Levi and the Lottery

*It is as rational to accept the hypothesis that ticket i will not win as it is to accept any statistical hypothesis that I can think of.*

Henry E. Kyburg

*It is as rational to suspend judgement regarding the outcome of a fair lottery as it is to suspend judgement in any case I can think of.*

Isaac Levi

Erik J. Olsson


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1. Introduction

Consider a lottery of 1,000,000 tickets where there is one and only one winner and where one ticket is as likely as any other to be the winner. In this sort of scenario it is extremely improbable that any given ticket will win. The probability that a given ticket will lose is as high as that of any statistical hypothesis one can think of. If we are ever allowed to accept an hypothesis as true, then surely we are
allowed to accept, of any given ticket, that it will lose. But to accept, for each
ticket, that it will lose is to commit oneself to there being no winning ticket. This
contradicts our background knowledge that there is a winning ticket. It seems,
then, that common sense gives us a license for holding contradictory beliefs.

The lottery paradox, first formulated by Henry Kyburg, is still a hotly disputed
subject that is thought to have all sorts of radical consequences for human
inquiry.\(^3\) Kyburg saw it as an argument for cultivating a tolerance for
inconsistency and against demanding logical closure of a rational agent’s beliefs.
Some have taken it as a reductio argument against quantitative probabilistic
accounts of inductive inference and as a positive reason for invoking qualitative
methods such as default logic instead. Others have arrived at the opposite
conclusion that it is the qualitative notion of full belief that is to be held
responsible for causing paradox, urging that quantitative degrees of belief are all
we need.

The main purpose of this paper is to assess Isaac Levi’s approach to the
lottery.\(^4\) Before I do this, I will briefly say why I am dissatisfied with Kyburg’s
account and also with the related position taken recently by Luc Bovens and

2. Kyburg on the lottery
 Given a theory \(K\), Kyburg suggests that one can construct a new more
comprehensive corpus by adding to \(K\) all sentences that are sufficiently probable.
In the case of the lottery, the inductively expanded set will be inconsistent.
Kyburg argues that this need not be a regrettable fact, provided that the agent is not allowed to add conjunctions of sentences in the set to the set itself. The resulting set is, to be sure, inconsistent in the sense that every sentence of the language can be derived from it. But it is not inconsistent in another sense: no single one of the 1 million statements is inconsistent with the initial corpus. According to Kyburg, all we require of a set of sentences representing beliefs is that it not be inconsistent in the latter sense.

To most philosophers, though, it is bad enough that our beliefs are inconsistent in the sense of allowing everything to be derived. If the inquirer’s belief set is inconsistent in this sense, then, for any given sentence A, both A and not--A will be derivable from the set. Since one of them is bound to be false, the agent is in certain error. It is difficult to see how it could be rational to enter such a clearly defective state of belief deliberately.

3. Bovens and Hawthorne on the lottery

Let me first put Bovens and Hawthorne’s account in perspective by relating it to Kyburg’s. In his illuminating criticism, Isaac Levi confronts Kyburg’s use of high-probability rules with the following trilemma: (1) Either one respects the concern to avoid error in induction and rejects such rules, or (2) one abandons avoidance of error as a concern in inquiry, or (3) one denies that inductive acceptance removes doubt (see Levi, 1996, p. 248). None of these three alternatives should be very attractive from Kyburg’s point of view. This should be obvious as regards the first two horns of the trilemma.
Although Bovens and Hawthorne do not relate their theory to Kyburg’s, their theory is, in effect, what Kyburg would get if he were to choose the third horn of the trilemma. For what they do is to combine a high probability rule with a notion of belief that does not require a belief to be free of doubt. For a person, they say, “belief is merely a convenient way to categorize those propositions for which her degree of confidence is no less than some threshold value $q$ that she considers significantly high.” (1999, pp. 245–6) Bovens and Hawthorne make clear that the threshold value $q$ need not coincide with the maximum degree of confidence, i.e. certainty (ibid., p. 246). That is to say, a person may, on their account, believe that a given proposition is true and yet not be entirely confident that it is true; some doubt may still remain as to its veracity. They emphasize that, as they use the term, categorizing something as a belief does not mean adding it to the stock of settled assumptions: “If asked whether she believes proposition $S$, $\alpha$ [the agent] may even explicitly report that her degree of confidence in $S$ is no less than $q$, and that since she takes $q$ to be an adequate threshold value for belief, she does indeed believe $S$.” (ibid., p. 246).

