Non-radiating sources in time-domain transmission-line theory

Sihvola, Ari; Kristensson, Gerhard; Lindell, Ismo V.

Published in:
IEEE Transactions on Microwave Theory and Techniques

DOI:
10.1109/22.643757

1997

Citation for published version (APA):

General rights
Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

• Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
• You may not further distribute the material or use it for any profit-making activity or commercial gain
• You may freely distribute the URL identifying the publication in the public portal

Take down policy
If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.
and
\[ \gamma_{EM} = T_m^{-1} P_m^{-1} T^{-1} e - Q_m^{-1} Q e^{-1} \]
\[ \alpha_E = -\frac{1}{\pi} \frac{1}{r} T_m^{-1} P_m^{-1} T^{-1} e - Q_m^{-1} Q e^{-1} \]
\[ \gamma_{EM} = T_m^{-1} P_m^{-1} T^{-1} e - T_m^{-1} T^{-1} e - T_m^{-1} Q e^{-1} . \quad (16) \]

**REFERENCES**


**Nonradiating Sources in Time-Domain Transmission-Line Theory**

Ari Sihvola, Gerhard Kristensson, and Ismo V. Lindell

**Abstract**—The concept of nonradiating (NR) sources is introduced to transmission lines in the time-domain analysis. A method is presented to construct localized voltage and current sources which do not produce any fields outside the source domain. These sources cannot, therefore, be detected by measurements made outside the source region. The importance of such sources for the uniqueness of the inverse-source problem is pointed out, and energy conditions for the uniqueness are discussed. The analysis can be advantageously used in the design and optimization of the electromagnetic compatibility (EMC) properties of transmission lines.

**Index Terms**—Nonradiating sources, partial differential equations, transmission-line theory.

I. INTRODUCTION

Direct problems in electromagnetics have unique solutions, which means that two different fields are necessarily generated by two different sources. However, the inverse problem is not unique without additional constraints. In other words, two different sources may radiate the same electromagnetic field outside the source region. One consequence of this nonuniqueness property of the inverse-source problem is that nonradiating (NR) sources exist. NR sources are such which do not generate any electric or magnetic fields outside their support.

The inverse-source problem in acoustics and electromagnetics has been studied by various authors [1], [3], [4], [7]. These papers treat currents and their radiation in free space from the NR point of view, and give conditions that the source distributions have to satisfy in order not to radiate electromagnetic energy. The construction of an NR-source distribution starts with choice of any function that vanishes outside a finite domain. Applying the wave operator to this function gives a certain source function. Because the resulting source function is a solution of the inhomogeneous-wave equation, it is an NR source because the field it corresponds to is zero outside the source domain.

One of the strong results of the NR-source studies is the following: a time-harmonic electric-current distribution \( J(r, \omega) \) does not radiate electromagnetic fields outside its support if

\[ k \times \int J(r, \omega) e^{ik \cdot r} dV = 0 \]

for \( \omega = c |k| \). The integral behaves well because the integration domain is the support of the current distribution, which is a finite domain. In fact, (1) is a necessary and sufficient condition for a dynamic current to be NR. In other words, the NR condition is that certain components vanish of the transverse part of the spatial Fourier transform of the current density; namely those components for which \( |k| = \omega/c \), where \( c = 1/\sqrt{\mu \epsilon} \) is the radiation velocity in the medium permeating the space [3]. To give one example of a single-frequency
current function satisfying this criterion, take \( \mathbf{J}(\mathbf{r}) = \mathbf{u}_c J(\mathbf{r}) \), which is a spherically symmetric vector function. This current source does not radiate in a homogeneous environment, as is well known.

The question of possible electron models that would be stable in the sense that the dynamics of the charges would not lose energy through radiation puzzled physicists like Sommerfeld, Herglotz, and Ehrenfest early in the beginning of this century. The famous result by Schott [6] is that a rotating spherical shell of charge is NR if the radius of the shell \( b \) satisfies the following condition:

\[
b = \frac{n \pi T}{2}
\]

where \( n \) is integer and \( T \) is the period of the charge motion. This result by Schott does not require that the rotation orbit be circular; as a matter of fact, it does not even have to be planar. Goedecke has generalized these results to more general rotating and spinning charge distributions [5]. This reference gives examples of various localized NR sources that can be asymmetric and nonspherical, and that could also include a spinning current contribution in addition to the orbital movement of the charge. The NR character of these charge constellations is connected to a quantized condition for the orbital and spinning motions, which as a result, leads to the temptation to hypothesize that all stable particles in nature would be “merely NR charge–current distributions whose mechanical properties are electromagnetic in origin,” although Goedecke in [5] seems to be very careful in propagating this suggestion.

