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2019

Document Version:
Peer reviewed version (aka post-print)

Link to publication

Citation for published version (APA):

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World Conference on Transport Research - WCTR 2019 Mumbai 26-31 May 2019

Overtakes and dwell time delays for Japanese commuter trains

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Abstract

Reducing train delays important in many countries, even in those like Japan, where punctuality is already high. There is a clear pattern across the literature that the delays typically occur at stations and are recovered on line sections. Previous work has shown that one explanation for this is that trains interact at stations. When trains have different speeds or stopping patterns, overtakes are important even on double-track lines. The latter is often the case in Japanese railways, and we can better understand their railway operations and delays by explicitly studying the way trains overtake each other. This paper uses historical train traffic records from three Japanese railway companies, in total 88 million observations, and finds both that most of the overtakes occur at a small subset of stations, and that only about seven percent of overtakes were executed as scheduled. We also found that the combined dwell time delays decreased in these rare, successful cases but increased in the other scenarios, with a high degree of statistical significance. Looking at the interactions and the resulting dwell time delays, it is also possible to show and evaluate the actions of dispatchers. We found that they often reduced the delays somewhat by shifting the location of overtakes between trains that were either early or delayed. Finally, we suggest that interactions like overtakes can be used to help calibrate and validate simulation models, as they provide another meaningful and quantifiable way to describe the performance of railways, much like delay distributions and punctuality.

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Keywords: Railway; railroad; train; delay; punctuality; train interaction; overtake; timetable; dwell time.

1. Introduction

Improving the punctuality on the railways is an important task in many countries. Even in Japan, where the punctuality is very high, delays are common and a big problem for commuter trains, especially during rush hours. While there are

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many reasons delays occur, the corresponding author’s previous work (Palmqvist et al. 2017a, 2017b) has shown that interactions between trains are an important driver for delays and reduced punctuality. Estimates from the Swedish railways are that every time two trains are at a station at the same time, their punctuality drops by about one percentage point. This result justifies looking closer at the different types of interactions that occur, and their effects on delays and punctuality. A further justification for focusing on dwell time delays is that previous work (Palmqvist et al. 2017c) has shown that the scheduled dwell times are exceeded far more often than the run times, a finding well in line with the international literature.

Just as trains on single-track lines can only meet at stations, on double-tracks the stations are usually the only places where trains can overtake one another. This is especially true in Japan, where bi-directional operation is not possible. By carefully planning or manipulating these overtakes, it is possible to both avoid and recover delays. Analogously, poorly handled overtakes can increase the delays throughout the system. The issue of train overtakes is thus of vital importance for the punctuality of trains on double-track lines, both from the perspective of timetable planners and from that of dispatchers. However, it has not received very much attention in the literature, especially using empirical data. This paper remedies this gap by answering the following research objectives: (1) Describe the possible scenarios in an overtake-type interaction. (2) Describe the distribution of these scenarios in real railway operations. (3) Describe the impact of the overtakes on delays using empirical data. (4) Describe methods to help analyze both real and simulated railway operations with regards to train interactions. (5) Describe the part that train interactions like overtakes play in the overall picture of train delays and punctuality.

To do this we have used extensive historical train movement records from Japanese railways. The hope is that a greater awareness of train interactions can lead to more punctual railway operations across the world.

2. Background

Figure 1. Timetable with a skip-stop operation. The light grey diagonal lines are local trains that stop at every station, while the darker diagonal lines represent rapid trains that only stop at bigger stations. Here the rapid trains only stop at the third station from the top, allowing passengers to transfer between the two trains. The rapid trains overtake the local trains at both the third and fifth stations from the top.

