>
<td>4.10</td>
<td>5.92</td>
<td>----</td>
<td>5.27</td>
</tr>
</tbody>
</table>

11Gavrieli and Aharon, 2005
16A. Vengosh, E. Rosenthal, 1994
19,20Bentor YK 1961 and 1969,
21M. Abu-Khader Mazen, 2006
22Katz and Nurit, 1981
23Yoram Eckstein, 1970

**Figure 2.** Dead Sea potential temperature, quasi-salinity, and potential density based on averages from June 1998 to December 2007 at Ein-Gedi 320 station.
1.3. Results from the previous studies

The decreasing water level in the DS has been a concern for many years and the idea of diverting seawater from the Mediterranean Sea or the Red Sea has been discussed several times. Two scenarios for the RSDSC project were studied by Al-Weshah [4] using a water balance model for the coming 50 years. The first scenario assumes a diversion capacity of 70 m³/s. It shows that the DS will recover its design level of 395 m in about 40 years. The second scenario assumes a diversion capacity of 60 m³/s and shows a level of 400.5 m after about 40 years. The net inflow to the DS from the Jordan River falls from 175 to 60 MCM per year. This decrease is because Jordan is permitted to utilize 20 MCM of the river water directly and to store another 20 MCM of excess river water in Lake Tiberias during winter for its consumption in the summer [24].

Gavrieli and Bein [25] studied a 40-year period for the DS assuming a diversion capacity of 60 m³/s and a starting elevation in 1995 of about 408.5 m bmsl. The new elevation after 40 years of discharge from a RO-plant was -400.5 m bmsl. In another scenario no project was assumed (no RSDSC), and the model showed that the DS level would fall to 444.4 m bmsl. Asmar and Ergenzinger [2] modeled a period of 100 years between 1989 and 2088 assuming that the current conditions would prevail. However, the industrial intake increased in the year 2000 by a projected amount of about 25%, leading to a water level change from 406 to 460 m bmsl during the study period.

1.4. Objectives

The extreme decrease in the DS water level, which is mainly caused by the reduced inflow from the Jordan River, high evaporation from the lake, and the intake of water to potash companies around the lake, has resulted in substantial damage to the development of the area around the DS.

The RSDSC has been considered to be one of the most important elements in a development strategy to provide desalinated drinking water to the area and brine discharge to the Dead Sea, which may be an important factor to recover the previous water level in the DS. The main aim of the present study is to investigate methods to maintain or improve the water level in the DS. However, it is also of interest to predict the impact of the water discharge depending on release point and density conditions. A major task of the research is to evaluate the exchange time or
mixing time for the DS depending on the properties of the inflow, which may vary with respect to density and salinity. Two models were developed, namely a single-layer (well-mixed) system and a two-layer (stratified) system, to assist in estimating the mixing time when taking into consideration changes in the elevation and surface area.

2. Overview of the study area

2.1. General

The principal development objective of the RSDSC initially was to provide desalinated drinking water for the people of the surrounding areas [26]. In contrast, the Mediterranean Dead Sea Canal, first studied in 1973, focused on the generation of electricity and this project was targeted to produce about 800 MW during peak hours [27]. At the end of the 1980s and in the beginning of the 1990s a re-evaluation of the major development goals for the region was undertaken. Thus, a project was initiated by the Israeli Ministry of Energy and Infrastructure involving hydrostatically driven desalination with reverse osmosis (RO) membranes to produce fresh water from Red Sea water [28,29].

The main objectives of the revised project are to provide a sustainable source of fresh water for the region and to halt the decline in the DS level. The total available hydrostatic head near the Dead Sea end ranges from 331 to 545m, depending on the alignment and the provided pumping head at the Red Sea end (Fig. 3) [4]. The capacity of such a conveyance system has been studied for different diversion flow rates ranging from 40 to 70m$^3$/s [24].

