Selling green transports to a retailer - investment, pricing, and contract choice

Berling, Peter; Eng Larsson, Fredrik

2014

Link to publication

Citation for published version (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
Selling green transports to a retailer – investment, pricing, and contract choice

Peter Berling
Department of Industrial Management and Logistics
Lund University, Sweden
e-mail: peter.berling@iml.lth.se

Fredrik Eng-Larsson
Department of Industrial Management and Logistics
Lund University, Sweden
e-mail: fredrik.eng_larsson@tlog.lth.se

Abstract

Inspired by the observation that capacity contracts are used by some retailers and manufacturers to increase their transport provider’s investments in green vehicles, we investigate and compare a service provider’s optimal investment, and its environmental implications, when the service is sold to a downstream retailer under a volume and a capacity contract respectively. We solve the service provider’s joint pricing and investment problem for the contracts, under the assumption that the retailer uses the service to replenish a warehouse with storable goods. We then show that a capacity contract leads to more transports being carried out using green vehicles, but not necessarily a larger investment in green vehicles. Instead, investment is done in more inventory. In fact, the investment in inventory is non-decreasing in the cost benefit of the green vehicles, which may have a significant negative environmental impact. The implication is that a capacity contract will be better than a volume contract from an environmental perspective only when the green vehicles’ cost benefit is within a given interval. Whether the capacity contract is the more profitable option for the service provider within this interval depends on inventory costs and the relative environmental costs from transportation and inventory. Interestingly, owing to this, regulation that target the price of the conventional vehicles, such as a carbon tax, may lead to both an increase or a decrease in environmental performance.

Key words: green transports, OM and the environment, contracting, capacity contracts
1 Introduction

With the increased interest in environmental sustainability, many firms express a desire to reduce greenhouse gas (GHG) emissions also from outsourced transport operations. In the 2013 Carbon Disclosure Project (CDP) report, more than 30% of the firms report that they measure and work with improving emissions from outsourced distribution and transports (CDP, 2013). Manufacturer Unilever, for instance, claims that “a major objective at Unilever is to reduce our carbon footprint in the distribution of our products...we need logistics providers that are not only capable of moving and storing our goods in a service-driven, cost-effective and reliable way, but also with the smallest carbon footprint possible” (Ehrhart, 2010). Retailer H&M has a similar point of view: “we know that the biggest climate impacts along our value chain happen outside of our operations”, adding that they therefore, “promote environmental consciousness at the transport companies we work with” (H&M, 2013).

As the pressure from large clients such as Unilever and H&M increases, transport service providers struggle to find a proper balance between conventional and green vehicles in their fleets. As one example, when Bring Frigo, a large European transport service provider, was pressured to offer a more carbon efficient replenishment solution for a customer, they decided to start investing in an intermodal truck-train solution (see e.g. Lammgård, 2012; Eng-Larsson, 2012). Similar to other green vehicles (see e.g. Wang et al., 2013), an intermodal solution offers lower operating costs than conventional vehicles due to lower fuel consumption, but requires a larger upfront investment. To find the proper balance in the fleet, the service provider must balance these costs. However, two complicating factors makes the problem challenging. First, compared to conventional vehicles, a large part of the upfront investment in green vehicles is in location-specific infrastructure and customer-specific assets. For instance, Bring Frigo would invest in railway and terminal access to fit the customer’s transport demand with regards to departure times, frequency, and priority. For plug-in hybrids or natural gas vehicles, location-specific investments in charging stations and gas pumps are required. As a result, the green transport capacity has a higher risk of being idle than the conventional capacity of a service provider’s fleet. In the words of Bring Frigo: “if the clients don’t have the volume, we’re still stuck with 49 trailers” (Eng-Larsson, 2012). Second, owing to scale economies and strategic interest, retailers increasingly work with only one service provider for a given replenishment flow, using long-term contracts (usually 1-2 years). Bring Frigo, for instance, is the sole contract service provider for their customer’s transports between the two regions. As a result, the contract, and the price it specifies, will impact the service provider’s optimal investment. To see why, consider a contract where the retailer pays for a fixed amount of capacity independently of utilization (a capacity contract), which is often used for transport services (see e.g. Henig et al., 1997). Under such a contract, a higher capacity price will make
it optimal for the retailer to purchase less transport capacity and instead hold more inventory. A higher inventory level may lead to a more stable ordering pattern, and a more stable ordering pattern will make a larger investment optimal. That is, the service provider’s optimal investment is affected by the service price. The service provider must therefore, for a given contract, decide how much green transport capacity to invest in while simultaneously setting the price of the greener service.

In this paper we investigate the service provider’s problem, and analyze the environmental implications of the optimal decisions. We analyze and compare investment under two long-term contracts often found on the transport market (see e.g. Mellin and Sorkina 2013; Lundin and Hedberg 2012): volume contracts, where the retailer pays for each unit of service when performed; and capacity contracts, where the retailer pays for a fixed amount of capacity in each period independently of utilization. Interestingly, some firms have argued that a shift to long-term capacity contracts will improve environmental performance. The argument typically mirrors that seen in the capacity management literature (Jin and Wu, 2007; Erkoc and Wu, 2005): by making a long-term commitment, the service provider receives a safe profit which enables investment in transport technology that, at scale, is both less costly and less polluting. Similar arrangements have been seen in other industries. For instance, Plambeck and Denend (2011) report that Walmart uses long-term commitments to increase suppliers’ incentives to invest in more sustainable production technology. They quote the executive Vice President of Private Brand Operations saying: “to get from here to scale might require an investment that takes two and a half or three years to pay off. Offering a two-year commitment gives a supplier enough incentive to make the investment.” But - taking into account the optimal decisions of the service provider - does a shift to long-term capacity contracts really lead to less environmental impact in the transport context? And what incentives does a carbon tax create under such an arrangement? In this paper we seek to answer these questions.

In the supply chain literature, volume and capacity contracts have been extensively studied in both one-period settings (e.g. Erkoc and Wu, 2005; Tomlin, 2003; Cachon and Lariviere, 2001; van Mieghem, 1999) and multi-period settings without inventory (Akşin et al., 2008). For instance, Akşin et al. (2008) analyze the same contracts as we do in the context of call center outsourcing. The major difference between these settings, however, is that in the transport service context, the buyer (i.e. the retailer) keeps an inventory which is affected by the contract parameters. This complicates the problem, since the service provider needs to consider how pricing affects the ordering pattern, which depends on downstream operations changes. Our first contribution is to extend this literature by considering a multi-period setting, where inventory can be kept between periods. In our model, a volume contract or a capacity contract (or a combination) is implemented before the start of the first period. From the first period on, in each period, the retailer observes demand and places a replenishment order which is transported to the warehouse by the service provider.
Thus, the work by Henig et al. (1997) and Serel et al. (2001) is perhaps most closely related to this work. In these papers, the authors consider a multi-period problem, where reserved capacity can be combined with a spot-market to replenish a storable good with stochastic demand. Henig et al. (1997) derive the retailer’s optimal inventory policy. Given the buyer’s response in Henig et al. (1997), Serel et al. (2001) determine the optimal pricing policy for the service provider numerically. We propose a slightly different approach to study the multi-period problem. To derive closed form expressions, we use tools from the literature on periodic review capacitated production-inventory systems (see e.g. Alp and Tan, 2008; Angelus and Porteus, 2002), which we embed in a Stackelberg contracting model. By proceeding in this way, we can extend previous work to compare contracts, and conduct sensitivity analyses to understand the impact of e.g. carbon taxes.

Our second contribution is in characterizing the players optimal decisions, and identifying some interesting structural properties. In our analysis, we derive closed form expressions for the retailer’s problem which is then used to solve the service provider’s joint investment and pricing problem under the different contracts. We illustrate how the investment in green transport capacity, under capacity contracts, is non-monotonic in a retailer’s capacity reservation. We show that this also implies that with capacity contracts, a carbon tax may, in fact, lead to less green capacity and a higher expected environmental footprint. This is of particular interest, since carbon taxes has gained traction in business media and among regulators (see e.g. Hargreaves, 2010). In fact, some of the sustainability policies of firms like Unilever and H&M are motivated by an expectation of higher future tax pressure. As discussed by Krass et al. (2013), carbon taxation is an indirect tool, through which regulators try to provide incentives for firms and people to make the “right” technology choice. However, what we see is that this is not necessarily the case: if a regulator decides to make conventional vehicles more expensive to operate, it will not create incentives for investments in green vehicles, it will only make transports more expensive. This finding is similar to the finding of Krass et al. (2013), although the underlying mechanism is slightly different. Here, the investment is non-monotonic because an increase in the cost of the conventional technology makes it optimal for the service provider to increase the price of the greener service. This reduces the retailer’s optimal capacity reservation which, under certain circumstances, reduces the service provider’s optimal green capacity investment as well.