The trouble with Kyburg’s original position, as we saw, is that is licenses the presence of inconsistency in the set of settled assumptions. Given their reinterpretation of the concept of belief Bovens and Hawthorne can consider it “rationally coherent” (ibid., p. 251) to believe both that some ticket will win and of every ticket that it will lose without thereby committing themselves to an inconsistent stock of settled assumptions. The reason, of course, is that the different lottery predictions are never added to that stock in the first place. Again,
being a belief, on the present proposal, just means being assigned a sufficiently high degree of confidence.

This advantage, however, comes with a price tag. The difficulty facing Bovens and Hawthorne is to explain why in the first place the agent should take the trouble of separating from other propositions those propositions in whose veracity she has a relatively high degree of confidence. What is the point of this separation business? After all, the agent is not actually doing anything with the resulting set of propositions. There would, to be sure, be a point to it if the purpose were to add the separated propositions to the stock of settled assumption. But this, again, is a practice which Bovens and Hawthorne do not want to engage in.

In not admitting inconsistencies into the set of settled assumptions, Bovens and Hawthorne do distance themselves successfully from one of the less attractive features of Kyburg’s original approach, that is to say, the legitimization of inconsistency among settled assumptions or full beliefs. But this benefit is attained at the cost of making beliefs irrelevant to inquiry and deliberation. In the final analysis, their proposal does not, in my view, represent a clear improvement upon Kyburg’s account.

4. Levi on the lottery

Let us see if Levi’s own theory fares any better with respect to the lottery. As Levi sees it, inquiry starts with a question. Given a question, it may be obvious what the potential answers to that question are. If I ask now, in July 2004, who will win the next election for the American presidency, it is clear that the potential answers
are “George W. Bush will win” and “John Kerry will win.” In other cases, it is less obvious what the potential answers are. In such cases, there is need for what Levi, following Peirce, calls abduction, i.e., the step in inquiry at which the potential answers are determined. Once the abduction step is completed, the stage is set for the identification of one answer as optimal in response to the inquirer’s question. This answer is inductively acceptable. Levi sees the identification of an optimal answer as a cognitive decision problem analogous to a practical decision problem, and he recommends using what is in its essence Bayesian decision theory. In a final step, the optimal answer is added to the inquirer’s stock of settled assumptions. This simplified account of Levi’s complex theory will do for present purposes.

In Levi’s view, inductive acceptability is relative to a question. What consequences does this have for the lottery paradox? Part of the formulation of the problem was that it appeared that all statement of the form “Ticket i will lose” are inductively acceptable. From the present perspective, this amounts to saying that “Ticket i will lose” is acceptable in response to the inquirer’s question Q. So, what is the question in response to which “Ticket i will lose” is an optimal answer?

Most people confronted with the lottery would presumably simply ask which ticket will win. The potential answers to this question, however, are “Ticket 1 will win,” “Ticket 2 will win,” and so on. Statements of the form “Ticket 1 will lose” are not potential answers to this question. Hence, trivially, none of those statements can be an acceptable potential answer.
However, the matter is more complicated than it appears on first sight. Levi may object that I have not represented his theory correctly here. According to him, “Ticket 1 will lose” is a potential answer to the question of which ticket will win. That ticket 1 will lose is equivalent to the disjunction “Ticket 2 will win or ticket 3 will win or ... or ticket 1000,000 will win.” All such disjunctions of “strongest potential answers” are, in Levi’s view, also potential answers to the inquirer’s question. I have argued against this proposal elsewhere and will not repeat the point here. Nonetheless, I do grant that adding “Ticket 1 will lose” to the body of evidence is one of the inquirer’s cognitive options in response to the question at hand, as indeed is adding any other disjunction of the same type. Adding it is surely a sensible reaction to the inquirer’s problem, and it does take him one step closer to a final solution, i.e., to accepting a genuine potential answer. Anyway, what comes out of Levi’s decision theory once we consider accepting any disjunction of potential answers is that the optimal choice is to suspend judgment, i.e., to accept only that some ticket will win but we do not know which one.

