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Published in:
IEEE Asia Pacific Conference on Circuits and Systems (APCCAS), 2002

DOI:
10.1109/APCCAS.2002.1114982

Published: 2002-01-01

Document Version
Peer reviewed version (aka post-print)

AN ALTERNATIVE APPROACH TO INTERPOLATED ARRAY PROCESSING FOR UNIFORM CIRCULAR ARRAYS

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ABSTRACT

In this paper, we propose an alternative approach to array interpolation for the sectorised array processing of a uniform circular array. While other approaches do not fully consider the influence of out-of-sector signals on the in-sector response, our approach explicitly addresses this issue. By controlling the out-of-sector response, our interpolated array demonstrates a significantly improved ability to cope with correlated signals which appear both in-sector and out-of-sector. Our approach also allows one to trade between various operating characteristics such as mean transformation error, in-sector error variance and out-of-sector signal suppression. In this way, a degree of versatility in the design method is provided.

1. INTRODUCTION

Because of their constant resolution coverage over the entire azimuth, uniform circular arrays (UCAs) are of great interest in array processing. In particular, much has been focused on the problem of direction of arrival (DOA) estimation in a correlated signal environment, where low complexity algorithms such as MUSIC can fail to resolve all signals. Although spatial smoothing [1] allows MUSIC to be applied successfully in a multipath environment, this technique can only be used on arrays, e.g. Uniform Linear Arrays (ULAs), whose steering vectors have a Vandermonde form. Also, root-MUSIC, which has been found to perform significantly better for short time-window estimates of the array covariance matrix, is only applicable to ULAs.

A popular approach to applying the above processing techniques to UCAs is to transform the UCA outputs so the effective steering vector has Vandermonde form. Currently, there are two main approaches. The first, pioneered by Davies [2] and applied to spatial smoothing in [3], is a modified spatial-DFT designed specifically for UCAs. It transforms the array response to a spatial "mode-space", which applies over the entire azimuth.

The second approach involves interpolating the response of a virtual ULA from the output of the UCA. This idea was first proposed by Bronez [4] and has been extended to spatially smoothed MUSIC by Friedlander and Weiss [5]. Note that, due to the significant difference between the UCA response and the ULA response over the azimuth, this approach is realistic only over a reduced range of angles (a sector). For 360° coverage, it is necessary to define multiple sectors, each with an appropriately rotated virtual array, and each processed separately.

In [5], the transformation matrix from UCA to virtual ULA is found as the least-squares solution which best maps a finite set of in-sector UCA response vectors to a corresponding set of ULA response vectors. However, this approach neglects the possible effects of out-of-sector signals on the in-sector response. By treating the out-of-sector region as a "don't care" region (in the least-squares sense), it is difficult to say what the response of the interpolated array will be to an out-of-sector signal. In fact, it can be demonstrated that out-of-sector signals can dramatically reduce the ability of MUSIC to resolve a correlated in-sector signal.

In this paper, we propose an alternative approach to the interpolated array transform, maintaining as much control as possible over the response of the interpolated array in the entire azimuth. By including out-of-sector suppression in the problem formulation, the technique demonstrates an improved ability to handle correlated signals which appear in both the in-sector and out-of-sector regions.

†This work was also supported by the Control Systems Research Group at the University of Western Australia

2. PROBLEM FORMULATION

2.1. Array Response Vector

The response at the n^{th} element of an N-element UCA to a far-field, narrow-band signal, incident from azimuth angle θ_ℓ is given by:

$$a_n(\theta_\ell) = G_n \exp\left\{j\frac{2\pi}{\lambda}r\cos\left(\theta_\ell - \frac{2\pi(n-1)}{N}\right)\right\} \quad (1)$$

where $n \in [1 \ldots N]$, λ is the wavelength of the signal of interest, r is the radius of the UCA, and $G_n = \alpha_n + j\beta_n$ is the complex gain factor of the n^{th} element.