The previous results of NR sources in the literature have dealt with waves in unbounded homogeneous space. In this paper, we concentrate on the problem of NR sources in transmission lines. The analysis allows arbitrary time dependence of the fields and sources. The sources in the transmission-line problem are enforced voltages and currents, which can be either distributed or lumped sources. The construction procedure of NR transmission-line sources will be presented. Also, power conditions are discussed because the power balance is obviously different for NR sources from ordinary sources. The construction procedure of NR transmission-line sources is based on the transmission-line equations:

\[
I(z,t) + R(z) I(z) + u(z,t) = 0
\]

The previous results of NR sources in the literature have dealt with waves in unbounded homogeneous space. In this paper, we concentrate on the problem of NR sources in transmission lines. The analysis allows arbitrary time dependence of the fields and sources. The sources in the transmission-line problem are enforced voltages and currents, which can be either distributed or lumped sources. The construction procedure of NR transmission-line sources will be presented. Also, power conditions are discussed because the power balance is obviously different for NR sources from ordinary sources. The construction procedure of NR transmission-line sources is based on the transmission-line equations:

\[
I(z,t) + R(z) I(z) + u(z,t) = 0
\]

bounded region in space, the source \( g_h = Lh \) is NR. This is because the corresponding field \( f_h \) satisfies

\[
L(f_h - h) = 0 \quad B(f_h - h) = 0
\]

assuming \( Bh = 0 \), i.e., the boundary of the problem is outside the support of \( h \). From uniqueness, we now have

\[
f_h - h = 0 \quad \Rightarrow \quad f_h = h
\]

which means that \( f_h = 0 \) outside the support of \( h \). Thus, the source \( g_h = Lh \) does not create (radiate) a field outside the support of \( h \).

III. APPLICATION TO TRANSMISSION LINES

In the time-domain transmission-line theory, the field \( f \) is a combination of the voltage \( U(z, t) \) and current \( f(z, t) \) functions, the source \( g \) is a combination of the distributed series voltage \( u(z, t) \) and shunt current \( i(z, t) \) functions, and the linear operator \( L \) is defined by the following transmission-line equations:

\[
L f = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \partial_z + \begin{bmatrix} 0 & L \\ C & 0 \end{bmatrix} \partial_t + \begin{bmatrix} 0 & R \\ G & 0 \end{bmatrix} \left( \begin{array}{c} U(z, t) \\ I(z, t) \end{array} \right)
\]

\[
= \left( \begin{array}{c} u(z, t) \\ i(z, t) \end{array} \right)
\]

Here, \( \partial_z \) and \( \partial_t \) denote differentiation with respect to \( z \) and \( t \) and the line parameters \( I, C, R, G \) (inductance, capacitance, series resistance, and leakage conductance, per unit length) may be functions of the position coordinate \( z \), but not the time \( t \). The circuit that obeys (2) is shown in Fig. 1.

A. Construction of NR Voltage–Current Combinations

After having written down the operator \( L \) for transmission-line dynamics (2), the NR voltage and current sources can be constructed using the principle presented in the previous section. Denote the function \( h \) by

\[
h = \begin{bmatrix} U_h \\ I_h \end{bmatrix}(z, t)
\]

and the NR source can be expressed as

\[
g_h = \begin{bmatrix} u_h \\ i_h \end{bmatrix}(z, t) = L h = \left( \begin{array}{c} \partial_z \\ G + C \partial_t \end{array} \right) \begin{bmatrix} U_h \\ I_h \end{bmatrix}(z, t)
\]

or, more explicitly, as

\[
\begin{aligned}
\quad u_h(z, t) = \partial_z U_h(z, t) + (R + L \partial_t) I_h(z, t) \\
\quad i_h(z, t) = \partial_z I_h(z, t) + (G + C \partial_t) U_h(z, t).
\end{aligned}
\]

Let us consider some basic examples for NR voltage–current distributions.