But perhaps the inquirer is actually asking a whole series of questions. Perhaps he is asking of each single ticket at a time whether that ticket will win or loose. He is asking of ticket 1 whether it will win or lose, of ticket 2 whether it will win or lose, and so on. In general, he is asking of ticket \( i \) whether it will win or lose. If so, the potential answers are “Ticket \( i \) will win” and “Ticket \( i \) will lose.” Let us, following Rabinowicz (1979), call a question of this kind a Hamlet question (“To win or not to win ...”). On this reconstruction attempt, the statement “Ticket \( i \) will
lose” does come out as a potential answer. This is promising news, although this does not by itself guarantee that it will also be an optimal potential answer.

Yet, given its initial high probability, it seems plausible that it could be optimal as well. This is conceded by Levi.⁷ That ticket 1 will lose may be the optimal answer to the question of whether it will lose or not; that ticket 2 will lose may be the optimal answer to the question of it will lose or not, and so on. What the lottery paradox illustrates, on this rendering, is that each statement in a set may be inductively acceptable even though the set as a whole is inconsistent with our background knowledge, for each statement of the form “Ticket \(i\) will lose” can be inductively acceptable relative to the corresponding Hamlet question, even though we are supposed to know that there is a winning ticket.

Does this mean that the inquirer is justified in actually accepting all the statements in the troublesome set by adding all of them to the stock of settled assumptions, thus making that stock inconsistent? No, this does not follow and in this I think Levi is right.

To see the point, consider again the statements of the form “Ticket \(i\) will lose” relative to their respective Hamlet questions. Suppose that the inquirer finds all these statements inductively acceptable and decides actually to accept them. Depending on how he attempts to accomplish this we get two sub-cases.

As for the first sub-case, suppose that the inquirer decides to proceed step-wise, starting with ticket 1 and updating his belief set with the new item “Ticket 1 will lose,” thus excluding ticket 1 from winning. As far as he can judge, the winning ticket could still be among the 999,999 remaining tickets. Accordingly
our inquirer now proceeds to the second ticket, noticing that its chance of losing is 999,998 in 999,999. As this is still a very high probability, our inquirer decides to exclude that ticket from winning as well, adding to his evidence that ticket 2 will lose.

However, it is not possible to continue excluding tickets in this way until no ticket is left. There will be a point at which further exclusion of tickets is no longer possible. The reason, obviously, is that as the inquirer accepts that a given ticket will lose, his evidence changes and with it the probability that a given remaining ticket will lose. This probability decreases.

I will confine myself to illustrating the claim just made for a simple high probability rule. Suppose that the inquirer accepts an hypothesis if and only if its probability is at least 0.99 and that he has excluded all tickets but the 99 last ones. At this point our inquirer must stop, as the probability that the first of these remaining tickets will lose is $98/99 \approx 0.98989... < 0.99$. He cannot conclude that this ticket, or any of the other remaining ones, will lose, and so he need not worry about causing inconsistency in his set of full beliefs as the result of making his different predictions.

As for the second sub-case, suppose that the inquirer does not proceed by adding statements of the type “Ticket $i$ will lose” to his evidence one at a time. Rather, when he has observed that it would be rational to accept, of any single ticket, that it will lose, he decides to add all statements of that form to his evidence in one swoop. This would indeed make his evidence -- his stock of
settled assumptions -- inconsistent, as he would then believe of each ticket that it will lose and at the same time believe that some ticket will win.

The impression of paradox vanishes once it is recognized that the last move is illegitimate. The inquirer should not be allowed, at a given point in time, to add several claims to his full beliefs, if those claims are answers to different questions. Thus, he should not be allowed to add, at a given time, both that ticket 1 will lose and that ticket 2 will lose, if the first statement is an answer to the question of whether ticket 1 will win or not and the second an answer to the question whether ticket 2 will win or not. These answers belong to different questions and hence to different inquiries. Nor should he be allowed to add as a new belief a conjunction of statements that are answers to different questions without first accepting each answer individually. The conjunction is not an answer to his current question. Either way, the lottery paradox suggests that pooling or simultaneous acceptance of inductively acceptable answers belonging to different questions should be forbidden.