In Japan, the railways are an important part of the urban transport systems. In the three major cities of Tokyo, Nagoya and Osaka, the mode share is as high as 50%. Tokyo alone has on average about 39 million train trips per day. To facilitate the volume of passengers, the trains are operated densely. During the morning rush hours, many double-track lines around Tokyo have 25-30 trains running per hour and direction. The timetables are also designed so that passengers from the suburban areas can reach the big terminal stations quickly. One way this is done is by using skip-stop operation, as illustrated in Figure 1. Stations on commuter lines are often quite near one another, within one or two kilometres, and are served by trains using two or more stopping patterns. Local trains stop at all the stations, while rapid trains only stop at bigger stations. The bigger stations are designed so that the transfer between local and rapid
trains is convenient, usually on the same platform. Smaller stations are instead designed so that the rapid trains overtake the local trains. In this way passengers who live near small stations can get on the local train, ride a few stops to a bigger station, transfer on the same platform to a rapid train, and quickly travel long distances. Even in the Japanese countryside where the frequencies are lower, and lines are sometimes single-track, and, there are rapid trains on the main lines that operate with a skip-stop operation. One of the drawbacks with skip-stop operations is that the overtakes provide a mechanism for the delays to spread from one train to another. This is one of the current problems for railways in the Tokyo area, as trains are often delayed by several minutes during the morning rush hours, and the delays easily propagate between the local and rapid trains.

Looking at the research literature, a recurring result is that train delays regularly occur at stations, and that the dwell times are often insufficient. Wiggenraad (2001) studied seven Dutch train stations and found that the dwell times were longer than scheduled, that the scheduled dwell times did not vary across peak and off-peak, and that passengers caused unnecessary congestion by concentrating around platform access points. Nie and Hansen (2005) studied trains in the station area of The Hague, also in the Netherlands, and found that trains ran at speeds lower than intended. They also found that the dwell times at platforms were systematically extended because of other trains blocking their routes, and because of the behaviour of train personnel. In Norway, Olsson and Haugland (2004) found that in congested areas, the management of boarding and alighting passengers was the key factor for punctuality, while on single-track lines the management of train crossings, or meetings, was the key success factor. Bender et al. (2013) drew similar conclusions: that the time needed for passenger boarding and alighting at stations is a critical element of overall train service performance. Continuing in Norway, Harris, Mjøsund and Haugland (2013) studied delays at stations in the Oslo area and claimed that the delays were often small in nature, poorly recorded, and not well understood. Peterson (2012) studied two Swedish train services and found that the dwell times were usually underestimated, without being sufficiently compensated by margins on the line. Further, Ceder and Hassold (2015) found that one of the main causes for delays in New Zealand was heavy passenger load, which increased the dwell times. Finally, Kim, Kang and Bae (2013) concluded that the important delay causes in South Korea were: short headways, short scheduled run times, delays of preceding trains, and excessive passenger loads. Thus, there is considerable evidence across the world that train delays often occur at stations, and that the scheduled dwell times are often too short.

Once delays occur they can easily spread and propagate to other trains. For instance, Wei et al. (2015) studied the cascade dynamics of delay propagation during severe weather in China. The spread of delays happens more easily in congested situations and when trains need to interact more with each other. One example is by Gorman (2009) who used statistical analysis to study which factors contributed the most to delays for freight trains in the US, and found that the number of meets, passes and overtakes consistently had the highest impact. However, as Parbo, Nielsen and Prato (2016) showed, timetable characteristics are important influencing factors for delays and robustness in railway traffic, and if set properly, can mitigate delay propagation. Nelldal et al. (2009) performed simulation experiments on high speed trains between Gothenburg and Stockholm and found that by ensuring that the minimum headway between these trains and the surrounding traffic was at least five minutes, punctuality would improve by five to ten percentage points. Yuan & Hansen (2008) also found headway times to be effective in reducing knock-on delays, and in another paper (2007) they proposed an analytic model for estimating knock-on delays of trains in complex stations. They found that as the buffer time between trains decreased, the knock-on delays increased exponentially. Similarly, Dewilde et al. (2013) introduced a method to increase timetable robustness in complex stations by maximising the minimum headway time between trains. They found that this approach improved the robustness in the station zone of Brussels by eight percent and reduced knock-on delays in the area by half. Thus, delays easily propagate when the trains interact with one another, and this propagation can be mitigated by changing the timetable. However, the focus has tended to be more on headway- and buffer times than on the successful execution of interactions.