![Fig. 1. The transmission line with series voltage and shunt current sources](image-url)
at the points $z_1 < z < z_2$, and in time, $t_1 < t < t_2$. More specifically, let us assume that there is only one nonzero function $U_h(z, t)$ defined by

$$U_h(z, t) = U_0 P(z_1, z, z_2) P(t_1, t, t_2); I_h(z, t) = 0.$$  

Here, the function $P(x, z, t)$ denotes the pulse function, which equals unity when $a < x < b$ and zero otherwise, and $U_0$ is a constant. Because the derivative of the pulse function is a combination of two delta functions, we have for the NR-source functions

$$u_h(z, t) = U_0 \delta(z - z_1) - \delta(z - z_2) P(t_1, t, t_2),$$
$$i_h(z, t) = U_0 P(z_1, z, z_2) \cdot \{ GP(t_1, t, t_2) + C \delta(t - t_1) - \delta(t - t_2) \}.$$  

According to Fig. 2, the source is composed of two series voltage generators plus distributed shunt current generators. The voltage generators are of opposite polarity, $\pm U_0$ at the points $z = z_1$ and $z = z_2$. These are turned on at $t = t_1$ and off at $t = t_2$. The current generators are distributed along the interval $[z_1, z_2]$, and flashing on with amplitude $CU_0$ at the moment $t = t_1$ and another time with the amplitude $-CU_0$ at the moment $t = t_2$. For a lossy transmission line, we also have a current generator distribution $G U_0$.

To understand that this combination of sources does not generate any fields propagating along the line, it is enough to note that during the time interval $[t_1, t_2]$, there exists a potential difference only within the space interval $[z_1, z_2]$. This will lead to leakage current, which is compensated by the continuous current source $G U_0$. The transient that is excited at $t = t_1$ as the voltage generators are turned on is equal and opposite to the effect of the simultaneous current flash with amplitude $CU_0$ and, therefore, no signal can be measured outside the region at any time. Similarly, this happens at the time $t = t_2$ with oppositely directed sources.

The complementary NR source constellation can be constructed with the following choice of the $U_h$ and $I_h$ functions:

$$U_h(z, t) = 0, \quad I_h(z, t) = I_0 P(z_1, z, z_2) P(t_1, t, t_2);$$

out of which we have

$$u_h(z, t) = I_0 P(z_1, z, z_2) \cdot \{ R P(t_1, t, t_2) + L [ \delta(t - t_1) - \delta(t - t_2) ] \},$$
$$i_h(z, t) = I_0 [ \delta(z - z_1) - \delta(z - z_2) ] P(t_1, t, t_2).$$  

This source is illustrated in Fig. 3. Two opposite shunt current sources at the points $z = z_1$ and $z = z_2$ create a circulating current which is limited within the region between these two points. For a lossy line, distributed series voltage generators $RI_0$ supply the voltage lost in the series resistance. Similar to the earlier case, to extinguish the effect of the transients at $t = t_1$ and $t = t_2$, voltage flashes have to be included with amplitudes $L I_0$ at these moments. These voltage sources are continuously distributed along the interval.

**B. Simple NR Sources**

Let us assume that the NR source is limited to an interval in space, $z_1 < z < z_2$, and in time, $t_1 < t < t_2$. More specifically, let us assume that there is only one nonzero function $U_h(z, t)$ defined by

$$U_h(z, t) = U_0 P(z_1, z, z_2) P(t_1, t, t_2); I_h(z, t) = 0.$$  

Here, the function $P(x, z, t)$ denotes the pulse function, which equals unity when $a < x < b$ and zero otherwise, and $U_0$ is a constant. Because the derivative of the pulse function is a combination of two delta functions, we have for the NR-source functions

$$u_h(z, t) = U_0 [ \delta(z - z_1) - \delta(z - z_2) ] P(t_1, t, t_2),$$
$$i_h(z, t) = U_0 P(z_1, z, z_2) \cdot \{ GP(t_1, t, t_2) + C \delta(t - t_1) - \delta(t - t_2) \}.$$  

According to Fig. 2, the source is composed of two series voltage generators plus distributed shunt current generators. The voltage generators are of opposite polarity, $\pm U_0$ at the points $z = z_1$ and $z = z_2$. These are turned on at $t = t_1$ and off at $t = t_2$. The current generators are distributed along the interval $[z_1, z_2]$, and flashing on with amplitude $CU_0$ at the moment $t = t_1$ and another time with the amplitude $-CU_0$ at the moment $t = t_2$. For a lossy transmission line, we also have a current generator distribution $G U_0$.

To understand that this combination of sources does not generate any fields propagating along the line, it is enough to note that during the time interval $[t_1, t_2]$, there exists a potential difference only within the space interval $[z_1, z_2]$. This will lead to leakage current, which is compensated by the continuous current source $G U_0$. The transient that is excited at $t = t_1$ as the voltage generators are turned on is equal and opposite to the effect of the simultaneous current flash with amplitude $CU_0$ and, therefore, no signal can be measured outside the region at any time. Similarly, this happens at the time $t = t_2$ with oppositely directed sources.

