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Decoding Procedure Capacities for the Gilbert-Elliott Channel

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Abstract — Sequential decoding for the Gilbert-Elliott channel is considered. The decoding procedure capacity \( C_0 \) is defined to be the supremum of the rates for which there exists a code that gives arbitrarily small decoding error probability. For different assumptions of the decoder's knowledge of the channel states expressions for \( C_0 \) are derived.

I. INTRODUCTION

Assume that a tree code is used together with sequential decoding to communicate over the Gilbert-Elliott channel. Let \( P(\mathcal{E}) \) denote the average probability of decoding error over the ensemble of random, infinite depth tree codes. In this paper we address the question: “When will \( P(\mathcal{E}) \to 0 \)?”

Consider the Gilbert-Elliott channel model and denote the error probabilities in the Good and Bad states by \( p_G \) and \( p_B \), respectively. Furthermore, let \( b_G \) and \( b_B \) denote the fraction of time spent in the Good and Bad states, respectively.

II. DECODING PROCEDURE CAPACITY

Let us define the decoding procedure assumptions, \( D \). The optimistic assumption, \( D = o \), assumes that the decoder has a complete knowledge of the channel state, which could be given by a genie. The pessimistic assumption, \( D = p \), assumes that the decoder neither is given any channel state information nor tries to make any estimate of it. Given the decoding procedure assumption \( D \) and the use of the Gilbert-Elliott channel, let \( C_0 \) denote the supremum of the rates for which we can guarantee that there exists a code that gives an arbitrarily small decoding error probability \( P(\mathcal{E}) \). We will call \( C_0 \) the decoding procedure capacity.

We have proved that the decoding procedure capacities are given by

\[
C_o = B_B \cdot G_{\text{sec}}(e_B) + B_G \cdot G_{\text{sec}}(e_G)
\]

and

\[
C_p = B_B \cdot (G_{\text{sec}}(e_B) - h(b)) + B_G \cdot (G_{\text{sec}}(e_B) - h(g))
\]

where \( b \) and \( g \) denote the transition probabilities from Good to Bad and from Bad to Good, respectively, in the channel model.

Theorem 1 Given the Gilbert-Elliott channel and the decoding procedure assumptions, the use of a rate \( R \) random, infinite depth tree code with the stack decoder, then for any rate \( R < C_0 \), and \( \eta \in \mathbb{Z}^+ \),

\[
P(N \geq \eta) \to 0 \quad \text{if} \quad \eta \to \infty,
\]

where \( N \) is the number of computations in an incorrect subtree.

When we wish to transmit over an ordinary Discrete Memoryless Channel at rates (above \( R_{\text{comp}} \) and) close to its capacity, it is sufficient to allow the number of computations of sequential decoding to go to infinity to be able to guarantee that \( P(\mathcal{E}) \) can be chosen arbitrarily small. We will show that this is also sufficient for transmission close to rates \( C_0 \), which is the motivation why we call these rates “decoding procedure capacities”.

Theorem 2 Given the assumptions of Theorem 1, then for any rate \( R < C_0 \) the average probability of decoding error

\[
P(\mathcal{E}) \to 0,
\]

if the number of computations, \( N \), is allowed to go to \( \infty \).

Since the important condition in Theorem 2 is that \( R < C_0 \), it is clear that the theorem's statement, given the decoding procedure assumptions, is equivalent to stating that the maximal transmission rate over the Gilbert-Elliott channel is at least the rate \( C_0 \).

In the pessimistic case we can interpret this as follows. For arbitrarily small \( P(\mathcal{E}) \), there exists a code such that the transmission rate will be (at least) \( C_0 \), even without any knowledge of the channel state or any attempt to estimate it.

III. CHANNEL CAPACITY

A common method to lowerbound \( C_{\text{sec}} \) is to calculate \( G_{\text{sec}}(e) \), where \( e = B_B \cdot e_B + B_G \cdot e_G \), but it turns out that \( C_0 \) is a better lower bound for channels with a stable behaviour. The optimistic case helps us to find a stronger result:

Theorem 3 Given that the receiver has a complete channel state knowledge, then the channel capacity for the Gilbert-Elliott channel \( C_{\text{sec}}^{\text{EE}} \) is equal to

\[
C_{\text{sec}}^{\text{EE}} = C_0.
\]

From the proof of Theorem 3 follows immediately

Corollary 4 Given that both transmitter and receiver have complete knowledge of the channel state sequence then for the channel capacity of the Gilbert-Elliott channel \( C_{\text{sec}}^{\text{EE}} \) we have

\[
C_{\text{sec}}^{\text{EE}} = C_{\text{sec}}^{\text{EE}}.
\]

It should be noted that the capacities \( C_{\text{sec}}^{\text{EE}} \) and \( C_{\text{sec}}^{\text{EE}} \), in contradiction to what is the case for \( C_0 \), are parameters purely dependent of the channel's properties and that nothing is assumed about the decoding method. In the derivations of \( C_0 \) we assume sequential decoding, but by deriving them we show that they are achievable rates as such, given the decoding procedure assumptions.

REFERENCES


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