Our third contribution is in conducting a numerical comparison between the volume and capacity contracts, as to understand when it is feasible and environmentally preferable to use each of the contracts. We show that the share of transports being carried out using green vehicles is higher with a capacity contract. Also, in line with previous research in the capacity management literature (e.g. Jin and Wu, 2007), the capacity contract typically leads to a larger investment in green capacity. However, a capacity contract also leads to more inventory at the retailer. This creates an interesting trade-off. According to a report from WEF (2009), warehousing accounts for roughly 10% of all logistics-related emissions. McKinnon et al. (2012)
argue that warehousing accounts for 2-3% of the world’s total energy related emissions. Consequently, the increase in inventory may even offset the reduction in transport related emissions. In the numerical study we show when this is the case. Through this analysis, we hope to add to the growing discussion about the effects of operations decision-making on environmental performance (see e.g. the reviews by Dekker et al., 2012 or Corbett and Klassen, 2006). Previous research on green versus conventional vehicles by e.g. Wang et al. (2013) and Kleindorfer et al. (2012) consider a centralized decision-maker. However, as shown by Mellin and Sorkina (2013), Jaafar and Rafiq (2005) and Hong et al. (2004), most transports are outsourced and controlled by different service providers. In this research we aim to show how this decentralized nature of transport operations is important in determining the environmental impact from a supply chain, through the optimal investment in green transport capacity as well as the effect of carbon taxes.

The remainder of the paper is organized as follows. Our multi-period model is described in greater detail Section 2. In Section 3, we characterize the players’ optimal decisions under a volume contract and a capacity contract, and identify structural properties regarding the sensitivity to changes from a carbon tax. Section 4 discusses an extension for the case when a volume contract and a capacity contract is combined, and in Section 5 we present a numerical illustration to compare contracts and elaborate on the environmental implications of the players’ optimal decisions. Finally Section 6 concludes the paper. All proofs can be found in the Appendix.

2 Model

We consider a service provider (she) and a retailer (he) that engage over multiple periods of equal length (Figure 1). Before the first period, a contract is implemented and the service provider chooses a green capacity investment level. We refer to this phase as the contracting and investment phase. From the first period on, in each period, the retailer observes demand and places a replenishment order, which is transported to the warehouse by the service provider. We refer to this as the replenishment phase. Since the contracting horizon (usually 1-2 years) is generally much longer than a replenishment period (usually one or a few days), we assume an infinite horizon in our dynamic program. In the following, we first explain the replenishment phase, before we explain the game played in the contracting and investment phase. The two phases are illustrated in the timeline in Figure 2.

2.1 The replenishment phase

In the replenishment phase, decisions are made with the contract type and service price as given. In each period of the replenishment phase, the retailer faces stochastic demand \( D_t = D \), which is assumed to be i.i.d.
Figure 1: The model

with strictly increasing cumulative distribution $F_D(\cdot)$, and $P\{D = 0\} = 0$. Let $d_t$ denote the realization of demand in period $t$. At the start of a period $t$, the retailer observes the demand $d_t$ and places a replenishment order of $Q_t$ units, which is transported to the warehouse by the service provider. At the end of the period, any remaining inventory incurs a holding cost for the retailer of $h$ per unit, while backorders incur a penalty $b$ per unit. We assume full backordering in case of any shortages in a period.

The choice of ordering quantity $Q_t$ is limited by the contract. In this paper we focus on volume contracts and capacity contracts and, as an extension, a combination of the two:

- With a volume contract, the retailer has made no capacity reservation, and $Q_t$ can be determined freely by the retailer.

- With a capacity contract, the retailer reserves a capacity $z$ per period, for all coming periods. The retailer is thus constrained by the reservation and can never replenish more than the reserved quantity in any period, i.e. $Q_t \leq z$.

- Under a capacity-and-volume contract, the retailer reserves a capacity $z$ per period, but can order additional units after demand has been observed. Therefore, no constraint exists on the replenishment quantity, and $Q_t$ can be determined freely.

Without loss of generality, we assume that each unit demanded by the end customer requires one unit of transport, so in the long run $E[D]$ must equal $E[Q]$.

2.2 The contracting and investment phase

In this phase the pricing and capacity terms for the contract is determined and the investment in green-capacity is made. This capacity and the contract terms are not changed later in the replenishment phase.
2.2.1 Investment-related costs

The service provider has access to an infinite pool of conventional vehicles. We refer to this pool of vehicles as conventional capacity. The pool of vehicles is used by the service provider and other firms to produce standard transport services. For such services, as well as other commodity services, barriers to enter the market are small, and firms tend to compete in a Bertrand-like fashion for any origin-destination pair on the market. This forces the price close to marginal production cost. To capture this, we let the service provider’s marginal production cost when using a conventional vehicle on the given route be the same as the market price for a standard transport service over the same route. We refer to this price as the conventional market price, or just market price, \( p \). A conventional vehicle that is not used by the service provider in a given period does not generate a cost for the service provider. This captures both cases when the service provider owns the asset and can use it for any other retailer, and cases when conventional capacity is subcontracted.

The service provider can make an investment in transport capacity from green vehicles. This will provide access to green vehicles – i.e. green capacity – starting at the beginning of the first period. The green capacity comes with a fixed cost \( c_F \) per unit and period, independently of whether or not that unit of capacity is used. This cost represents all fixed costs of the capacity, e.g. investment financing, monthly lease payments, the depreciation of vehicles, or the cost to ensure railway access. We assume that \( c_F \) is constant and non-negative. In addition to the fixed cost there is a variable cost, \( c_V \), per unit that is only paid when the green vehicles are used. This corresponds to the operating cost, e.g. the fuel and driver cost.
Furthermore, to avoid non-trivial solutions, we assume that the total cost per period of investing and using the green vehicles is less than the cost of using subcontractor market capacity, i.e. $c_F + c_V < p$. Clearly, this leads to a situation where green transport capacity comes at a cost also when idle.

### 2.2.2 Sequence of events

The contract and investment phase starts when the retailer chooses the service provider as the sole provider for the replenishment. As is common in the supply chain contracting literature, we assume full information, that is, we assume the service provider has full information about the demand distribution and the relevant costs. Consequently, before setting the price and choosing investment level, the service provider observes the retailer’s demand distribution (or, equivalently, she receives a forecast). Based on this information, she anticipates the retailer’s ordering behavior under both types of contracts in line with Section 2.1, and decides on an investment level, $K$, and service price, $w$, to maximize profit. Implicitly, this means that she is also choosing contract (unless that has been specified in advance).

By making the investment, the service provider creates a “technology monopoly” as the only service provider that can serve the retailer at the lower marginal cost, selling the greener service. This position means that she can set the service price freely. However, since the pool of conventional vehicles is available to any firm, it is also available to the retailer. The conventional capacity thus serves as an outside option for the retailer: If he is unhappy with the service provider’s offer, replenishment can be carried out using standard services. Consequently, the price of the greener service can be set freely, but only as long as the retailer’s total costs do not exceed those from using the outside option. For instance, if the service provider decides to invest in an intermodal truck-train solution to serve the retailer, the price of the intermodal service (greener service) can be set freely as long as the retailer is not less profitable using the intermodal service compared to using full truckloads (standard service) from the market. In the words of Wolf and Seuring (2010): “Customer demands for environmentally adapted transport and logistics is rising, but as soon as the question of costs comes up, transport buyers put environmental criteria in second or third line, if at all”. Consequently, as long as the retailer’s expected cost does not exceed the outside option, $pE[D]$, the retailer chooses the contract with the lowest expected cost. If a capacity contract is chosen, the retailer also specifies the capacity reservation level $z$ that minimizes his long-run expected costs. Costs and profits are then realized in each of the subsequent periods, which we assume to continue indefinitely.
3 Analysis

In the following section, we derive the players’ optimal decisions under a volume contract (Section 3.1) and a capacity contract (Section 3.2). Thereafter we analyze the sensitivity of the results with regards to regulation through a carbon tax (Section 3.3). In each section, the replenishment phase is addressed first, after which the contracting and investment phase is addressed. When needed, we let subscript “U” and “C” denote volume and capacity contracts respectively, while subscripts “S” and “R” denote service provider and retailer respectively. When applicable, we will consider the difference in forced and voluntary compliance.