The complementary NR source constellation can be constructed with the following choice of the $U_h$ and $I_h$ functions:

$$U_h(z, t) = 0, \quad I_h(z, t) = I_0 P(z_1, z, z_2) P(t_1, t, t_2);$$

out of which we have

$$u_h(z, t) = I_0 P(z_1, z, z_2) \cdot \{ R P(t_1, t, t_2) + L [ \delta(t - t_1) - \delta(t - t_2) ] \},$$
$$i_h(z, t) = I_0 [ \delta(z - z_1) - \delta(z - z_2) ] P(t_1, t, t_2).$$  

This source is illustrated in Fig. 3. Two opposite shunt current sources at the points $z = z_1$ and $z = z_2$ create a circulating current which is limited within the region between these two points. For a lossy line, distributed series voltage generators $RI_0$ supply the voltage lost in the series resistance. Similar to the earlier case, to extinguish the effect of the transients at $t = t_1$ and $t = t_2$, voltage flashes have to be included with amplitudes $L I_0$ at these moments. These voltage sources are continuously distributed along the interval.

**C. Voltage and Current Sources**

In the previous examples, the NR sources consisted of both voltages and currents. We can also design an NR source which only consists of voltage functions. In the above example, we did not use the function $I_h(z, t)$ at all, and the result was a combination of voltage and current sources. With a suitable choice for $I_h(z, t)$, the current source $i_h(z, t)$ can be required to vanish. With this requirement, we find the following condition from (3):

$$I_h(z, t) = -(G + C \delta_t) \int_{t_1}^{t} U_h(z', t) d z'$$  

for the current function. Denoting the integral of the voltage by $M(z, t)$ as follows:

$$M(z, t) = \int_{t_1}^{t} U_h(z', t) d z'$$

we have from (3)

$$u_h(z, t) = \partial_z M(z, t) - (R + L \delta_t) \{ G + C \delta_t \} M(z, t)$$
$$i_h(z, t) = 0$$

which is an expression for an NR purely voltage-type source. The function $M(z, t)$ is an arbitrary function with compact support in space: $M(z, t) \equiv 0$ for $z < z_1$ and $z > z_2$. This is sufficient to guarantee that the $I_h$ and $U_h$ functions also vanish outside the support.

Similarly, we can write an expression for an NR source of a purely current type, by requiring $u_h(z, t) \equiv 0$ from (3), and using the condition for the current function. The result is

$$u_h(z, t) = 0$$
$$i_h(z, t) = \partial_z N(z, t) - (R + L \delta_t) \{ G + C \delta_t \} N(z, t)$$

where again, $N(z, t)$ is an arbitrary function with bounded support in space.

As an example of an NR voltage-only source, choose the following function:

$$M(z, t) = \sin^2 (k z) P(0, z, \pi/k) \sin (\omega t).$$  

This is a monochromatic function, vibrating with the angular frequency $\omega$. Therefore, in time it is not bounded, but spatially it is restricted by the pulse function within a finite interval. The $z$-dependence has been intentionally chosen in a manner such that the function and its first derivative are continuous, with the goal of finding a “soft” source function. The source function contains second-order derivatives of the test function in this case.

The source function is

$$u_h(z, t) = \left[ (2k^2 \cos 2k z - RG \sin^2 k z + \omega^2 LC \sin^2 k z) \cdot \sin \omega t - (RC + LG) \omega \sin^2 k z \cos \omega t \right] \cdot P(0, z, \pi/k)$$

and is illustrated in Fig. 4. The transmission-line parameters in this example are chosen to be those of a 75-Ω coaxial cable with 1-cm diameter and dielectric insulator of relative permittivity $\varepsilon_r = 2.5$.
and loss tangent $\tan \delta = 0.001$. The conductor is assumed to be copper with conductivity $\sigma = 5.7 \times 10^7$ S/m. Microwave-engineering formulas [2, Sec. 9.3] give the following transmission-line parameters for this cable at 1-GHz frequency:

$$C \approx 7.9 \varepsilon_0, \quad L \approx 0.32 \mu_0,$$

$$G \approx 4.4 \times 10^{-1} \text{S/m}, \quad R \approx 2.2 \Omega/\text{m}$$

where $\varepsilon_0 \approx 8.854 \times 10^{-12}$ As/Vm and $\mu_0 = 4\pi \times 10^{-7}$ Vs/Am are the free-space permittivity and permeability, respectively. The width $L = \pi/k$ of the source function is chosen to be 30 cm. Note that this length does not need to have any connection to the frequency of the wave, nor to its free-space wavelength. $L$ is only the width of the support of the NR source function and it is determined by the manner the source is excited.