The desalination plant is proposed to produce fresh water using the RO method. The estimated annual fresh water production of this plant is a function of the conveyance system capacity. In one design it can reach up to 850 MCM, where the design and economy are limited by the diversion capacity. It is expected that two-thirds of this production will go to Jordan and the rest will go to Israel and the Palestine [24]. Total dissolved solids (TDS) from the desalination plant are expected to be between 200-300 mg/l. The DS level restoration is designed to produce a water level at around 400m bmsl after approximately 50 years.
Tables 3a and 3b summarize some properties of the DS and its output/input, where the river inflow represents the total water received from rivers, springs, and infiltration [25,30,31].

Table 3a
Dead Sea properties [25,30,31]

<table>
<thead>
<tr>
<th>Year</th>
<th>Vol. km³</th>
<th>Area km²</th>
<th>Level m</th>
<th>Ppt. mm</th>
<th>Evapn. m</th>
</tr>
</thead>
<tbody>
<tr>
<td>1997</td>
<td>131</td>
<td>640</td>
<td>-411</td>
<td>70</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 3b
Dead Sea annual average of output/input [25,30,31]

<table>
<thead>
<tr>
<th></th>
<th>Annually (MCM)</th>
<th>Density (kg/m³)</th>
<th>Salinity (ppt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial outtake (outtake)</td>
<td>531</td>
<td>1300</td>
<td>337</td>
</tr>
<tr>
<td>Industrial brine (inflow)</td>
<td>225</td>
<td>1350</td>
<td>500</td>
</tr>
<tr>
<td>Brine disposal (inflow) RO</td>
<td>1100</td>
<td>1045</td>
<td>68</td>
</tr>
<tr>
<td>Rivers inflow</td>
<td>375</td>
<td>1025</td>
<td>20</td>
</tr>
<tr>
<td>Evaporation</td>
<td>928</td>
<td>1000</td>
<td>---</td>
</tr>
<tr>
<td>Rainfall</td>
<td>51.2</td>
<td>1000</td>
<td>---</td>
</tr>
</tbody>
</table>

2.2. Potash and salt works

An important part of the salt balance for the DS is the contributions by potash companies and salt extraction. In the early part of the 20th century, the DS began to attract interest from chemists who concluded that the lake was a natural deposit for potash and bromine. The Palestine Potash Company was chartered in 1929 and two more plants were established after 1934, the first plant
on the north shore of the DS at 'Kalia' and the second in the Sodom area south of the 'Lashon' region. The Dead Sea Works Ltd. was established in 1952 as a state-owned company.

From the DS brine, Israel produces 1.77 million tons potash, 206,000 tons elemental bromine, 44,900 tons caustic soda, 25,000 tons magnesium metal, and sodium chloride (year 2001). On the Jordanian side of the DS, Arab Potash (APC), formed in 1956, produces 2.0 million tons of potash annually, as well as sodium chloride and bromine. The power plant on the Israeli side allows production of magnesium metal (by a subsidiary, Dead Sea Magnesium Ltd.).

2.3. Environmental impact

One of the major ideas of the RSDSC project is to compensate for the negative water balance, which strongly affects the DS water level (see Fig. 4). The lake level recommended for planning for the Katif alignment was 390 m bmsl [27] and only 12 years later for the RSDSC it was 400 m bmsl [32]. The only way to overcome the falling water level problem is by adding sea water, which creates a positive balance between the net inflowing water and the loss by evaporation. However, the inflow of sea water or of concentrated rejects could build up an upper layer of lower density that may create a hypolimnion with reduced environmental conditions and intensive precipitation of gypsum, at the same time as the biological environment of the epilimnion changes [30]. A schematized description of the DS for the case of discharge to upper layer, including the different inputs and outputs, is given in Fig. 5.

