

Research project by Carlo Proietti

Ceteris paribus modal logics for
epistemology and philosophy of
science.

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1 Aims and background

The broad aim of this research project is to develop formal logical methods for clarifying and solving various problems that arise in different areas of philosophical investigation, i.e; epistemology, philosophy of science and temporal reasoning. More specifically, I employ formal methods from modal logic to investigate a certain number of paradoxes and puzzles which can be formalized in logical language.

The development of axiomatic systems featuring modal operators \Box (called *necessity* operator) and \Diamond (the *possibility* operator), with applications to philosophy and other disciplines (mathematics and computer science) is a relatively recent phenomenon which emerged in the second half 20th century. Nevertheless, the modal notions of necessity and possibility, and the problems they involve, are deep-rooted in the philosophical discussion since the Aristotelian analysis of these notions, proposed in *Prior Analytics* and *On Interpretation*. A second key landmark in this two-thousand year history was Leibniz' definition of necessity as truth in all possible worlds. The latter idea was a crucial premise to the development of what is known as possible world semantics, notably due to Saul Kripke and Jaakko Hintikka, during the 1960s. In particular, the key feature of the new semantics consisted in the *relativization* of the universal quantification in Leibniz's definition. More precisely, given a space W of worlds, or points, and a binary accessibility relation R between them, necessity is now defined as "truth in all accessible worlds", i.e.

(\Box) $\Box\phi$ is true in w if and only if ϕ is true in all v such that wRv

and possibility is defined as truth in some accessible world.

(\Diamond) $\Diamond\phi$ is true in w if and only if there is some v such that wRv and ϕ is true in v

Modal logics based on possible world semantics were widely applied in philosophy not only to the analysis of so-called alethic notions such as necessity and possibility, but also to a vast range of concepts such as epistemic ones ("I know that", "I believe that"), deontic ones ("it is obligatory that", "it is permitted that"), temporal ones ("It will be the case that", "It was the case that") and many others. In almost every one of these cases, the possible world approach faced many paradoxes and puzzles which can be formally rendered as derivations of a contradiction or of a counterintuitive conclusion. Many of these have been open problems for centuries; yet others have only recently become an object of philosophers' attention. The antinomies discussed in my PhD thesis are paradigmatic in this sense: on the one hand,

there is the problem of future contingents, dating back to Aristotle in *On Interpretation IX*; on the other, Fitch's paradox was first discovered and formalized in 1963, by Frederic B. Fitch (3).

The future contingents problem is represented by the statement: "if something will be the case then it will necessarily be the case". This directly follows from some accepted premises and the application of standard rules of normal modal logic, but also appears highly problematic, inasmuch as the consequent can be seen to imply the negation of free will. In order to obtain Fitch's paradox, the starting point is the verification thesis: "Every truth is knowable". Againg by applying basic rules of modal logic, one can then derive the clearly untenable conclusion: "Every truth is known".

Insofar as both paradoxes can be derived by the standard rules of classical modal logic, they indicate the inadequacy of this system. Notwithstanding the wide agreement on this diagnosis, there have been many suggestions of widely diverging solutions. Among these, we can distinguish two broader strands, or logic-philosophical approaches:

- a) One can question the entire framework of classical logic and classical modal logic with it. For example, the mainstream logical solutions of the future contingents problem deny the validity of the *law of excluded middle*, looking to alternative logics – such as many-valued logic or super-valuations – for a solution. In the same spirit, solutions to the Fitch paradox are often sought in alternative fields such as intuitionistic or paraconsistent logic, much as for other logico-philosophical paradoxes.
- b) One can instead take a more conservative standpoint, leaving aside non-classical solutions, by observing that in most cases the problem lies specifically in the modal component; in particular, it has been suggested that the modal language and semantics, in which the paradoxes are formalised and investigated, are too *coarse-grained* for the purpose of providing a solution.

In my research project I intend to take this second stand. Among many reasons for endorsing this view, here I single out two - one more pragmatic, the other more theoretical. On the one hand, it seems desirable to withhold classical logic and its toolbox of well-established results and methods; moreover, classical modal logic has been studied extensively and is thus relatively well known.

The theoretical motivation stems from the thesis that the paradoxes of modal logic would benefit from a *generalization* and a further development of the original idea of simple relational structure.

In areas such as dynamic logic for computer programs (see (5)) or dynamic epistemic logic (see (10)) for information change in multi-agent systems, the simple necessity operator \Box is replaced by operators of kind $[\psi]$, where ψ is some formula, possibly containing other encapsulated modal operators. The semantics of these operators is based on multi-relational structures, allowing to express more complex situation features.

In my PhD thesis I developed an axiomatic system with modalities of type $[\Gamma]$ and $\langle\Gamma\rangle$, where Γ is a set of temporal formulas and the accessibility relation is defined as modal equivalence (\sim_Γ) with respect to formulas in Γ . Modal clauses are defined as follows:

$[\Gamma]$ $[\Gamma]\phi$ is true in w if and only if ϕ is true in all v such that $w \sim_\Gamma v$

and

$\langle\Gamma\rangle$ $\langle\Gamma\rangle\phi$ is true in w if and only if there is some v such that $w \sim_\Gamma v$ and ϕ is true in v

This logic is strongly inspired by the logics for *ceteris paribus* preferences, first developed by J. van Benthem, P. Girard and O. Roy (4). In my thesis, I reformulated the paradoxes of Fitch and of future contingents within such a framework; while I retained those basic intuitive features of the relevant modal concepts, I also avoided any counterintuitive results, so as to ultimately weaken (and, in the latter case, nullify) the paradoxical effects.