5. How restrictive are Levi’s assumptions?

The two assumptions that make Levi’s model immune to the lottery paradox are: 1) that inductive acceptability is relative to a question and 2) that one should not be allowed to pool answers to different question by adding these answers simultaneously to one’s stock of settled assumptions.

How serious are these restrictions? The first restriction -- that acceptability is to be seen as relative to a question -- does not seem serious at all. Human inquiry is
always driven by a question. There is always an issue in response to which things are accepted or rejected. Hence, to borrow C. S. Peirce’s celebrated metaphor, no roadblock of inquiry is introduced by assuming acceptance to be question relative. A critic must show that it is important to human inquiry to have a notion of acceptance that is not question relative. But it is difficult to see why anyone would care about inductive acceptance in the absence of an issue.

The second restriction -- that the inquirer should not be allowed to pool answers to different questions in the manner described above -- is less obviously an innocent one. I believe, however, that a strong case can be made for it. I fail to see how an inquirer could derive any advantages from pooling answers to different question as compared to an inquirer who proceeds in an incremental fashion, answering each question as it arises. If so, in forbidding pooling we are not obstructing the path of inquiry.

6. On knowing that one’s ticket will lose

On Levi’s account, an inquirer may be justified in adding to his stock of full beliefs that a given single ticket will lose. In particular he may add that the ticket he himself has drawn will lose. I do not think this is problematic in itself. Yet problems arise, I submit, when we combine this with Levi’s theory of knowledge.

Levi has taken the controversial position that knowledge is just true belief. As soon as a person believes something fully and what he believes is true he can rightly be said to know. Against this it may be objected that knowledge requires in addition that the knower has some sort of justification for his belief. Unimpressed
by this type of criticism, Levi urges that an inquirer who already believes fully that something is the case is in no need of justifying his belief to himself. After all, he is not in doubt regarding its truth.

This does not mean that an inquirer is never required to justify things to himself. On the contrary, a rational agent is obliged to justify to himself why a given belief change should be carried out. For instance, before adding a proposition to his full beliefs as an answer to a question, he is obliged to justify to himself why that answer is better than other competing ones. Once the proposition has been accepted there ceases to be a need for justification. There is no need for the inquirer to justify to himself beliefs already held.

Now, as C. S. Peirce insisted, from the believer’s own perspective all beliefs are true. If I believe something, then trivially I take what I believe to be true. Hence, “[f]rom X’s point of view at t, there is no difference between what he fully believes at t and what he knows at t.” (Levi, 1980, p. 28) Knowledge in the sense of true belief reduces to mere belief if the perspective taken is that of the believer himself. If I fully believe that it rains, then, as far as I can judge, I know that it rains. By the same token, if I fully believe that my lottery ticket will lose, then, from where I stand, I know that it will lose.

But while one may claim to be certain that one’s ticket will lose, in the sense of excluding it as a serious possibility, it seems awkward to claim to know that one’s ticket will lose. Our lack of knowledge in this regard is as clearly a part of common sense as any other claim I can think of.$^8$
The obvious reaction is to assign blame to Levi’s already suspect minimal theory of knowledge. The suggestion would then be that we would do better if we were to adopt the standard justified–true–belief analysis of knowledge (JTB). This, however, is not the case. Again, Levi’s theory of induction allows that a person can predict, in the sense of adopting as a full belief, that her ticket will lose if, roughly speaking, losing is much more probable than winning. Presumably, most defenders of the JTB analysis would say that in these circumstances the person is justified in believing that her ticket will lose. So, an inquirer’s true belief that her ticket will lose may well be a case of justified true belief.

There are two main ways to react to this problem: (1) by rejecting the legitimacy of adding to one’s full beliefs in single cases that a given ticket will lose or (2) by devising an alternative theory of knowledge. I have already said, in connection with Bovens and Hawthorne, why I find the first alternative unattractive. As I see it, the second path is the one to take. Knowledge is neither true belief nor justified true belief, if justification is understood, as it usually is, in terms of the individual’s personal reasons for holding the belief in question.