Figure 4. Dead Sea water volume, surface area, and level [30]
The rate of water level decrease over the last 10 years is about 0.9 m/yr, representing an annual water loss of about 600 million cubic meters. The sharp decrease in water level is a function of the annual withdrawal of water by neighboring countries (over 1000 MCM of freshwater), which in the past supplied the DS. Approximately 200-250 MCM/yr of this withdrawal, corresponding to about 35 cm/yr water level drop, is attributed to the activities of the Israeli and Jordanian potash and salt works. These industries pump together 400-450 MCM from the DS into evaporation ponds located in the southern basin, where halite (NaCl) and carnallite (KMgCl$_3$·H$_2$O) precipitate. At the end of the process, less than 200 MCM of concentrated brines (density = 1.35 kg/l; TDS = 500 g/l) are returned to the DS [25].

3. Methodology

3.1. Single-layer system (well-mixed system)

The mathematical model used in this study is based on the LOICZ Biogeochemical Modeling Guidelines [33] and validated by comparing its performance with other modeling studies of the DS. The model is employed to describe the dynamic behavior of the DS using data available at 1997 as initial conditions and simulating the evolution for a 100-year period. In a second modeling effort, historical data from the period 1976 to 2006 are used for comparison with simulation results from the model.
A general mass balance equation for the DS system yields (see Fig. 6 for a definition sketch explaining the various terms):

\[
dM/dt = \Sigma \text{Inputs} - \Sigma \text{Outputs} + \Sigma (\text{sources-sinks})
\]  

(1)

where, \(dM/dt\) represents the change in mass of any particular material in the system with respect to time and \(\Sigma \text{sinks}\) is the mass of chemical precipitated per unit time. The amount of salt production was found in previous studies to be approximately 0.1 m annually, and this rate was included as a sink in the calculations [34]. The DS is not in steady-state condition, but it was assumed to be close to steady state during the first year. Water and salt balances may have internal inputs and outputs; this is valid mostly in the two-layer approach. We may thus simplify equation (1) to become:

\[
dM/dt = \Sigma \text{Inputs} - \Sigma \text{Outputs} - \Sigma \text{sinks}
\]  

(2)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{dead_sea_mass_balance.pdf}
\caption{Schematization of mass balance for the Dead Sea using a single-layer system}
\end{figure}

The definition of the various DS inputs/outputs are displayed in Figure 6, where the total mass flow of water received from rivers, springs, and infiltration are denoted by \(Q_{Ri}\), and direct precipitation on the system by \(Q_P\) (likely to be the major freshwater inputs to the system). The water used by the Potash Company and DS industrial works is denoted as \(Q_{Ind-o}\) and the brine discharged by \(Q_{Ind-i}\) (also called industrial outtake and industrial brine input for the system). An important part of this study is the brine discharge from the desalination plant \(Q_{RO}\), which will come through the RSDSC. Evaporation \(Q_E\) is a significant freshwater output which is considered the most important factor in causing a decrease in the DS level. In principle, the net
flow \((Q_N)\), also called the "residual flow" \((Q_N = Q_{Ri} + Q_P + Q_{Ind-i} + Q_{RO} - Q_{Ind-o} - Q_E - \text{sinks})\), can be either to or from the system. It is algebraically treated as an input/output. We can re-write equation (2) to describe the water budget of the system as:

\[
(dM/dt)_N = \Sigma(Q_{RO} + Q_P + Q_{Ri} + Q_{Ind-i}) - \Sigma(Q_{Ind-o} + Q_E) - \Sigma \text{sinks}
\]  

(3)

where "\((dM/dt)_N\)" is the time change in the net mass of water in specific time period which is also denoted as \((Q_N)\). This equation describes the water mass budget for the system of interest. It is also possible to write a second equation describing the salt mass budget. For the salt budget, an average salinity is assigned to each one of the water inputs and outputs.