3. Method

In this paper we are focus on trains overtaking one another at stations on double-track lines. Other types of interactions include: meetings on single-track lines, crossing train paths, and headway-type interactions. These interaction types are also very interesting and ought to be studied, but they are not included in this paper. Meetings were excluded
because they are usually only relevant on single-track lines, which are not common in the dataset used for this paper, while crossing train path- and headway-type interactions were excluded because the methods required to identify them in the data are quite different. In future work we will return to these other interaction types, but here we focus on overtaking on double-track lines. We define an overtaking-type interaction as when two trains are at the same station at the same time, are headed in the same direction, and when the order of the train shifts from arrival to departure. Overtaking is relevant on both single- and double-track lines, but it will rarely be an issue on quadruple-track lines, because the trains there will likely use different tracks even when traveling in the same direction. As single-track lines only make up a very small portion of the dataset available for this paper, we focus here on the double-track lines. Thus, in this paper, we study overtaking-type interactions on double-track lines.

For this study we have used data from three Japanese railway companies. Company 1 connects suburban areas with the centre of Tokyo, with some trains going directly to subway lines. Their trains run very densely and make heavy use of skip-stop operations. They provided data from December 2012 through June 2018. Company 2 is a subway company in Tokyo with very dense operation, with skip-stop operation on only a couple of the lines. We used data from ten of their lines over the period of April 2012 to June 2018. Company 3 mainly has lines in the countryside, with some single-track lines in rural areas. The frequencies are lower than for the other companies, and they only use skip-stop operations on the busier lines. Here we only had data from March of 2017 through May of 2018. All companies’ lines are exclusively for passenger trains, including some deadhead runs. The total dataset has about 88 million observations in total, with the share from each company being respectively 73, 20 and seven percent. Although the companies have different characteristics, together they give a more holistic picture. Each observation contains information on the following: which company and line it concerns, the train number, date, station number, track number, and the arrival and departure times both in the timetable and in actual operation. The times are specified using hours, minutes and seconds.

To identify the interactions using this empirical data, we used the following method. As the definition of an overtaking-type interaction requires two trains to be at the same station at the same time, the first step is to identify all such cases. This was carried out by first creating two identical tables, like the one illustrated in Table 1 below, A and B, to hold all the data. A third table, C, was then created to hold all combinations of A and B that match the following set of seven criteria. (1) Either the train number or the date should be different, as the train does not interact with itself. (2) We required that the company and line should be the same, as they all use different infrastructure. (3) The departing station should be the same, otherwise the trains are not in the same place. (4) Neither train should leave before the other has arrived, otherwise they would not be in the same place at the same time. (5) To limit the number of duplicates, we specified that A should arrive before train B, otherwise we would get two copies of each interaction, with each train being found both in the A and B positions. This means that in each record, train A is being overtaken by train B. (6) Train B should depart before train A, because if train A left before B, the train order would be maintained instead of shifting, and no overtake would take place. (7) Simultaneous arrivals or departures are not allowed, as it should not be physically possible using the same infrastructure and only one-directional operation, and both the scheduled and actual times are specified in intervals of seconds. Points (4)-(7) leads to the following inequality between the arrival and departure times for trains A and B:

\[
\begin{align*}
  t_{\text{arrival}}^A &< t_{\text{arrival}}^B \\
  t_{\text{departure}}^B &< t_{\text{departure}}^A
\end{align*}
\]