In a real array, G_n may be a function of θ_ℓ. However, for simplicity, we will assume identical, omnidirectional sensor elements. In the case of an M-element ideal ULA, oriented along the y-axis with element spacing d , the response at the m^{th} element is given by:

$$b_n(\theta_l) = \exp\left\{j\frac{2\pi}{\lambda}(n-1)\,d\sin\theta_l\right\} \quad (2)$$

2.2. System Model

The array response vector of an N-element array is defined as:

$$\mathbf{a}(\theta) = \begin{bmatrix} a_1(\theta) & a_2(\theta) & \cdots & a_N(\theta) \end{bmatrix}^T \quad (3)$$

For an N-element array with L incident narrow-band, plane-wave signals, we write the output of the array as:

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\Theta})\,\mathbf{s}(t) + \mathbf{n}(t) \quad (4)$$

where $\boldsymbol{\Theta} = \begin{bmatrix} \theta_1 & \theta_2 & \cdots & \theta_L \end{bmatrix}^T$ is the vector of signal DOAs, $\mathbf{A}(\boldsymbol{\Theta}) = \begin{bmatrix} \mathbf{a}(\theta_1) & \mathbf{a}(\theta_2) & \cdots & \mathbf{a}(\theta_L) \end{bmatrix}$ is the $N \times L$ array response matrix, the signal source vector is $\mathbf{s}(t) = \begin{bmatrix} s_1(t) & s_2(t) & \cdots & s_L(t) \end{bmatrix}^T$ and the noise vector is $\mathbf{n}(t) = \begin{bmatrix} n_1(t) & n_2(t) & \cdots & n_N(t) \end{bmatrix}^T$. The noise at the i^{th} sensor element, $n_i(t)$, is assumed to be spatially and temporally uncorrelated additive white Gaussian.

2.3. Previous Approaches

In [4], the transformation matrix ($\mathbf{T}$) was calculated on a row-by-row basis by minimising the overall response of the interpolated array, subject to an equality constraint which fits the interpolated response to that of a ULA at discrete points in a sector. However, the response of the array is minimised in the least-squares sense, meaning other than at the constraint points, the response is unknown.

In [5], the authors transform an N-element physical array to an M-element ULA by matching the desired response at S discrete points. This is best described as the solution to the following equation:

$$\min_{\mathbf{T}} \|\mathbf{TA} - \mathbf{B}\|_F^2 \quad (5)$$

where $\mathbf{T}$ is the $M \times N$ unknown transformation matrix, $\mathbf{A}$ is an $N \times S$ array containing physical array response vectors and $\mathbf{B}$ is an $M \times S$ array containing ULA response vectors.

By treating the out-of-sector region as an LS don't care region, the transform in [5] was able to achieve very low in-sector transformation error. However, by ignoring the out-of-sector response, it creates a situation where one is unable to say exactly how out-of-sector signals will be handled. An interpolated array with a known response over the entire azimuth would not suffer from this problem.

2.4. Proposed Approach

In this paper, we propose a least-squares problem which attempts to fit the transformed UCA response vector to a "target response", continuously, over the entire azimuth (rather than in-sector and at discrete points):

$$\min_{\mathbf{T}} \int_{-\pi}^{\pi} \|\mathbf{Ta}(\theta) - g(\theta)\,\mathbf{b}(\theta)\|^2\,\partial\theta \quad (6)$$

where $\mathbf{T}$ is the array transformation matrix, $\mathbf{a}(\theta)$ is the UCA response vector, $\mathbf{b}(\theta)$ is the ideal ULA response, and $g(\theta)$ is a real function which "shapes" the magnitude of the target response over θ. The *shaping function* used depends on the operating requirements and thus falls to the designer's discretion. In this paper we propose a shaping function based (somewhat arbitrarily) on a Gaussian pulse:

$$g(\theta) = \begin{cases} \exp\left\{-\alpha(\theta - \theta_b)^2\right\} & \theta > \theta_b \\ 1 & \theta_a \le \theta \le \theta_b \\ \exp\left\{-\alpha(\theta - \theta_a)^2\right\} & \theta < \theta_a \end{cases} \quad (7)$$

where θ_a and θ_b are the clockwise and anti-clockwise edges of the optimisation sector, respectively, and α is a rolloff factor which defines the transition width of the shaping function.

The optimum solution to Problem (6) can be found by taking the derivative of the integrand with respect to $\mathbf{T}$ and equating it to $\mathbf{0}$.