For some of the terms it is sufficiently accurate to assume that the salinity is zero (e.g., for the precipitation and evaporation [33]). The average salinity of the whole system \((S_N)\) is the salinity near the boundary between the output and the input to the system; often it is adequate to assign this salinity as the average of the adjacent water and the system water. Re-writing equation (3) for a salt budget:

\[
Q_N S_N = \Sigma(Q_{RO} S_{RO} + Q_P S_P + Q_{Ri} S_{Ri} + Q_{Ind-i} S_{Ind-i}) - \Sigma(Q_{Ind-o} S_{Ind-o} + Q_E S_E) - \Sigma \text{sinks}
\]  

(4)

We now simplify this equation by leaving out those terms with salinity likely to be close to zero (all salinities units are in part per thousand):

\[
Q_N S_N = \Sigma(Q_{RO} S_{RO} + Q_{Ri} S_{Ri} + Q_{Ind-i} S_{Ind-i}) - \Sigma(Q_{Ind-o} S_{Ind-o}) - \Sigma \text{sinks}
\]  

(5)

where, \(S_N = (S_{syst} - S_{ave}) / 2\).

The residual flow (exchange flow) determines the water residence time, which is obtained from the system volume \((V_{syst})\) divided by the absolute value of net surface flow which is estimated from net flow rate through the receiving water as a result of water mass balance. Thus, the water residence time (exchange time) is given by:

\[
\tau = V_{syst} / |Q_N|
\]  

(6)
3.2. Two-layer system (stratified system)

The DS shows relatively strong vertical stratification that can be assumed to resemble a two-layer system [2,35,36]. Such a system is likely to develop in the case of relatively large freshwater input. Figure 7 illustrates a modified water budget for a two-layer system (stratified system), which includes entrainment flow \( Q_{\text{Deep}'} \), vertical exchange volume \( Q_z \), and a modified exchange time \( \tau \). The calculations are quite similar to the single-layer system calculation. In addition, three assumptions are needed for the two-layer system:

- Outflow volume associated with freshwater inputs (i.e., \( Q_N \)) occurs in the surface layer, displacing water in the surface portion of the system \( S_{\text{Sys-1}} \).
- River and industrial flow (i.e., \( Q_{Ri}, Q_{\text{Ind-1}} \)) enters the deep layer, flows upward into the surface layer, and out again from the surface layer. The combined outflow from the surface layer \( Q_{\text{surf}} \) and \( Q_{Ri} \) has positive sign, whereas both \( Q_N \) and \( Q_{\text{surf}} \) have negative signs.
- The salt balance is maintained through a vertical exchange flow \( Q_z \) between the surface and deep layer.

**Figure 7.** Schematization of mass balance for the Dead Sea using a two-layer system (stratified system), including entrainment flow \( Q_{\text{Deep}'} \), vertical exchange volume \( Q_z \), and modified exchange time \( \tau \)

The calculation of \( Q_{\text{Deep}'} \) is possible only if there is a salinity difference between the inflowing deep water and the surface water of the system \( Q_{\text{Sys-1}} \), where \( Q_{\text{Sys-1}} \) is about 10% of the total volume of the system. The top layer of the DS approximately encompasses the first 30 m out of the total depth of 300 m. Note that this geometry creates a vertical "loop circulation," with \( Q_{\text{Deep}'} \).
flowing upward within the system (sometimes called entrainment flow), carrying water of deep system salinity ($S_{Sys-2}$) to the surface, and then outward on the surface. This circulation represents an analogue to the so-called "estuarine circulation" characteristic of many estuaries.

To balance the salt, there is one additional term: the vertical mixing term ($Q_Z$) which exchanges surface and deep water within the system. This vertical mixing term only contributes if there is a vertical salinity difference between the surface and deep water. The salt balance of the surface layer is given by:

$$Q_{ROSRO} + Q_{PSP} + Q_{PS} + Q_{Ri}S_{Ri} + Q_{Ind-o}S_{Ind-o} + Q_{Deep'}S_{Deep'} - \Sigma sinks + Q_Z(S_{Sys-2}-S_{Sys-1}) = 0$$

(7)

To find the vertical exchange volume ($Q_Z$) we have to eliminate the terms that are equal to or near zero with respect to the salinity. The system inflow is balanced by a vertical flow to the surface layer ($Q_z$); it does not enter the calculation because its magnitude is the same in both vertical directions,

$$Q_z = \frac{-Q_{Ind-o}S_{Ind-o} + Q_{ROSRO} + Q_{Ri}S_{Ri} - \Sigma sinks + Q_{Deep'}S_{Deep'}}{S_{Sys-2}-S_{Sys-1}}$$

(8)

where $Q_{Deep'}$ is the flow that enters the deep layer, flows upward into the surface layer, and out again from the surface layer:

$$Q_{Deep'} = \frac{Q_N S_1 / S_{Deep'}}{S_{Deep'}}$$

(9)

in which, $S_{Deep'} = S_{Sys-2}$.