Beyond these seven criteria, three additional pieces of information are important to classify the interactions: the arrival station, the number of tracks, and the arrival delay at the departure station. The arrival station is used to keep track of direction. If the two trains have the same arrival station, they are traveling in the same direction, and the interaction is of the overtaking-type. However, if the trains have different arrival stations, the interaction is considered a meeting and is not considered in this paper. The number of tracks is used to classify the interactions, as the interactions can be relevant on either single- or double-track, but not quadruple-track lines, and we choose to focus on the double-track lines. Finally, the arrival delay to the departure station is used to identify the trains that were on time to the interaction, and to help isolate the impact of the interaction itself on delays from the effect of any incoming delay on the interaction.
Table 1. Data structure used to find interactions. In the dataset we used there were about 88 million rows following this structure, from three companies and 16 lines. To identify the overtakes, two copies of this data were matched together in the following way. Columns 1-4 were used to ensure that the trains were in the same place and headed in the same direction, so that they would use the same track out of the station. Columns 5-6 were used to make sure that a train was not paired up with itself across the two tables, as the train would not overtake itself. Columns 6-10 were used to ensure that the times overlapped, either in the timetable or in actual operation, and that the order of the trains shifted from arrival to departure. Column 11 was used to specify whether the trains were early, on time or delayed to the interaction. Column 12 was used to select only trains that ran on double-track, as single-track was quite rare in this dataset, and because overtakes are less relevant on quadruple-tracks.

<table>
<thead>
<tr>
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<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>110</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>92</td>
<td>12:01:50</td>
<td>12:02:10</td>
<td>12:03:25</td>
<td>12:04:03</td>
<td>95</td>
<td>82</td>
<td>2</td>
</tr>
</tbody>
</table>

Comparing the timetable to actual operations, there are several possible outcomes with regards to the train interactions, even when only considering overtaking-type interactions. For instance, an overtake can be scheduled but cancelled (1), or it may occur without being scheduled (2). Even when an overtake occurs and is scheduled, the interacting train may be another one than scheduled (3), or the intended one. If the interacting train in actual operations is the same one as in the timetable, it may either arrive to the interaction early (4), on time (5) or delayed (6). Of course, the most common case is that there is no overtaking in either the timetable or actual operations (7). There are thus seven distinct possibilities to consider, even when limited to overtaking-type interactions, all of which are illustrated in Figure 2.

Figure 2. Seven different possible scenarios for train A being overtaken at X-station. From top left: not at all, successfully according to the timetable, by a planned but delayed train, for a scheduled overtaking to be cancelled, for an unscheduled overtaking to take place, for a different train than scheduled to perform the overtaking, and for the train to be overtaken by a train that is planned but early. The possible delay implications are not shown.
4. Results and analysis

Punctuality is a popular way to aggregate delay data into a more easily understood format. The measure can be defined in many ways, by using different subsets of stations or delay thresholds. Using one common definition which states that trains arriving at their destination should be at most five minutes delayed, the punctuality is 97% across the Japanese datasets in this study, which is considered very high. In the research literature, most delays are often found to occur at stations, in the form of dwell time delays. This is also the case in the Japanese datasets used for this study. On average, dwell times are delayed by 15 seconds, while runtimes are on average 13 seconds shorter than scheduled. The pattern of delays mostly occurring at stations and being recovered on line sections thus remains true, despite the remarkably high punctuality of the Japanese railways.

We find that overtakes are relatively rare even on the densely operated Japanese double-track lines. About two percent of all train movements on double-tracks contain an overtaking, either in the timetable or actual operations, and Figure 3 shows that they are mainly centered around a small number of stations. Table 2 below shows the overall breakdown between cancelled and unscheduled overtakes, as well as those that happen with a delayed, early, timely or unintended train. The key point here is to get an understanding of the proportions, how common the different scenarios are. We have not seen this approach used before and believe it may also prove useful when analysing the output of simulation models, in addition to understanding the real operations. In total, we found that there were slightly more overtakes in actual operations than in the timetable, which can be seen by comparing the figure of 23% for unscheduled overtakes with that of 22% for cancelled ones. The least common scenario, at two percent, is that a train is overtaken in a location that was scheduled, but by a different train. Unfortunately, it is also rather rare that overtakes happen with the right train at the right time (here measured with a threshold of 60 seconds), only seven percent of the overtakes are of this type. It is far more common for the interacting train to either be early or delayed, at 14% and 33%, respectively. Thus, despite a very high punctuality, many of the key interactions between trains do not happen according to the timetable.