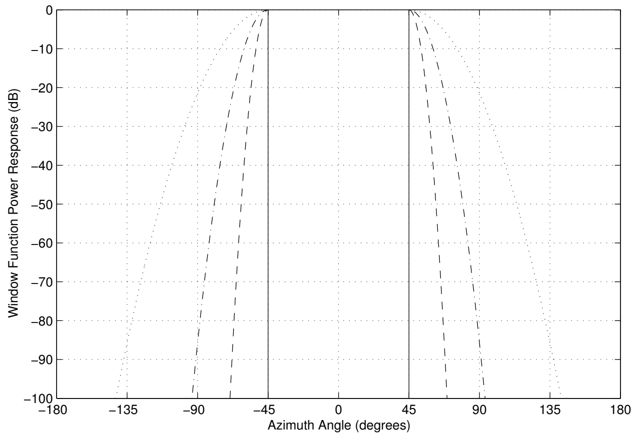

Figure 1: Interpolated array response shaping function. The solid line is $\alpha \to \infty$, the dashed line is $\alpha = 8$, the dot-dahsed is $\alpha = 4$ and the dotted line is $\alpha = 2$

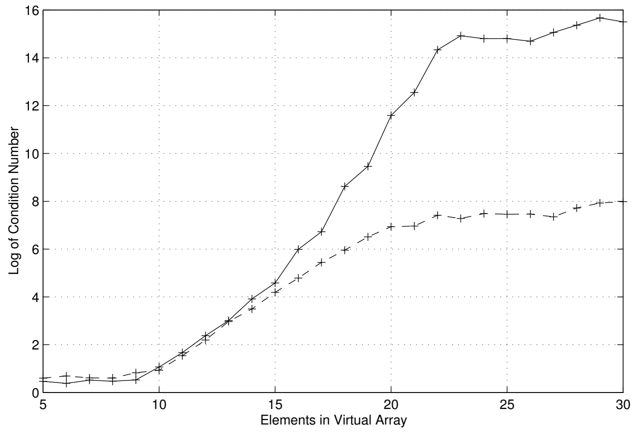

Figure 2: Log of condition number of $\mathbf{T}$ for our technique (solid line) and the method of [5] (dashed line) as a function of number of elements in the virtual array.

3. DESIGN OF THE TARGET RESPONSE

3.1. The Response Shaping Function

There are two main issues to address when designing the shaping function. The first is to keep the transformation error as low as possible. A window function which equals 1 in-sector and 0 out-of-sector may seem ideal, but the sharp discontinuity from full to null response can cause the transformation error to peak significantly at the sector boundaries (Figure 3). The peaks in the error curve can be reduced by smoothing the transition from in-sector to out-of-sector response.

The second issue is concerned with the *operating* and *optimisation* sectors. The virtual ULA loses the ability to resolve closely spaced signals as they deviate from broadside. This places a bound on the largest acceptable operating sector width. The optimisation sector width (defines where $g(\theta) = 1$) and the rolloff factor must then be tailored to the desired operating sector width. While a wider transition region will generally reduce the maximum interpolation error, we also need to suppress sufficiently signals incident in the image of the operating sector (the *image sector* on the "back" of the ULA). Figure 1 shows the Gaussian shaping function (7) for different values of α using an optimisation sector width of $90°$.

Note the value of the shaping function for a particular θ does not directly translate into an actual suppression value. That is, an operating sector width of $70°$ using an optimisation sector width of $90°$ and $\alpha = 2$ will not equate to a minimum image sector suppression of 100dB as Figure 1 may suggest. The actual suppression depends on the optimisation problem, which finds the best transformation in the least-squares sense.

3.2. Virtual Array Size

We define the *aperture ratio* r_a as the ratio of the broadside aperture of the virtual ULA to the aperture of the UCA. $r_a > 1$ means the virtual ULA is larger than the UCA, suggesting a better resolution (signal separation) than the UCA. However, we would expect such a result to increase the transformation error. In general, an r_a of 1 was found to be satisfactory.

In [6] it is stated that $\mathbf{T}$ must be full rank and should be well conditioned, otherwise, subspace methods such as MUSIC can become numerically unstable and fail under certain signal scenarios. In general, transforming from a UCA to a ULA with equal apertures and equal numbers of elements may introduce redundancy, that is, some rows of $\mathbf{T}$ may be linear combinations of other rows. This is because we are interpolating spatial samples which are more closely spaced than our original samples. As a guideline, the elements in the virtual array should be spaced similarly to the elements in the UCA. Since we want the same aperture for both virtual ULA and UCA, this means a reduction in the number of elements in the virtual array.