My future research would move from these background premises, towards a twofold goal. The first aim would be to carry out a more general logical investigation of systems containing $[\Gamma]$ modalities, i.e. axiomatization and completeness results, decidability and other technical results about correspondence w.r.t. first order logic. For example, it is a very well known result (see (8)), that standard modal logics with a \Box operator correspond to a decidable fragment of first order logic with a relational symbol R . Intuitively, as van Benthem, Roy and Girard seem to suggest in their paper, logics with new modalities should correspond to fragments of infinitary logic, but this is still an open question.

A second goal would be to apply this new approach to other logico-philosophical problems. I give two examples of such possibilities:

- (a) The semantics of $[\Gamma]$ and $\langle\Gamma\rangle$ modalities has potential applications to the theory of counterfactual conditionals. The standard Lewis-Stalnaker approach is also based on possible world semantics, where counterfactual conditionals (with a false antecedent) are evaluated with respect to “most similar possible worlds where the antecedent holds”. Similarity between worlds is given as a specific metric relation. The equivalence

relation defining the semantic of $[\Gamma]$ and $\langle \Gamma \rangle$ modalities implicitly encompasses a similarity relation between worlds and could then serve as an alternative account for expressing counterfactual reasoning. Exploring the semantics of counterfactuals is an important task in contemporary philosophy, due to their use in the definition of various concepts in philosophy of science and epistemology. For example, thought experiments are often introduced by counterfactual reasoning (see (7)).

- (b) Newcomb’s paradox in decision theory. This problem is based on a very simple game, in which the player is presented with two opaque boxes labeled A and B; he is then given the choice of taking either the contents of both boxes, or just that of box B. Box A contains 1,000 dollars. The contents of box B, on the other hand, are determined as follows: before the game starts, an infallible *Predictor* foresees the player’s choice. If the player is predicted to choose both, then box B will contain nothing, while in the other case it will contain 1,000,000 dollars. The paradox lies in the fact that a case can be made for evaluating either option as the most rational. This clash of intuitions is due to the presence of the predictor in the game. Some philosophers argue that the paradox is caused by the contradictory assumptions, i.e. the presence of the Predictor and the assumption that the player has freedom of choice. In the modal approach I developed for the problem of future contingents a sharp distinction is made between truth and foreknowledge on the one hand and necessity and the absence of free-will on the other. The use of these modalities has a potential application for the clarification of this problem and his philosophical aspects.

2 Survey of the field and preliminary results

Much of the contemporary literature on modal logic (e.g. (1)) highlights the importance of studying general multi-relational structures for complex modal languages. For this purpose, new mathematical instruments have also been introduced.

The modal systems I study have only very recently become object of attention, since the publication of the seminal (4). In a paper co-authored with Gabriel Sandu (6), we used *ceteris paribus* modalities to attempt to solve Fitch’s paradox. In my PhD thesis I gave a further application to the problem of future contingents, and moreover formalized this modal semantics axiomatically.

3 Project description

In the first phase of my research I propose to develop the technical results about systems containing $[\Gamma]$ modalities such as soundness, completeness, decidability and correspondence w.r.t. first order logic.

In a second stage the goal will be to exploit this results from a more philosophical point of view, moving from the premise that the notion of *ceteris paribus* alternative possible situation underlies several other key philosophical concepts, such as the notion of scientific law.

The philosophical community of the University of Lund is an ideal environment for such a research project, due to the presence of leading logicians working in modal and philosophical logic, such as Professors Bengt Hansson and Wlodek Rabinowicz, and a large active group of researchers in this field. Collaboration with Prof. Hansson would be particularly fruitful owing to his competences in areas of epistemology and decision theory in which the formal methods I am developing are intended to apply.

4 Significance of the subject

Modal logic is applied every day in an increasing number of fields, ranging from philosophy to computer science and information theory. Although modal methods are widely employed in the context of philosophical investigations, they are however often restricted to very basic systems, containing just the (inter-definable) operators \Box and \Diamond . Problems due to the limited expressivity of this language are in most cases the source of alternative non-classical logical approaches. However, many of these alternative approaches come with disadvantages. As an example, consider Fitch's paradox: one possible solution is based on the use of intuitionistic logic; however, this solution strategy overlooks the fact that, given a translation from intuitionistic to classical modal logic (see for instance (2)), one could also render the intuitionistic solution in classical multi-modal logic based on possible world semantics. A chapter of my thesis was precisely dedicated to such problem.

There are many examples in the literature of contexts in which the application of complex modal methods is of particular interest: one such is provided by (9), which discusses how the application of dynamic epistemic logic to the analysis of Fitch's paradox can shed new light on some aspects of the problems which are usually neglected by traditional modal treatments.

In general, the development of more complex modal logics based on possible world semantics, is a useful tool both for deeper investigation of problems and puzzles within classical logic and for understanding alternative non-

classical solutions.

5 Ethical considerations

My research is mainly theoretical and doesn't involve experimental studies on humans or animals. I believe that there are no particular ethical issues or relevant ethical problems related to it.

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