To understand the impact on delays we focus on the approximately 10% of the trains in the dataset that arrived exactly on time to the station of the overtaking, as shown in Table 2, with a strict delay threshold of 0 seconds. This is done because it eliminates the following sources of uncertainty: (1) the impact of train A’s delay on the type of overtake scenario that is realised, (2) that train A, being on time, has not transferred any delay to train B, and finally that (3) any delay transfer from B to A will happen here, during the impact of the interaction. While combining the dwell time delays in this way may be a small innovation, we believe that it makes the analysis of train interactions much simpler, and that it may be especially useful in evaluating the actions of dispatchers more generally. These delays are also presented in Table 2 below. In the table, the result for the scenario when train B is on time sticks out. The combined dwell time delay is then nine seconds, which is significantly less than for all other scenarios – and the difference is statistically significant to a very high degree (see Table 2). The delays for the three scenarios where train B is either different, delayed, or early, are of similar size, but their standard deviations are quite different. Also, it is
Table 2. Breakdown of overtake scenarios, along with the average and standard deviation of the combined dwell time delay of the trains involved. The distribution is calculated across all trains with overtakes, while the delays are calculated only considering cases where train A arrived exactly on time to the overtake-interaction, with a threshold of 0 seconds. The threshold for when train B is considered early or delayed is 60 s. Displayed are also the results of one-tailed Welch’s t-tests (taking into account different sample sizes and variances, see Welch, 1947) on the statistical significance of the difference in delays, compared to the scenario when train B is on time. The results are highly significant with p-values approaching 0 for all other scenarios.

<table>
<thead>
<tr>
<th>Overtake scenario</th>
<th>All trains with overtakes of some sort (N = 1 201 594)</th>
<th>Where train A is exactly on time (N = 125 103)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution of scenarios</td>
<td>Combined dwell time delays</td>
<td>Average Standard Deviation</td>
</tr>
<tr>
<td>Share</td>
<td>Degrees of freedom</td>
<td>p-value</td>
</tr>
<tr>
<td>Cancelled</td>
<td>22%</td>
<td>24 s   195 s</td>
</tr>
<tr>
<td>Unscheduled</td>
<td>23%</td>
<td>33 s   264 s</td>
</tr>
<tr>
<td>Scheduled, but train B is different</td>
<td>2%</td>
<td>47 s   340 s</td>
</tr>
<tr>
<td>Scheduled, but train B is delayed</td>
<td>33%</td>
<td>51 s   125 s</td>
</tr>
<tr>
<td>Scheduled, but train B is early</td>
<td>14%</td>
<td>46 s   74 s</td>
</tr>
<tr>
<td>Scheduled, and train B is on time</td>
<td>7%</td>
<td>9 s    104 s</td>
</tr>
<tr>
<td>All scenarios</td>
<td>100%</td>
<td></td>
</tr>
</tbody>
</table>

Notable that delays increase by almost half a minute both when overtakes are cancelled, and when they happen without being scheduled. Shifting an overtake from one station to another thus leads to some delays at both stations. All in all, the overtakes lead to delays increasing, on average by a little less than a minute, but sometimes substantially more.

To put the delay figures presented in the previous section into some context, they can be compared to the average dwell time delay of 15 seconds per train, calculated across all trains, without regard for any interactions. Thus, two trains that enter a station at separate times would have an expected combined dwell time delay of around 30 seconds, which is more than three times larger than the case when both trains arrive to the overtake-interaction on time. This suggests that, when overtakes are carried out as scheduled, overtakes do not cause delays that would not happen otherwise, quite the contrary. However, when the overtake does not go fully according to schedule, and either occurs with a different train than intended, or when the overtaking train is either early or late by as little as a minute, the expected delay, across both trains, increases by about 50%. The combined dwell time delays also increase somewhat when an overtake is cancelled at one station and moved to another one, illustrated in Figure 4: in the rescheduled case train A is delayed by 24 s and train B by 15 s at station X (where the overtake was cancelled), and the combined delay at station Y (where the overtake took place instead) is 33 s, for a total of 72 s. In a case with no interactions at all, also pictured in Figure 4, both trains C and D would be expected to be delayed 15 s each at both station X and Y, for a total of 60 s. The combined delay in the rescheduled overtake scenario is thus 20% larger than if no interaction had taken place at all, which is not at all as good as if the overtake had gone as scheduled, but still better than if the trains had been ordered to wait for one another. This suggests that the dispatchers, on average, manage to reduce the delays somewhat by shifting the location of overtakes among trains that are not exactly on time.