The water residence times (exchange time) for the two layers are:

$$\tau_1 = \frac{V_1}{Q_{Ind-o} + |Q_z|}$$

(10a)

$$\tau_2 = \frac{V_2}{|Q_{Deep'}| + |Q_z|}$$

(10b)
4. Result and Discussion

4.1. Single-layer versus two-layer system

Two different mathematical models were developed to describe the dynamic behavior of the Dead Sea (DS) based on the above-discussed mass balance equations for single-layer and two layer systems. The models were employed using data from 1997 and forecasts were made for a 100-year period. The two models were formulated following the LOICZ Biogeochemical Modeling Guidelines and validated by comparing the results with other modeling studies for the DS. The models can be used to predict the future behavior of the DS based on the current conditions and under different alternatives, including the proposed RSDSC project.

The first model employed encompassed a single box (a well-mixed system) for which the water and salt mass balance was derived. Salinity variations and water discharged from the desalination plant and salinity variations without brine discharge were taken into consideration. In this study there are significant variations between salinities for both the output from and input to the system, especially when the RSDSC project is considered. In Fig. 8 a single box model for a well-mixed system is used to calculate the water and salt mass balances, which gives as a result the residual flow ($Q_N$) and the residence time ($\tau$) for: a) salinity variations including RO discharge; b) salinity variations excluding RO discharge.

Considering the significant differences in the salinities and densities of the input/output and the DS itself with respect to depth, a two-layer system was judged to be a better description than the single-layer system. The upper layer is about 10% of the total depth, whereas the rest of the lake constitutes a rather homogenous lower layer, as shown in Figure 2. The mixing time (exchange time, $\tau$) between the two layers, determined by the flow that enters the deep layer ($Q_{Deep}$) and the vertical exchange volume ($Q_Z$), is an important parameter to characterize the lake conditions.

Table 5 shows a comparison between the calculation results for the single-layer model and the two-layer model based on given data [30]. The water and salt mass balances included salinity variations with and without RO discharge. In these calculations, the brine disposal from the desalination plant through the RSDSC project was included together with other inputs and outputs (evaporation, rainfall, industrial intake, rivers inflow and industrial brine). Predicted DS volume, surface area, elevation, and cumulative levels for a 100-year period for the single-layer
4.2. Result for single-layer system

After a period of 100 years the elevation is calculated to change from 411m bmsl (in 1997) to 391.4m and 479m bmsl in the two cases (with and without RO discharge) based on a water mass balance, respectively, and to reach 401.9m and 443.6m bmsl in the two cases based on a salt mass balance, respectively.

Discharge from the reject of a RO-plant affects the water level and volume of the DS significantly. The net residual flow volume is 292.2 and 807.8 MCM/yr, and its residence time increased by 48.2% in the water mass balance. Also the net residual flow volume in the salt mass balance is 132 and 427 MCM/yr and its residence time increased from 58 to 116 years, which is about 50% when excluding RO discharge from the system.

4.3. Result for two-layer system

After the same period the calculated elevation has changed from 411m bmsl (in 1997) to 397m and 488m bmsl in the two cases based on a water mass balance, respectively, and to 387m and 425m bmsl based on a salt mass balance, respectively. In this model the exchange time for the upper mixed layer are 1.2 and 1.7 years including RO discharge and excluding RO discharge, respectively, and 11 and 15.3 years for the lower layer including RO discharge and excluding RO discharge, respectively. It has been observed that the exchange time decreased by 29.8% in the upper layer including RO discharge. If excluding the RO discharge the exchange time increased by 28% in the lower layer, as presented in Table 5. For the two-layer system it was found that the amount of volume needed to mix ($Q_z$) the two layers was $10.56 \times 10^9$ m$^3$ with and without RO discharge, respectively, which represents an increase by 31% that will increase the exchange time.
Fig. 8. Simple single-layer (well-mixed) system results by using the modified equations for water-salt balance for $Q_N$ and $r$: by a) salinity and RO discharge included and; b) salinity and RO discharge excluded.