3.1 Volume contract

With a volume contract, the retailer is fully flexible in his replenishment quantity \( Q_t \). Since the replenishment order can be placed after demand has been observed, the optimal replenishment policy for the retailer is simply to let \( Q_t = d_t \). In this way, all of the period’s demand can be satisfied during the period and no extra safety stock is needed. The retailer’s expected profit is thus given by

\[
C_{R,U}(w) = wE[D]. \tag{1}
\]

Knowing the optimal replenishment policy, we move to the contracting and investment phase. We know that the demand is drawn from a stationary distribution \( D \), so the service provider’s expected profit per period is stationary, and given by

\[
\pi_{S,U}(w, K_U) = wE[D] - cV E[\min (D, K)] - pE \left[ (D - K)^+ \right], \tag{2}
\]

where \( pE \left[ (D - K)^+ \right] = pE \left[ \max (D - K, 0) \right] \) is the cost of using conventional capacity. Hence, for a given \( w \), the investment in green capacity is a newsvendor decision: \( \pi_{S,U}(K_U) \) is concave in \( K_U \), maximized at \( K_U^* \) and increasing over \([0, K_U^*]\). It is also clear that the revenue and cost parts of the function are not connected. This means that the price can be optimized independently of the green capacity investment. The optimal price maximizes \( \pi_{S,U}(w, K_U) \) subject to \( C_{R,U} \leq pE[D] \), which yields \( w^* = p \). That is, the retailer will pay the same price for the greener service and the conventional service. This is also what Bring Frigo did: “The pricing towards the customers is the same independently of whether we use the train or trucks. Our customers don’t buy an intermodal solution - they buy a transport solution.” (Eng-Larsson, 2012).

We thus get that the retailer’s long-run expected cost per period is

\[
C_{R,U} = pE[D]. \tag{3}
\]
and the service provider’s optimal green capacity investment is given by the newsvendor fractile,

\[
F_D(K_U^*) = \frac{p - c_F - c_V}{p - c_V}.
\] (4)

Consequently, the volume contract will always lead to a positive investment except in the trivial case where \(c_V + c_F \geq p\). We will use this as a benchmark for the more complicated capacity contract.

### 3.2 Capacity contract

With a strict capacity contract, the retailer cannot order more than the reserved capacity \(z\) in any period, i.e. \(Q_t \leq z\). This means the retailer is facing a capacitated inventory decision, for which a modified base stock policy has been shown to be optimal, see Federgruen and Zipkin (1986). That is, in each period it is optimal to order so that the inventory level reaches the base-stock level \(s\) if possible, otherwise order the capacity limit \(z\). (Note that in this case, by design, \(z \geq E[D]\) or the expected backorder cost will grow to infinity.)

To solve the retailer’s replenishment problem we will use the shortfall, \(V_t = s - IL_t\), i.e. the difference between the desired inventory level, \(s\), and the actual inventory level \(IL_t\). Previous work by e.g. Tayur (1993) and Glasserman (1997) have shown that by considering the shortfall instead of the inventory level, the analysis of a capacitated system is simplified significantly.

The shortfall at the start of period \(t + 1\) can be expressed recursively as \(V_{t+1} = \max(V_t + D_t - z, 0)\). If demands are i.i.d. and \(E[D] < z\), the shortfall converges to a random variable \(V\),

\[
V =^d \max (V + D - z, 0),
\] (5)

where \(=^d\) denotes equality in distribution. This means that we can express the retailer’s long-run expected cost per period by using the shortfall,

\[
C_{R,C}(s, w, z) = wz + hE \left[(s - V)^+\right] + bE \left[(V - s)^+\right]
\]

\[
= wz + h(s - E[V]) + (h + b)E \left[(V - s)^+\right],
\] (6)

where \(s - E[V]\) is the expected inventory on-hand at the end of a period, and \(E \left[(V - s)^+\right] = E \left[\max(V - s, 0)\right]\) is the expected backorder log at the end of a period.

Due to the complexity of capacitated systems, we introduce two assumptions regarding the distribution of the shortfall.
Assumption (A1). The complementary distribution of the shortfall is

\[ P\{V > v\} = \bar{F}_V(v) = \begin{cases} e^{-\lambda(v+z)}, & v \geq 0, \\ 1, & \text{o.w.,} \end{cases} \tag{7} \]

where \( \lambda \) is the conjugate point, that is, the (strictly positive) point where the moment generating function of the tail distribution of the demand is 1,

\[ E\left[e^{\lambda(D-z)}\right] = 1. \tag{8} \]

Assumption (A2). The conjugate point can be expressed as a function of capacity,

\[ \lambda(z) = \frac{W_0(-\mu z e^{-\mu z}) + \mu z}{z}, \tag{9} \]

for \( z > E[D] \), where \( W_0(\cdot) \) is the single-valued Lambert’s W-function.

A major reason for defining the conjugate point \( \lambda(z) \) according to (9) is that the expected shortfall simplifies to \( E[V] = 1/\lambda(z) - 1/\mu \). This makes the analysis tractable, but it also provides a convenient interpretation of the conjugate point: \( \lambda(z) \) can be interpreted as the desired rate of replenishment, since \( 1/\lambda(z) = E[D] + E[V] \). Note that the expected replenishment in each period \( E[\min(V + D, z)] = E[D] \) is less than the desired replenishment in each period \( 1/\lambda(z) \) due to the capacity limit \( z \). Interestingly, more reserved capacity leads to a smaller desired replenishment (“the more you have the less you need it”). That is, the desired replenishment per period is decreasing in \( z \). This is because more capacity leads to less shortfall that needs to be satisfied. A more technical motivation of the assumptions is found in the appendix.

For analytical purposes, we will assume that A1 and A2 hold for the remainder of the paper. As seen in the appendix, this means that the results are exact for exponential demand.

The solution to the retailer’s replenishment problem is summarized in Lemma 1 below.

**Lemma 1.** If assumptions A1 and A2 hold, then under a capacity contract,

i. the retailer’s optimal base stock is

\[ s^*(z) = \max \left( \frac{1}{\lambda(z)} \ln \left( \frac{h+b}{h} \right) - z, 0 \right), \tag{10} \]

which is a convex function in \( z \), decreasing from \( \infty \) at \( z = E[D] \) to 0 at \( \hat{z} = \frac{1}{\mu} \ln \left( \frac{h+b}{h} \right) \), after which it remains constant; and

ii. the retailer’s long-run expected cost with the optimal base stock level, \( C_{R,C}(s^*(z), w, z) \), is convex and
Theorem 1. Suppose assumptions A1 and A2 hold. Then, under a capacity contract, the following hold.

i. The service provider’s optimal price for a given $z$ is unique and given by

$$w^*(z) = \begin{cases} 
    h \left( 1 - \ln \left( \frac{b+h}{h} \right) \right) \cdot \frac{d}{dz} \left( \frac{1}{\lambda(z)} \right), & z < \hat{z} \\
    -b \cdot \frac{d}{dz} \left( \frac{1}{\lambda(z)} \right), & \text{otherwise}
\end{cases}$$

which is a continuous decreasing function in $z$.

ii. The service provider’s long-run expected revenue per period with the optimal price, $R_{S,C}(w^*(z), z) = w^*(z)z$, is convex in $z$ for $E[D] < z < \hat{z}$, and convex decreasing in $z$ for $z \geq \hat{z}$.

The first part of the theorem follows from Lemma 1, since a one-to-one-mapping between price and capacity reservation is guaranteed.