4. NUMERICAL EXAMPLES

Consider a 30-element UCA with $d = 0.4\lambda$[1], and aperture ratio $r_a = 1$. Figure 2 shows how the condition number of $\mathbf{T}$ for both our approach and that of [5]

[1]Note: the inter-element spacing referred to in this paper is the spacing between adjacent elements in the UCA and is related to the physical radius by $r = \frac{d}{2\sin(\pi/N)}$

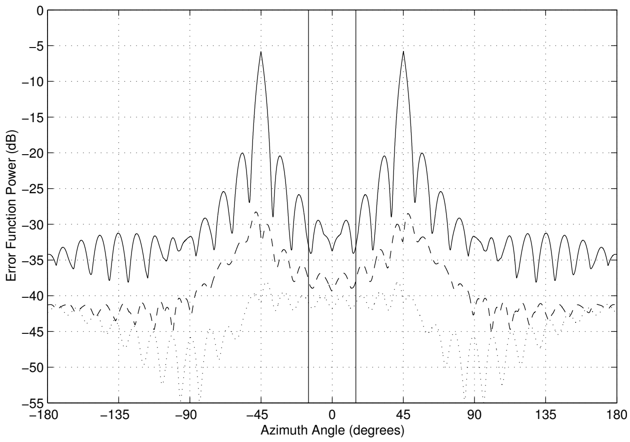

Figure 3: Error function power for $\alpha \to \infty$ (Solid), $\alpha = 4$ (dashed) and $\alpha = 2$ (dotted)

varies with M. We selected $M = 13$ for further analysis as this gives comparable condition numbers of $\mathbf{T}$ for both approaches. Note, in applying the approach of [5], we fitted 8192 points within a sector width of 30°.

We define the transformation error function as the integrand of the LS problem (6):

$$e(\theta) = |\mathbf{Ta}(\theta) - g(\theta)\,\mathbf{b}(\theta)|^2 \qquad (8)$$

Figure 3 plots the transformation error function vs. azimuth angle (θ) for different values of α and an optimisation sector of 90° (The 30° *operating* sector is indicated on figures with solid vertical lines). Ideally we want the error function to be as low as possible in order to reduce the bias in the estimated DOAs.

Figure 4 shows a possible indoor multipath environment with four fully correlated signals incident on the UCA: two in-sector at -8° and 8° and two out-of-sector at 45° and -165° (which is in the image sector). We performed forward/backward spatial smoothing using 3 sub-arrays so the closely spaced correlated signals could be resolved successfully. In this situation, our technique can be seen to provide sharper peaks with less DOA estimation error than the technique of [5]. The smoother rolloff (smaller α) is also shown to improve the peaks.

5. CONCLUSIONS

In this paper, we presented an alternative approach to deriving an interpolated array transform for sectorised array processing. By controlling the out-of-sector response, the approach demonstrates a significantly improved ability to cope with correlated signals which appear in both the in-sector and out-of-sector

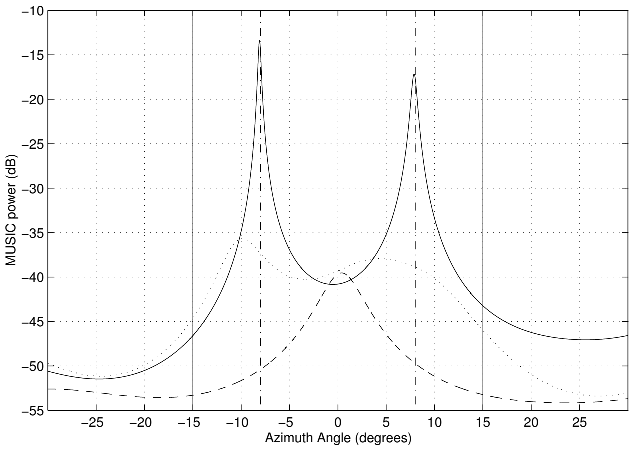

Figure 4: Spatially smoothed MUSIC spectrum using our technique with $\alpha = 2$ (solid line) and $\alpha \to \infty$ (dotted line) and the technique of [5] (dashed line).

regions. Also, by controlling the shaping function, one may trade between relevant factors such as mean transformation error, in-sector error variance and out-of-sector suppression.

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