Fig. 9. Two-layer (stratified) system results of entrainment flow ($Q_{Deep}$), vertical exchange volume ($Q_Z$) and exchange time $r$: by a) salinity and RO discharge included and; b) salinity and RO discharge excluded.
Table 5
Results for both single-layer (well-mixed) system and two-layer (stratified) system after one year

<table>
<thead>
<tr>
<th></th>
<th>Water mass balance:</th>
<th>Salt mass balance:</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RO discharge</td>
<td>RO discharge</td>
<td></td>
</tr>
<tr>
<td></td>
<td>included</td>
<td>excluded</td>
<td></td>
</tr>
<tr>
<td>Simple</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Single Box</td>
<td>Residual Volume, $Q_N$ (MCM/yr)</td>
<td>292.2</td>
<td>807.8</td>
</tr>
<tr>
<td>Residence Time, $\tau$ (year)</td>
<td>57</td>
<td>110</td>
<td>48.2</td>
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<tr>
<td>Salt mass balance:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual Volume, $Q_N$ (MCM/yr)</td>
<td>132</td>
<td>427</td>
<td>69.1</td>
</tr>
<tr>
<td>Residence Time, $\tau$ (year)</td>
<td>58</td>
<td>116</td>
<td>50.0</td>
</tr>
<tr>
<td>Two Layers Box</td>
<td>Entrainment Volume, $Q_{Deep}$ (MCM/yr)</td>
<td>138.3</td>
<td>426</td>
</tr>
<tr>
<td>Vertical Exchange Volume, $Q_Z$ (MCM/yr)</td>
<td>10561</td>
<td>7281</td>
<td>31.1</td>
</tr>
<tr>
<td>Exchange Time, $\tau$ (year)</td>
<td>$\tau_1$</td>
<td>1.18</td>
<td>1.68</td>
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<tr>
<td></td>
<td></td>
<td>$\tau_2$ 11.02</td>
<td>15.3</td>
</tr>
</tbody>
</table>

Table 6
Dead Sea simulations for different scenarios, Data based on 1997 [23]

<table>
<thead>
<tr>
<th>Year</th>
<th>Vol. km$^3$</th>
<th>Area km$^2$</th>
<th>H (m) (±)</th>
<th>El. m bmsl</th>
<th>Vol. km$^3$</th>
<th>Area km$^2$</th>
<th>H (m) (±)</th>
<th>El. m bmsl</th>
<th>Vol. km$^3$</th>
<th>Area km$^2$</th>
<th>H (m) (±)</th>
<th>El. m bmsl</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>131</td>
<td>131</td>
<td>0.0</td>
<td>411</td>
<td>131</td>
<td>131</td>
<td>0.0</td>
<td>411</td>
<td>131</td>
<td>131</td>
<td>0.0</td>
<td>411</td>
</tr>
<tr>
<td>30</td>
<td>139.5</td>
<td>136.96</td>
<td>6.745</td>
<td>404.3</td>
<td>131</td>
<td>131</td>
<td>0.0</td>
<td>411</td>
<td>131</td>
<td>131</td>
<td>0.0</td>
<td>411</td>
</tr>
<tr>
<td>60</td>
<td>148.2</td>
<td>143.13</td>
<td>13.3</td>
<td>397.7</td>
<td>131</td>
<td>131</td>
<td>0.0</td>
<td>411</td>
<td>131</td>
<td>131</td>
<td>0.0</td>
<td>411</td>
</tr>
<tr>
<td>90</td>
<td>157.0</td>
<td>149.29</td>
<td>19.63</td>
<td>391.4</td>
<td>131</td>
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<td>0.0</td>
<td>411</td>
<td>131</td>
<td>131</td>
<td>0.0</td>
<td>411</td>
</tr>
</tbody>
</table>
Fig. 10a. Predicted Dead Sea volume, surface area, elevation, and cumulative height for a 100-year period using a single-layer and two-layer model for the water mass balance considering salinity variations including RO discharge and excluding RO discharge.
Fig. 10b. Predicted Dead Sea volume, surface area, elevation, and cumulative height for 100-year period using a single-layer and two-layer model for the salt mass balance considering salinity variations including RO discharge and excluding RO discharge.
4.3. Historical data comparison