The second part of the theorem implies that if the service provider’s cost function, $C_{S,C}(z)$, is concave and increasing in $z$, there are two possible solutions to her pricing problem. The optimal price is either the highest price (or, correspondingly, the lowest capacity reservation, $z_{\min}$) where the participation constraint $C_{R,C} \leq pE[D]$ is binding, or the price for which the retailer’s best response is $\hat{z}$. That is, there are two possible pricing strategies that may be optimal for the service provider: 1) to set the price sufficiently low to cover the retailer’s additional costs from inventory holding/backorders (equivalently, $z = z_{\min}$), or 2) to set the price sufficiently low for the retailer to reserve enough capacity to make the holding of inventory uneconomical ($z = \hat{z}$). Both strategies can be seen as a price reduction compared to the market price, $p$. That is, the greener service is sold at a lower price than conventional transport services, but require a long-term commitment.

Which price and corresponding capacity reservation that is optimal for the service provider depends on the outside option $pE[D]$ as well as the inventory costs of the retailer. For intuition, these can be translated to a
Figure 3: Illustration of different regions for equilibrium

cost for transportation and a cost for holding inventory. Lower inventory costs relative to the transportation costs will lead to a higher price and less reserved capacity. If the inventory cost is sufficiently low compared to the transportation cost, the equilibrium will be in $z = z_{\min}$, where $z_{\min}$ falls within Region I in Figure 3. In this region, the retailer holds inventory, and the optimal price covers the retailer’s additional costs from inventory holding. If the inventory cost is sufficiently high compared to the transportation cost, then again $z = z_{\min}$, but $z_{\min}$ will fall within Region III. In this region, no inventory is held, and a very small price reduction is given to cover expected backorder penalties. Clearly, this is a much less profitable scenario for the service provider. Only in cases when the relative inventory costs are somewhere in the middle will the solution be $z = \bar{z}$ and fall within Region II. In this case the retailer will get some of the financial benefits of the implemented contract since $C_{R,C} < pE[D]$.

We can now proceed to the cost-part of the service provider’s problem.

**Theorem 2.** Suppose assumptions A1 and A2 hold. Then the service provider’s optimal green capacity investment under voluntary compliance is

$$K_C^*(z) = F_{\bar{z} D}^{-1} \left( \frac{p - c_F - c_V}{p - c_V} \right) = \min \left( z, F_{V+D}^{-1} \left( \frac{p - c_F - c_V}{p - c_V} \right) \right),$$

which is linearly increasing in $z$ for $z < \bar{z} = F_{V+D}^{-1} \left( \frac{p - c_F - c_V}{p - c_V} \right)$ and decreasing towards $F_D^{-1} \left( \frac{p - c_F - c_V}{p - c_V} \right) = K_U$ for larger values of $z$.

From the theorem, we get that the service provider’s long-run expected cost, with the optimal green
capacity investment, is given by

$$C_{S,C}(K^*_C(z),z) = \begin{cases} c_F z + c_V E[D], & z < \hat{z}, \\ c_F z + (p - c_V) \left( \int K F(x) dx + z \bar{F}_V + D(z) \right) + c_V E[D], & \text{otherwise} \end{cases} \quad (13)$$

Under forced compliance, $C_{S,C}(K^*_C(z),z) = c_F z + c_V E[D]$ for all $z$ which is trivially concave in $z$. Under voluntary compliance the investment will be less than $z$ if $z$ is sufficiently high. Nevertheless, the cost function still concave increasing in these cases if the demand is exponentially distributed (and for several other demand distributions under assumption A1 and A2). This means that the abovementioned possible equilibria are unchanged by adding the cost-part of the problem, despite the fact that the cost and revenue parts are connected for this contract.

Note that the critical fractile of (12) is the same as for the unit price contract (equation (4)), but for this contract it is subject to another cumulative distribution. This has an interesting effect: an increase in the capacity reservation does not always lead to a larger green capacity investment. Instead, as the capacity reservation increases, the green capacity investment first increases linearly in the reservation, up to a point at which it starts to decrease. Put differently, the capacity reservation leads to an investment that matches the reserved capacity even under voluntary compliance but only up until a certain point: the non-compliance breakpoint, $\hat{z}$. As soon as this point is reached, any increase in the reservation quantity will lead to a smaller green capacity investment. This is due to the composition of the replenishment quantity, and can be thought of as a pooling effect. The replenishment order consist of both shortfall-orders and the period’s demand. When the capacity reservation is low, the shortfall makes up a larger share of the replenishment quantity than when the capacity reservation is large, which has a moderating effect on the variability of the replenishment orders, $Q$. With larger variability in these orders – i.e. with a more unstable ordering pattern – the optimal green capacity investment level is reduced. This effect is, in fact, more general than the lemma states since it applies to all situations where the shortfall is stochastic decreasing in the capacity reservation.

As a consequence of Lemma 1 and Theorem 2, the green capacity investment and the base stock level are positively correlated at higher reservation levels. This is the mirrored opposite of the optimal decision in a centralized system, as seen in, for instance, Angelus and Porteus (2002). The difference is due to the incentives created by the capacity reservation. More inventory at the retailer is advantageous for the service provider from a cost perspective, since it enables more leveled replenishments which, in turn, reduces the risk of having idle green capacity.
3.3 Sensitivity to changes from a carbon tax

In the following section we will focus on the sensitivity of the results with regards to the conventional market price $p$. By changing $p$ while holding other parameters constant, we get an understanding of how sensitive the results are to changes in the cost advantage of the green capacity. That is, an increase in $p$ means that it becomes increasingly advantageous for the service provider to invest in and use the green vehicles since the cost of using conventional vehicles from the vehicle pool increases. The parameter $p$ is, however, of particular interest since it has a policy implication: a regulator can modify this cost upward through taxation that target the transport market. As seen in the introduction, carbon taxes has gained traction in business media and among regulators (see e.g. Hargreaves, 2010). For instance, the German “Maut” tax, and the state of California carbon tax, have been implemented aiming to (among other things) increase the cost of using conventional vehicles. In Europe, the EU is advocating increased road tolls through “a strategy to ensure that the prices of transport better reflect their real cost to society” (EU, 2008). Such, and similar, initiatives will lead to an increase in $p$.

For a volume contract, changes in $p$ are easily evaluated. An increase in $p$ leads to a larger underage cost which increases the optimal investment in green capacity. This change is continuous. Consequently, a higher carbon tax reduces the environmental impact. This is well in line with conventional wisdom and requires no further explanation.

For a capacity contract it is less straight-forward. That is shown in the following Theorem.

**Theorem 3.** Suppose assumptions A1 and A2 hold. Then, under a capacity contract, the optimal capacity $z^*$ is non-increasing in $p$. This implies that

i. $K_C^*(z^*)$ is non-increasing in $p$ for $z^* \leq \tilde{z}$ and non-decreasing in $p$ otherwise under voluntary compliance, and non-increasing in $p$ for all $z^*$ under forced compliance.

ii. $s^*(z^*)$ is non-decreasing in $p$ for all $z > E[D]$.

iii. an increase in $p$ leads to a smaller green capacity investment as well as more inventory if $z^* \leq \tilde{z}$ and $z^* \leq \bar{z}$ or if forced compliance is assumed; changes in outcomes under other circumstances are summarized in Table 1.

<table>
<thead>
<tr>
<th>$z^*$</th>
<th>$z^* \leq \tilde{z}$ or forced compliance</th>
<th>$z^* &gt; \tilde{z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z^* \leq \tilde{z}$</td>
<td>Smaller green capacity investment</td>
<td>Larger green capacity investment</td>
</tr>
<tr>
<td></td>
<td>More inventory</td>
<td>More inventory</td>
</tr>
<tr>
<td>$z^* &gt; \tilde{z}$</td>
<td>No change</td>
<td>Larger green capacity investment</td>
</tr>
</tbody>
</table>
As seen in the theorem, when a capacity contract is used, the effect an increase in $p$ has on the environmental impact depends on the optimal capacity reservation and whether or not a matching green capacity investment can be enforced by the retailer. As seen in Table 1, in several instances an increase in $p$ actually leads to a *smaller* investment in green capacity. Further, in several instances it leads to more inventory. None of these changes improve the environmental performance. In certain instances, an increase in $p$ leads to both a smaller investment and more inventory, which means that the environmental impact is guaranteed to increase. That is, making the conventional vehicles more expensive to operate through e.g. a carbon tax leads, surprisingly, to worse environmental performance.