In the numerical calculation of Fig. 4, the pulse function is approximated by a combination of hyperbolic tangent functions as follows:

$$P(x_1, x, x_2) \approx \frac{1}{2} \{ \tanh [a(x - x_1)] - \tanh [a(x - x_2)] \}$$

where the parameter $a$ adjusts the steepness of the step. In the calculations, the value $a = 200$ was used.

IV. POWER CONDITIONS

A. Lossless Transmission Line

It was pointed out earlier that the existence of NR sources is tantamount to the nonuniqueness of the inverse-source problem. To ensure uniqueness for this inverse-source problem, certain conditions for the source must be imposed. In fact, the existence of an NR source implies that no energy is given by the source to propagate along the transmission line. Thus, if part of the source gives energy to the line, another part must absorb the energy, or all supplied energy shall have to be dissipated into losses in the source region. Therefore, one may suggest that NR sources that are nonabsorbing at the same number, i.e., the condition

$$Q(z, t) = u(z, t) \bar{U}(z, t) + i(z, t) \bar{U}(z, t) \geq 0$$

should be valid for all points $z$ and all times $t$.

Let us consider this condition for the NR source (3):

$$Q_h(z, t) = \left( \begin{array}{c} U_h \\ \bar{I}_h \end{array} \right) \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right) \left( \begin{array}{c} U_h \\ \bar{I}_h \end{array} \right)$$

$$= \left( U_h \right)^T \left( G + C \partial_t \frac{\partial}{\partial_z} + R + L \partial_z \right) \left( U_h \right)$$

$$= \partial_z \left( U_h I_h \right) + \frac{1}{2} \partial_t \left( C U_h^2 + L I_h^2 + G U_h^2 + R I_h^2 \right) \geq 0.$$

(5)

For the previous example with $I_h(z, t) = 0$, $u_h(z, t) = U_0 P(z_1, z_2) P(t_1, t, t_2)$, we have

$$Q_h(z, t) = \frac{1}{2} \partial_z \left( C U_h^2 + L I_h^2 \right)$$

$$= U_0^2 \left[ C \delta(t - t_1) - \delta(t - t_2) \right] + G P(t_1, t, t_2)$$

$$= U_0^2 \left[ C \delta(z - z_1) - \delta(z - z_2) \right] + G P(t_1, t, t_2)$$

$$= \partial_z \left( U_0 P(z_1, z_2) \right) \geq 0.$$
information about the scatterer. This problem has large similarities to the inverse-source problem due to the presence of equivalent induced sources in the scatterer.

The aim of this paper has been to present a method to construct NR-source distributions for transmission-line problems. The inverse-source problem does not seem to have been previously studied in transmission-line theory. An application of the partial differential operator \( \frac{\partial}{\partial x} \) on a localized function yields source functions that are bounded in space and which do not generate any voltage and current waves traveling along the transmission line. These sources cannot be detected with any voltage or current measurements on the transmission line, external to the source region.

With simple generating functions, such NR sources could be created that can be intuitively understood as being NR. Two basic examples of such type with both voltage and current sources were discussed. The idea behind these source combinations was that the current (voltage) was confined within the source region by a certain source, and to extinguish the transients resulting from the onset and offset of this source, voltage (current) flashes of opposite polarity have to be added. Power conditions were also discussed because energy balance leads to certain requirements for the character of NR sources. If no power is flowing out of the source region, the power emitted by the source either has to be absorbed by another part of the same source, or be dissipated in the losses of the transmission line. This would suggest that a nonabsorbing source and an NR source cannot exist in a lossless transmission line.

This paper also presents a way of constructing NR sources that consist of only voltage functions or only current functions. In practical applications, this type of source description may be more useful. The forced voltage-only source could be thought of as a slot in the wall of a waveguide which an external plane wave is illuminating. The amplitude can be controlled by the width of the opening. A pure current source would be the result of surface currents induced on an unshielded microstrip line under a similar exposure to an incident field. The present analysis of NR transmission-line sources is hopefully useful for EMC applications, eliminating interference, and the design of microwave equipment in difficult electromagnetic environments.

ACKNOWLEDGMENT

This paper was inspired by the Ph.D. dissertation of Dr. J. Lundstedt of the Department of Electromagnetic Theory, Royal Institute of Technology, Stockholm, Sweden. The authors acknowledge the helpful discussions with Prof. S. Ström and Dr. S. He, also from the Department of Electromagnetic Theory. The authors also thank Dr. W. R. Stone and Prof. K. J. Langenberg for advice concerning the existing literature of NR sources.

REFERENCES