Comparison was also made between historical data and model results using a starting level of 420.6 m bmsl in 2006. Results from a 30-year calculation based on water and salt mass balances, including both the single-layer and two-layer system, are presented in Figure 11. The historical data display significant variations during some years, for example, in the 1991 and 1992 there was a large amount of rainfall in the DS catchment causing strong runoff to the DS. The water and salt mass balance for the single-layer model yielded a water level in the year 1976 of about 398.4 and 398.5 m bmsl, respectively, compared to 399 m bmsl from the historical data.

Employing the two-layer model for the water and salt mass balance, a water level of about 401.9 and 400.9 m bmsl, respectively, was found in the year 1976 compared with the observed level 399 m bmsl in the data. A better agreement was obtained when comparing with historical data for the single-layer model both with regard to the water and salt mass balance calculations.

Small differences in results were found with the two-layer model depending on the output/input location to the system and because all water with close or near to zero salinity was excluded, thus omitting the impact of the water with low salinity on the sea level but including all input/output in the water mass balance. Also, differences may be caused by uncertainties in the potash company production and salts extracted from the Dead Sea as industrial work resulting from poor control of the input/output water to the basin. The amount of salt production was found to be approximately 0.1 m annually based on previous studies.

![Comparison between historical data and water/salt balance in the Dead Sea for water level from year 2006 back to 1976](image-url)
5. Conclusion

Two models, namely a single-layer and a two-layer model, were used to predict the behavior of the Dead Sea for shorter and longer periods. It was shown that the results strongly depend on differences in the salinity of the input and output water, and the possible disposal of brine from a planned RO desalination plant, which is a part of the Red Sea-Dead Sea Canal (RSDSC) project. This study included simulations of the conditions in the DS with the models over a period of 100 years assuming that the current conditions continue or that brine discharge from the RO desalination plant is added to the system.

The exchange time or mixing time for a part of the system and the whole system were significantly different: the two-layer model displayed much lower values than the single-layer model when including the RSDSC project. From the results of the two models, the two-layer (stratified) system description seems to give better results than the simple single-layer (well-mixed) system. The exchange time in the upper layer is approximately one year, as presented in Table 5.

It is important to have a mixing time of less than one year in the system in order to avoid less dense input floating in the upper layer, which may affect the evaporation rate. Less dense fluid in the upper layer implies a higher evaporation rate. Compared to previous studies, the single-layer and two-layer box models proved to be robust alternatives to the traditional water and salt balance techniques, enabling successful calculations of the water exchange through a relatively simple description of a very complex and dynamic system such as the Dead Sea.

It was found that the single-layer model predicts a 1.4% and 2% higher lake level than the two-layer model through the water mass balance with and without RO discharge, respectively, and that the two-layer model yields a 3.7% and 4% higher value than the single-layer system through the salt mass balance with and without RO discharge, respectively. Uncertainties in the extracted amount and production of salt and other chemicals from industrial works caused by poor control of the input/output waters in the basin could be a reason for the discrepancies.
7. References

[18] Israel Oceanographic and Limnology Research, Israel Marine Data Center (ISRAMAR), http://isramar.ocean.org.il/21


[34] N. G. Lensky, Y. Dvorkin, and V. Lyakhovsky, Water, salt, and energy balances of the Dead Sea Water Resources Research, 2005, VOL. 41.