The intuition is the following. Due to the technology monopoly situation, the service provider can set the price of the service freely, as long as the retailer’s costs do not exceed the costs from using the outside option. Consequently, when the cost from using the outside option increases, the service provider can increase her profit by increasing the service price. When the service price increases, it is optimal for the retailer to reserve less capacity and increase the inventory base stock (or rather, to not reduce it). That is, $z^*$ decreases and $s$ increases (or stays the same). The change in $z^*$ translates to different effects on the service provider’s optimal green capacity investment depending on the situation. For $z^* \leq \bar{z}$, there is inventory in the system, and a reduction in $z^*$ will lead to a build-up of inventory and an increase in inventory related environmental impact. If $z^* \leq \bar{z}$, or if forced compliance is ensured, then all transports are already made using green capacity, so the increase in inventory related impact cannot be offset by a reduction in the transport related environmental impact. If $z^* > \bar{z}$ a decrease in $z^*$ will lead to a larger green capacity investment and more transports made using green vehicles. This can partly offset the possible increase in inventory related emissions. If $z^* > \bar{z}$ an environmental improvement will be seen, since their will be no increase in inventory.

It shall be noted that the above assumes that a capacity contract is used when $p$ changes. However, this choice may be endogenous to the service provider. For small values of $p$ it is likely that a volume contract provides the service provider with a higher profit, particularly if the inventory costs are high. The contract preference, and its implication on the environmental performance, must be taken into consideration if this is endogenous. This is further elaborated upon in the numerical illustration in the coming sections.

### 4 Extension: capacity-and-volume contract

We now consider the case where a capacity contract and a volume contract is combined. We refer to this as a *capacity-and-volume contract*. With such a contract, the retailer reserves a capacity $z$ but, after the demand has been observed, has the option to order additional volume at a fixed price. Under such a contract, the retailer’s optimal replenishment policy is a two critical levels policy. That is, the retailer uses the available
capacity to order up to $s'$ if possible, otherwise orders $z$, and then use additional volume to order up to $s''$ to ensure that the shortfall does not exceed $\Delta = s' - s''$. Naturally, if demand in any period is less than the reserved capacity, no additional volume is used.

The fact that the shortfall is capped by $\Delta$ makes the analysis considerably more complicated. Denote the new shortfall $V'$, and note that this will depend on the maximum shortfall, $\Delta$, as well as the amount of reserved capacity, $z$. While the shortfall process $V'$ converges, its distribution in stationarity is not known for any demand distribution, and we will therefore resort to numerical techniques to solve the problem. If it is assumed that the stationary complementary distribution of the shortfall is

$$P\{V' > v\} = \bar{F}_{V'}(v) = \begin{cases} 
0, & v \geq \Delta, \\
\alpha e^{-\beta v}, & 0 \leq v < \Delta, \\
1, & \text{o.w.,} 
\end{cases} \quad (14)$$

it can be shown that as $\Delta$ increases, the shortfall approaches that of the shortfall without a cap in (7). For smaller $\Delta$, we can numerically evaluate $\alpha$ and $\beta$ to provide a good fit for several types of demand distributions. In our numerical study, we will however stick to exponentially distributed demand.

While we cannot provide explicit formulations for the two critical levels, we note that the total order quantity in each period, $Q' = \min(V' + D, z) + \max(V' + D - z - \Delta, 0)$, has complementary CDF

$$\bar{F}_{Q'}(x) = \begin{cases} 
\bar{F}_D(x) + \int_0^x \bar{F}_{V'}(x-y)f_D(y)dy, & x < z, \\
\bar{F}_D(\Delta + x) + \int_0^\Delta \bar{F}_{V'}(\Delta-y)f_D(y+x)dy, & \text{o.w.,} 
\end{cases} \quad (15)$$

which is independent of $s'$. This means that the optimal base stock $s'^*$, is easily found, given a reserved capacity $z$ and a maximum shortfall $\Delta$. The retailer’s replenishment problem is therefore reduced to a two-dimensional search over $z$ and $\Delta$.

This provides some structure for the contracting and investment phase. As for the pricing of the additional volume, the logic from the volume contract still applies, which means that the optimal price for the additional volume is $p$ per unit. When it comes to the capacity part, numerical techniques are necessary. The service provider’s optimal capacity price and green capacity investment can be found through a search over $w$, with $K_C^*$, given as the solution to newsvendor problem with $F_{Q'}(x)$ as the distribution:

$$K_C^*(z) = F_{Q'}^{-1} \left( \frac{p-c_F-c_V}{p-c_V} \right) = \min \left( z, F_{Q'}^{-1} \left( \frac{p-c_F-c_V}{p-c_V} \right) \right). \quad (16)$$

As we shall see, the key results from the previous section still holds for our examples, despite the increased
flexibility for the retailer. While the mechanisms seem to be the same, the green capacity investment, as well as inventory, is significantly reduced as \( \Delta \) becomes large.

## 5 Numerical Illustration

In this section we present a numerical study to corroborate the analytical findings, highlight some policy-relevant results, and compare volume contracts and capacity contracts. The baseline parameters (Table 2) are based on discussions with industry and chosen to capture the case of a third-party logistics service provider and a retailer importing goods from southern to northern Europe. The service provider handles the transports, and can choose between "conventional" truck transports or invest in a "green" intermodal truck-train solution. A longer discussion on the parameter calibration and the computation approach can be found in an online companion.

To analyze the environmental implications we introduce the corresponding environmental costs \( e_p, e_V, \) and \( e_h \). These costs are the external cost faced by society but can also be seen as an arbitrary measure of environmental impact reported by either player. For instance, if greenhouse gas emissions are measured and reported by the retailer, \( e_p \) captures the greenhouse gas emissions from the replenishment using one service unit (e.g. one truckload) of the conventional capacity.

### 5.1 Drivers of environmental impact

In our model, three measures drive the expected environmental impact: expected use of truck transports (per period), expected use of intermodal truck-train transports (per period), and expected inventory-on-hand. A

---

Table 2: Specification of baseline parameters for the study

<table>
<thead>
<tr>
<th><strong>Parameter</strong></th>
<th><strong>Value</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected demand in each period, ( E[D] )</td>
<td>20</td>
</tr>
<tr>
<td><strong>Intermodal truck-train transport:</strong></td>
<td></td>
</tr>
<tr>
<td>Slot-fees and equipment leasing fee in each period, ( c_F )</td>
<td>$1,000</td>
</tr>
<tr>
<td>Fuel, handling, and driving cost per trailer moved, ( c_V )</td>
<td>$400</td>
</tr>
<tr>
<td>Emissions per trailer moved, ( e_V )</td>
<td>44</td>
</tr>
<tr>
<td><strong>Truck transport:</strong></td>
<td></td>
</tr>
<tr>
<td>Price for full truckload transport over the same distance, ( p )</td>
<td>$1750</td>
</tr>
<tr>
<td>Emissions per truckload transport, ( e_p )</td>
<td>100</td>
</tr>
<tr>
<td><strong>Inventory holding:</strong></td>
<td></td>
</tr>
<tr>
<td>Inventory holding cost per unit and time unit, ( h )</td>
<td>$50</td>
</tr>
<tr>
<td>Backorder penalty per unit and time unit, ( b )</td>
<td>$500</td>
</tr>
<tr>
<td>Emissions per unit and time unit in warehouse, ( e_h )</td>
<td>1</td>
</tr>
</tbody>
</table>
change in the cost parameters have the potential to change the environmental performance of the system, by changing the policies that control these drivers. This change can be gradual if it leads to minor adjustments in e.g. the investment in intermodal transports, but also large and instantaneous if the optimal contract choice changes, i.e. the service provider’s preference shifts from a volume contract to a capacity contract or vice versa. The existence of the break-points $\hat{z}$ (no inventory) and $\check{z}$ (non-compliance) clearly complicates the analysis. In the following, we will in more detail investigate how the market price, $p$, and the inventory cost, $h$ and $b$, impact the equilibrium and thus the drivers of environmental impact.

The market price $p$ represents the cost of using conventional truck transports. An increase in $p$ will lead to a number of effects as seen in Section 3.3. For volume contracts we know that an increase in $p$ would lead to incrementally more intermodal transports. But what happens when capacity contracts are used?