5. Conclusions

We found that delays in Japanese railways are mostly caused at stations and recovered on the line sections. This is the same pattern that is found in the research literature, suggesting that despite a higher punctuality, the mechanisms and patterns are essentially the same that are found elsewhere. We have described the seven possible scenarios relating to train overtakes and found that, despite high punctuality and mostly very small delays, these interactions between trains rarely unfolded according to the timetable. Only seven percent of overtakes were realised as scheduled. All other scenarios, where at least one of the trains was not on time, lead to increased delays. Even small deviations from the timetable, in either direction, thus leads to situations where the delays increase, which suggests that a high-precision approach is required in both timetabling, dispatching, and driving of trains.
Figure 4. Illustration of the expected dwell time delays for train A, B, C and D, depending on the overtake scenarios. Trains A and B do not have any interactions, so their expected delays are 15 s at each station. Train D is scheduled to overtake train C at X-station, but as its arrival is delayed, the overtake is shifted to Y-station. This means that at X-station, train C has a cancelled overtake, associated with an average delay of 24 s, while train D does not have an interaction, and an average delay of 15 s. At Y-station, both trains are involved in an unscheduled overtake scenario, with an expected combined dwell time delay of 33 s, here split up evenly across both trains. In this case, the combined delays would clearly have been even larger, if train C had been forced to wait for train D at X-station, suggesting that the dispatcher was correct in shifting the overtake, even if the amount of delays would have been even lower, if there had been no interaction between the trains at all.

The paper also introduced some new methods and measures which may be of use in the analysis of railway traffic. While the concept of train interactions is by no means new, the different scenarios that may occur, along with their likelihoods and consequences, are rarely discussed either in theory or in practice. This concept may be particularly useful when simulating train operations, as it provides a good complement to the more commonly used comparisons of punctuality and delay distributions. By looking at how the interactions unfold, both in real and simulated operations, a greater understanding can be reached. Similarly, looking at the interaction scenarios and the combined dwell time delays of interacting trains provides a way to study and evaluate the actions of dispatchers using widely available train traffic records. Cases where the dispatchers intervene to change or maintain the train order can be identified, and the impact on delays can be measured. These measures thus allow for novel kinds of useful analyses.

Finally, we concluded that the overtakes only explain a small part of the overall delays. In the datasets we studied, about two percent of train movements were part of an overtake. This mechanism cannot explain the delays found in the other 98% of cases, and only a small part of the total average dwell time delay of 15 s. Other types of interactions between trains, like meetings on single-track lines, crossing train paths, or headway-type interactions, may all help explain more of the variation, as may the number of passengers, or other causes entirely. Nonetheless, overtakes are a key feature of double-track lines and the backbone of many timetable concepts, and it is important to consider them, even if they only help explain a small part of the overall problem of train delays.

Acknowledgements

The first author is a JSPS International Research Fellow, and gratefully acknowledges the JSPS Summer Program for the funding the research visit during which this work was done. Funding for the first author research is provided in part by the Swedish Transport Administration. The second author is partly supported by the Ministry of Education, Science, Sports and Culture, Grant-in-Aid for Scientific Research (C) 15K01199 and 16K06428. This research was done in collaboration with K2 – The Swedish Knowledge Centre for Public Transport. The authors gratefully acknowledge the constructive comments from the two anonymous reviewers.
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