In Figure 4 the expected use of truck transport and intermodal truck-train transport as a function of $p$ is illustrated. Figures 4a-c show a capacity contract, and Figures 4d-f show a capacity-and-volume contract. Starting with the capacity contract, the figures illustrate what we know from Theorem 3. In the first two figures, the non-compliance break-point, $\hat{z}$, is clearly visible. As stated in the theorem, for a market price that leads to a capacity reservation higher than this break-point, an increase in $p$ leads to a larger investment.
(the upper right hand cell of Table 1). In the figure, we see that for small market prices, this is the situation we get. As the market price grows larger, the break-point is reached and, as predicted by the theorem, we see a smaller green capacity investment (we move to the upper left-hand cell of Table 1). In Figure 4c the inventory cost is too large to reach the break-point over the selected values for \( p \). However, we see that the other break-point, i.e. the no-inventory break-point, \( \hat{z} \), is crossed. For low market prices, the solution belongs to the lower right-hand cell of Table 1. As the market price increases, the solution moves to the upper right-hand cell. As we see in the figures, even though the green capacity investment becomes smaller, it does not necessarily affect the environmental impact, since all transports at that point use green capacity. Rather, as also highlighted in theorem, the negative environmental impact in these cases come from increased inventory holding. Nevertheless, this impact may be significant (see e.g. McKinnon, 2010).

For the capacity-and-volume contract in Figures 4c-d, the same major mechanisms as in Figures 4a-c are seen, albeit without the clear break-points. Inventory increases in \( p \), while the expected use of capacity shifts from conventional to green capacity. This means that the main findings for the capacity contract seem to apply also for the much more complicated capacity-and-volume contract, only with a smaller optimal green capacity investment and less inventory on hand. Also, due to the construction of the contract, there will always be an expectation of some conventional capacity being used, which is not the case for a capacity contract.

Figures 4a-f also show the impact of inventory costs on the three drivers. This can be seen by comparing the different figures. As can be expected, the amount of inventory is decreasing with the inventory costs. Since, for the retailer, inventory and reserved capacity are substitutes, this is compensated for by a larger capacity reservation \( z \). The increase in the capacity reservation will make the replenishment, \( Q \), more variable, which leads to a smaller investment and thus lower usage of intermodal transport and an increased reliance on truck transports. The reduction in inventory and the increase in demand variability is faster under the capacity-and-volume contract, as the retailer can use additional volume at cost \( p \), while the difference between \( p \) and \( b \) is decreasing. As a result, there is a faster shift from green to conventional vehicles. The inventory costs has no direct influence on the volume contract. However, an increase of these costs make this contract a more profitable choice for the service provider compared to a capacity contract. This is further explored next.

5.2 Contract comparison

We saw in the introduction that some firms have argued that a shift to long-term capacity contracts will improve environmental performance compared to volume contracts. The argument mirrored that seen in
the capacity management literature (Jin and Wu, 2007; Erkoc and Wu, 2005): by making a long-term commitment, the service provider receiving a safe profit which enables investment in transport technology that, at scale, is both less costly and less polluting.

So what is the benefit of using a capacity contract compared to a volume contract, and how is this benefit impacted by the above-mentioned changes in market price and inventory costs? In this section we try to answer this question by comparing first the financial performance of the contracts, and then the environmental performance of the contracts.

5.2.1 Financial performance

If contract choice is endogenous, the service provider will set contract parameters so that the retailer chooses the contract that is more profitable for the retailer. The service provider’s profit will, clearly, increase with the revenue and decrease with the cost independently of what contract that is used. However, to understand how profit differs between contracts, we need to know what drives revenue and what drives cost. Typically, the revenue will increase with $E[D]$ and $p$ and decrease with $c_F$ and $c_V$, but not in the same manner for capacity contracts as for volume contracts. There are two factors that are driving the difference in profit between the two contracts. I) The capacity contract will lead to a more stable demand for the service provider, so it can be expected that the benefit of using this contract will increase if the underage cost, $c_F$, and/or the overage cost, $p - c_V - c_F$, increases. II) There is a cost associated with ensuring a more stable demand, as it leads to more backorders and/or inventory at the retailer that he must be compensated for.

The financial benefit of using a capacity contract can thus be expected to decrease with $h$ and $b$. That is, when inventory holding becomes more expensive, it becomes less profitable to use a contract that increases inventory holding. However, this is not always the case. If, for instance, $z > \hat{z}$, then an increase in $h$ will not make a difference, as no inventory is kept.

Figure 5a illustrates how the relative profitability for the service provider changes in $h$ and $p$. As the market price increases, the lowest holding cost for which a capacity contract provides a higher profit for the service provider increases. For the service provider, capacity contracts are more profitable for higher market prices and low holding costs. In a transport services context, this translates to situations where the demand is for transportation of commodity goods, in a transport market segment with few competitors and/or a high variable transport cost due to, e.g., a long distance between the origin and the destination. That also means that if a retailer wants to use capacity contracts to incentivize investment, this is the region where the service provider can make profit under such an agreement.
Comparison of a) the service provider’s profit, and b) the total emissions; volume contract vs. capacity contracts, as a function of the market price, $p$, and the holding cost, $h$ (and backorder cost, $b = 100h$).

### 5.2.2 Environmental performance

Figure 5b illustrates the relative environmental performance ($E_i$ denotes the environmental cost for contract $i$) of the capacity contract and the volume contract. Due to the above-mentioned changes in capacity and inventory, we see that capacity contracts lead to better environmental performance for low market prices and low-to-medium inventory holding costs. In a transport services context, this means that incentivizing though capacity contracts, similar to that reported in Plambeck and Denend (2011), is likely to improve the environmental performance if competition among service providers is fierce and the goods are of low-to-medium value. In a less suitable market, the intentions of such schemes may backfire. First because it may be optimal for the service provider to only make a very small investment in green vehicles; second because it will lead to inventory build-up. As seen in Figure 5b, this qualitative insight does not change as the relation between capacity related and inventory related environmental impact increases, only the break-point at which the change occurs. Already at very modest environmental impact from inventory, in this example when inventory related impact per unit in relation to the transport related impact per unit is 4:100, is the zone in which capacity contracts outperform volume contracts very small. The zone in which capacity contracts outperform volume contracts both environmentally and financially is even smaller. This points to the difficulties of implementing such a scheme sucessfully.

A perhaps more realistic alternative is to use the capacity-and-volume contract, as it provides more flexibility for the retailer. However, from Figures 4c-d it is clear that this is, from an environmental point of view, rarely a better choice. Such contracts are environmentally ourperformed by volume contracts at high market prices; and, if inventory related impact is sufficiently low, outperformed by capacity contracts at low
market prices.

It should also be noted that even if an increase in $p$ leads to more green transports under a capacity contract than under a volume contract, volume contracts may still be the more environmental option as it does not lead to inventory build-up. The result is that by increasing $p$ one might first see a gradual improvement of the environmental performance as the amount of transports using the environmentally superior technology is increasing with $p$ under a unit price contract. If the contract choice is endogenous, then, at a certain $p$, the service provider’s contract preference will change, and at this point there will be a jump in the environmental performance. This jump can be upwards or downwards depending on the relationship between $e_p$, $e_V$ and $e_h$. After the jump one might continue to see a gradual improvement but eventually the environmental performance will start to gradually decrease with $p$ and one will start to “push” the equilibrium out of the overlapping area where capacity contracts are more profitable and provide a better environmental performance than unit prices.

6 Discussion and conclusion

This paper studied a system where a service provider sells a transport service to a downstream retailer, and has the option to invest in green transport capacity that will be dedicated to the retailer. We investigated the service provider’s optimal green capacity investment and its environmental implications under volume and capacity contracts, taking into account the special structure of the transport market and the fact that the price of the service should be set simultaneously with the investment decision. To solve the service provider’s problem, we used literature on capacitated production-inventory systems to develop a multi-period model, where the retailer could keep inventory between periods. We showed that by proceeding in this way, we could extend previous research and investigate sensitivity to changes in carbon taxes and compare different contracts.

While the model is stylized in nature, we believe that it captures a few fundamental aspects of the transport market not well researched in the past. Previous research on investments in green transport capacity has largely ignored that most retailers (and manufacturers) have outsourced their transport operations. When an operation is outsourced, incentives to make a certain investment will be dictated by the downstream contract as well as market interactions, which may sometimes be difficult to overview. In this research we have aimed to provide a first step in this direction, by explicitly trying to model the service provider-retailer relationship.

In the sustainability literature, long-term relationships between service providers and retailers is often advocated. For instance, Plambeck (2012) states that “[firms] must find ways not only to reduce emissions
under their direct control but also to influence emissions caused by their suppliers and customers—by providing them information and incentives, and collaborating or even vertically integrating with them”. In this research we see that if firms such as Unilever and H&M decide to work with their service providers in long-term relationships through capacity contracts, this may indeed be a way forward. As shown in this paper, such contracts often lead to more investments in green transport capacity. However, as long as the greener service cannot be sold at a higher price, there are often not enough incentives for the service provider to transfer all freight to green vehicles. The operating cost for green vehicles may be lower, but the dedicated nature and the higher fixed costs of green vehicles create a larger risk for the service provider than conventional vehicles. This risk needs to be compensated for, in some way, if a large scale shift is to be seen. Bring Frigo has tried to move in this direction with little results: “Goods owners are not willing to take this risk at all”, they have stated, “we have tried. But right now it is a buyer’s market.” (Eng-Larsson, 2012). Nevertheless, successful examples exist, although these seem to be initiated by the retailer. And there is more hope for service providers. According to Ehrhart (2010), “what will further drive the demand for greener [logistics] products is the very fact that sustainability has become a key success factor in shaping the reputation of a company and its brands. For multinational companies, in particular, it is important to have a consistent approach to sustainability along the whole value chain”. If this increased focus will change transport purchasing behavior remains, however, to be seen.

That investments in green vehicles often involves a rather complex set of motivators also needs to be understood by the policy-makers. Transport services may be much like other commodities, but there are several aspects of the market that means the actors on the market may react differently than expected to carbon taxes or other regulation. One aspect was highlighted in this paper. When capacity contracts are used, a carbon tax will in many cases not lead to more green transports, only more expensive transports and more inventory.

The system can be made more realistic and several ways. This usually involves providing more options for the players of the game. In reality, there are more contract types, as well as more technology types. There may also be asymmetric information, even though, for an initiated seller, it should not be too difficult to get a good approximation of the true costs of the retailer. Nevertheless, including such aspects may enable us to study a more realistic system, and this is an interesting venue for further research. Such studies could corroborate the findings and investigate the generality of the results. Another direction would be to make a thorough welfare analysis of transport market regulation, where the micro-level decisions that couples inventory and transportation are taken into account. We hope and believe that this study will be of use for such endeavours.
References


Ehrhart, C. E., 2010. Delivering tomorrow - towards sustainable logistics, how business innovation and green demand drive a carbon-efficient industry. DHL.


EU, 2008. Greening transport: new commission package to drive the market towards sustainability.


Mellin, A., Sorkina, E., 2013. The role of contractual and non-contractual relations between transport buyers and providers, in an environmental context.


**Appendix**

For the derivations and proofs, we will use the single-valued Lambert’s W-function, which is defined as the solution to

\[ x = W_0(x)e^{W_0(x)}, \]  

(17)

with the constraint that \( W_0(x) > -1 \). The constraint makes the function single-valued and the unique solution obtained corresponds to the largest solution to the unconstrained definition. Note that

\[ W_0(-xe^{-x}) \in (-1, 0), x > -1, \]  

(18)

and

\[ \frac{dW_0(-xe^{-x})}{dx} = \frac{W_0(x)}{x(1 + W_0(x))}, x > -e^{-1}. \]  

(19)

To simplify notation we let \( g(z) = W_0(-\mu ze^{-\mu z}) \), with \(-1 < g(z) < 0\), for \( z > 1/\mu \). Using the above we get that:

\[ \frac{1}{\lambda(z)} = \frac{z}{W_0(-\mu ze^{-\mu z}) + \mu z} = \frac{z}{g(z) + \mu z}. \]  

(20)
and the three first derivatives;

\[
\frac{d(1/\lambda(z))}{dz} = \frac{g(z)}{(g(z) + 1)(g(z) + \mu z)} < 0, \quad (21)
\]

\[
\frac{d^2(1/\lambda(z))}{dz^2} = -\frac{g(z)}{z(g(z) + 1)^3} > 0, \quad (22)
\]

\[
(1/\lambda(z))^{(3)} = \frac{g(z)(g(z)^2 + g(z)(4 - 2\mu z) + \mu z)}{z^2(g(z) + 1)^5} < 0. \quad (23)
\]

### Assumptions A1 and A2

Glasserman (1997) has shown that the approximation in (7) is effective for demand distributions from the Erlang and hyperexponential families, especially when the coefficient of variation is low. For all distributions that belong to the NBU-class (New Better than Used) the approximation provides a relatively strict lower bound while for distributions that belong to the NWU-class (New Worse than Used) it provides a relatively strict upper bound. In particular, if demand follows an exponential distribution, (7) is exact, and \( \lambda \) is given as the smallest root to \( \lambda = \mu (1 - \exp(-\lambda z)) \). The approximation in (7) can also be slightly modified to provide a good approximation for normal demand, as shown in e.g. Glasserman (1997) and Roundy and Muckstadt (2000). Similar approximations have been used and evaluated by e.g. Toktay and Wein (2001) and Roundy and Muckstadt (2000).

The second assumption concerns how the conjugate point \( \lambda \) depends on \( z \). It holds for exponential demand, and for several other distributions under constraints on the relation between \( z \) and the demand parameters. For example, if demand is normal (with \( E[D] = 1/\mu \)), A2 holds for \( z = \lambda(z)(\sigma^2/2) + 1/\mu \), while it provides an approximation for other cases.

### Proof of Lemma 1

**Proof.** Part i. From (7), \( F_V(v) \) is increasing in \( v \), which means that (??) is convex in \( s \). The solution to the inventory problem of (??) is thus given by \( \tilde{F}_V(s) = h/(h + b) \), which yields

\[
s^*(z) = \frac{1}{\lambda(z)} \ln \left( \frac{h + b}{h} \right) - z. \quad (24)
\]

Let us define \( \hat{z} \) as the capacity level where \( s^*(\hat{z}) = 0 \) from (24). By solving for \( \hat{z} \) we get

\[
\hat{z} = \frac{1}{\mu} \cdot \frac{ke^k}{e^k - 1}, \quad (25)
\]

28
where \( k = \ln((h + b)/h) \).

By construction, \( s^*(z) < 0 \) for \( z > \hat{z} \). This is not a viable solution to (32), since a negative base stock means that there are backorders in each period by design, so costs can be reduced by setting the base stock to zero (which does not lead to more inventory but less backorders). For \( z > \hat{z} \) the base stock level is thus constant at zero. These observations together prove the first part of the lemma.

Part ii. We prove this part of the lemma by first showing that \( C_{B,C}(s^*(z), w, z) \) is convex and continuous in \( (E[D], \hat{z}) \), as well as in \( [\hat{z}, \infty) \). We then show that \( \lim_{z \to \hat{z}^-} \frac{\partial}{\partial z} C_{B,C}(s^*(z), w, z) \) is the same whether the limit is taken from the left or the right. We have

\[
E[V] = \int_0^\infty x \lambda(z) e^{-\lambda(z)(z+x)} \, dx = \frac{1}{\lambda(z)} e^{-\lambda(z)z} = \frac{1}{\lambda(z)} - \frac{1}{\mu}, \tag{26}
\]

and

\[
E[(V - s)^+] = \int_s^\infty (x-s) \lambda(z) e^{-\lambda(z)(z+x)} \, dx = \frac{1}{\lambda(z)} e^{-\lambda(z)(s+z)} = \left( \frac{1}{\lambda(z)} - \frac{1}{\mu} \right) e^{-\lambda(z)s}. \tag{27}
\]

Using (10), we get

\[
E[(V - s^*(z))^+] = \begin{cases} \frac{1}{\lambda(z)} \cdot \frac{h}{h+b}, & z < \hat{z}, \\ \frac{1}{\lambda(z)} - \frac{1}{\mu}, & o.w., \end{cases} \tag{28}
\]

which yields

\[
C_{B,C}(s^*(z), w, z) = w \cdot z + \begin{cases} h \left( \frac{1}{\mu} + \frac{1}{\lambda(z)} \ln \left( \frac{h+b}{h} \right) - z \right), & z < \hat{z} \\ b \left( \frac{1}{\lambda(z)} - \frac{1}{\mu} \right), & o.w. \end{cases} \tag{29}
\]

Since \( s^*(z) \) is continuous, \( C_{B,C}(s^*(z), w, z) \) is continuous. Differentiation over \( z < \hat{z} \) gives us

\[
\frac{\partial C_{B,C}(s^*(z), w, z)}{\partial z} = (w - h) + h \ln \left( \frac{h+b}{h} \right) \frac{d(1/\lambda(z))}{dz}, \tag{30}
\]

which is increasing in \( z > E[D] \) if \( h \ln((h+b)/h) > 0 \), which holds for all \( h > 0 \) and \( b > 0 \). Differentiation over \( z \geq \hat{z} \) yields

\[
\frac{\partial C_{B,C}(s^*(z), w, z)}{\partial z} = w + b \frac{d(1/\lambda(z))}{dz}, \tag{31}
\]

which is increasing in \( z \).

Recall that \( \hat{z} = \mu^{-1} k e^k/(e^k - 1) \), where \( k = \ln((h+b)/h) \). It follows that \( g(\hat{z}) = -hk/b \), and thus \( \mu \hat{z} = k + \hat{h}k/b \), which gives us

\[
\frac{d(1/\lambda(\hat{z}))}{dz} = - \frac{h}{b-hk}. \tag{32}
\]

Inserting (32) into (30) and (31) yields the same result, which means that the left and right derivatives
are the same in \( \hat{z} \). Since \( d(1/\lambda(\hat{z}))/dz < d(1/\lambda(z))/dz \) for all \( z > \hat{z} \), it follows that the retailer’s expected cost is convex for all \( z > E[D] \).

\[ \square \]

**Proof of Theorem 1**

*Proof.* Part i. Continuity and convexity was proven in Lemma 1, which means there is a unique minimizer \( z^* \) for each price \( w \). \( z^* \) is found from first order conditions. Solving for \( w \) gives the result.

Part ii. Differentiating \( R_{S,C}(w^*(z), z) \) over \( z \) yields,

\[
\frac{dR_{S,C}(w^*(z), z)}{dz} = \frac{dw^*(z)z}{dz} = \begin{cases} 
    h - hkJ \left( \frac{1}{\lambda(z)} \right) + \lambda \left( \frac{1}{\lambda(z)} \right) z, & z < \hat{z}, \\
    -b \left( \frac{1}{\lambda(z)} \right) + \lambda \left( \frac{1}{\lambda(z)} \right), & o.w.,
\end{cases}
\]

(33)

where \( k = \ln((h + b)/h) \). Using equations (21)-(22), we get

\[
\frac{dw^*(z)z}{dz} = \begin{cases} 
    h - hkj(z), & z < \hat{z}, \\
    -bj(z), & o.w.,
\end{cases}
\]

(34)

where \( j(z) = \left( \frac{d}{dz} \left( \frac{1}{\lambda(z)} \right) \right) \left( \frac{(g(z)+1)^2-(g(z)+\mu^2)}{(g(z)+1)^2} \right) \). Since \( 0 < (g(z)+1)^2 < 1 \), and \( \frac{d}{dz} \left( \frac{1}{\lambda(z)} \right) < 0 \), \( j(z) \) is non-negative. Consequently, for \( z \geq \hat{z} \) the revenue is strictly decreasing in \( z \). The first case may be increasing in \( z \) since \( h \geq 0 \). To show that both segments are convex, we differentiate \( j(z) \) with respect to \( z \),

\[
\frac{d}{dz}j(z) = \frac{d}{dz} \left( \frac{1}{\lambda(z)} \right) + \frac{d^2}{dz^2} \left( \frac{1}{\lambda(z)} \right) z = 2 \cdot \frac{d^2}{dz^2} \left( \frac{1}{\lambda(z)} \right) + (1/\lambda(z))(3).z.
\]

(35)

For convexity, it should hold that

\[
2 \cdot \frac{d^2}{dz^2} \left( \frac{1}{\lambda(z)} \right) + (1/\lambda(z))(3).z < 0,
\]

(36)

which can be written as

\[
- \frac{2g(z)}{(g(z)+1)^3} < - \frac{g(z)(g(z)^2 + g(z)(4 - 2\mu z) + \mu^2)}{(g(z)+1)^5} \iff 0 < g(z)^2 + 2g(z)\mu z - \mu z + 2. \]

(37)

Note that \( G(z) = g(z)^2 + 2g(z)\mu z - \mu z + 2 < g(z)^2 + 2g(z)\mu z - \mu z + 2 \) for all \( z \geq 0 = 3/\mu \). Using this we have

\[
G(z) = g(z)^2 + 2g(z)\mu z - \mu z + 2 < 3 - (1 - 2g(z_0))\mu z, z < z_0
\]

(38)
with \( G(z) \leq 0 \) for all \( z \geq z_1 = 3/((1 - 2g(z_0)) \mu) \), where \( z_1 < z_0 \). This procedure can continue, so that

\[
G(z) = g(z)^2 + 2g(z)\mu z - \mu z + 2 < 3 - (1 - 2g(z_n)) \mu z, z < z_n,
\] (39)

with \( G(z) \leq 0 \) for all \( z \geq z_{n+1} = 3/((1 - 2g(z_n)) \mu) \), where \( z_{n+1} < z_n \), and \( \lim_{n \to \infty} z_n = 1/\mu \). Thus both segments are convex which concludes the proof.

\[\square\]

**Proof of Theorem 2**

*Proof.* The service provider’s cost is

\[
C_{S,C}(K, z) = c_F K + c_V (K - \int_0^K F_{Q|z}(x)dx) + p(z - K - \int_0^K F_{Q|z}(x)dx).
\] (40)

Since \( F_V(x) \) and \( F_D(x) \) are increasing in \( x \), \( F_{Q|z}(x) \) is increasing in \( x \), and thus the cost is convex in \( K \). First-order conditions then give us

\[
F_{Q|z}(K^{*}_C) = \frac{c_F + c_V - p}{c_V - p} = \frac{p - c_F - c_V}{p - c_V}.
\] (41)

Now, the complementary CDF of \( Q = \min(V + D, z) \) is given by

\[
\bar{F}_{Q|z}(x) = \begin{cases} 
\bar{F}_D(x) + \int_0^x f_D(y) \bar{F}_V(x - y)dy, & 0 \leq x < z, \\
0, & o.w.
\end{cases}
\] (42)

Differentiation over \( z \) yields,

\[
\frac{d}{dz} \bar{F}_{Q|z}(x) = \begin{cases} 
\int_0^x f_D(y) \frac{d}{dz} \bar{F}_V(x - y)dy, & 0 \leq x < z, \\
0, & o.w.,
\end{cases}
\] (43)

which is non-positive, since

\[
\frac{d}{dz} \bar{F}_V(x) = - \left[(x + z) \frac{d\lambda(z)}{dz} + \lambda(z)\right] e^{-\lambda(z)(x+z)} \leq 0.
\] (44)

Consequently, \( F_{Q|z}^{-1}(\cdot) \) is decreasing in \( z \) for \( F_{Q|z}^{-1}(\cdot) < z \). The theorem follows.

\[\square\]
Proof of Theorem 3

Proof. Part i. Differentiation of $C_{B,C}(s^*(z), w^*(z), z)$ over $z$ yields

$$
rac{dC_{B,C}(s^*(z), w^*(z), z)}{dz} = \begin{cases} 
  h \frac{g(z)}{(g(z)+1)^2}, & z < \hat{z}, \\
  b \frac{g(z)}{(g(z)+1)^2}, & \text{o.w.,}
\end{cases}
$$

which is strictly negative for all $z > 1/\mu$. Using (23) we see that $C_{B,C}(s^*(z), w^*(z), z)$ is convex. Since the cost is convex decreasing, an increase in $pE[D]$ will lead to a decrease in $z$ whenever $z^* \neq \hat{z}$. If $z^* = \hat{z}$, an increase in $pE[D]$ will leave $z^*$ unchanged. Thus, $z^*$ is non-increasing in $p$. Theorem 2 established that $V + D$ is stochastic decreasing in $z$. This means that $F_{V+D}^{-1} \left( \frac{p-c_F-c_v}{p-c_v} \right)$ is decreasing in $z$. Consequently, $K_C^*$ is decreasing in $z$ and hence non-increasing in $p$ when $K_C^* \neq z^*$, and non-decreasing otherwise.

Part ii. Follows from above and the fact that $s^*$ is decreasing in $z$ (Lemma 1). Part iii follows. \qed

32