According to the social account of knowledge that I favor, it is not sufficient for knowledge that the inquirer has her own personal reasons for believing what she does believe. Those reasons must be valid for others as well. Knowledge requires that the individual has a socially acknowledged right or even duty to believe in the circumstances in question. What she believes is what anyone in her position would believe as well. Wittgenstein put the matter as follows:
When we say that we know that such and such, we mean that any reasonable person in our position would also know it, that … anyone endowed with reason … would know it just the same. (1977, § 325)

Beliefs arrived at via direct observation are of this kind: if I believe something as the effect of seeing it in broad daylight, then anyone standing where I were standing would arrive at the same belief. The same holds for beliefs based on clear memory and on the testimony of recognized experts on non-controversial issues, just to mention two other traditionally celebrated sources of knowledge that also come out as such on the social view.

Contrary to what Levi maintains, then, when I say that I know I am not just expressing my own personal certitude. I am also committing myself to the existence of grounds that are socially recognizable as such. I am giving others a license to take on the same view as I have. I am assuring them that there is no need on their part to bother with the details of justification. That part, I am promising, has already been taken care of. If it turned out that my belief was based on, say, reading tea leaves I would, in claiming to know, be open to charges of misleading my audience.

The notion that knowledge is essentially social is certainly not new. We have seen that Wittgenstein held this view. Traces of it can be found in an early paper by Gilbert Harman in connection with his well known newspaper example and in a recent paper by Robert Shope. Levi does mention a social conception as an alternative to his own: “Of course, the claim that X knows that h at t may mean
even for a pragmatist that $X$ not only truly believes that $h$ at $t$ but does so authoritatively in the sense that in some way or other he can justify to others the truth of what he fully believes.” (1997, pp. 67--68) In referring to others, this conception is clearly social in nature. As far as I can judge, Levi does not present any reasons for thinking that the social conception is inferior to his own individualistic analysis.

In urging that one should make a distinction between what one fully believes and what one knows I am not introducing what Levi calls a “double standard of serious possibility.” Levi is right in insisting that what is seriously possible for a person is that which is compatible with the person’s full beliefs. This raises the question of what role knowledge could play in inquiry and deliberation. I do not claim to have a complete account of what this role might be. I do tend to think that knowledge has a function in collective inquiries and, more specifically, in the efficient exchange of information between different inquirers. My knowledge is that part of my belief system which I, by means of making knowledge claims, can efficiently share with others. There is obviously more to be said about this, but that would have to await another occasion.

The social conception of knowledge qualifies as “pedigree epistemology,” a term Levi uses for theories that are concerned with the origins of beliefs (e.g. Levi, 1980, section 1.1.). On the social view, whether or not a given belief counts as knowledge depends upon how it was arrived at. Only belief acquisitions mechanisms that are common to all inquirers give rise to knowledge. Those that are peculiar to a given individual inquirer do not. If, as I have suggested,
knowledge is useful because of its role in the efficient sharing of information among different inquirers, it makes sense to separate one’s knowledge from one’s mere beliefs. This process does require some concern with the pedigree of one’s beliefs.

Levi thinks that pedigree epistemology is bad, urging that “[w]e ought to look forward rather than backward and avoid fixation on origins.” (1980, p. 1) The pedigree theories he discusses in this connection are traditional foundationalism and what I have called JTB. Against the first, he objects that “[t]here are no immaculate preconceptions.” (ibid.). Against the second, he holds that there is no need to justify the truth of one’s full beliefs to oneself *ex ante* (ibid., p. 28). I fail to see how any of this would count against the social theory of knowledge that I favor. Such a theory need not and, in my view, should not be associated with the notion of an incorrigible foundation. Nor is it part of such a theory that an inquirer needs to justify the truth of his beliefs to himself after he has acquired them. Contrary to what Levi maintains, a concern with pedigree is not bad in itself -- far from it -- although he is certainly right in pointing out that some variations on that theme have little to recommend them.

What are the consequences of the social conception of knowledge for the lottery? Consider my full belief, arrived at by means of induction in response to the Hamlet question, that this ticket which I am holding in my hand (we pretend) will lose. Does this belief of mine satisfy Wittgenstein’s social criterion? No. Surely, not everyone in my position -- holding a ticket in her hand and contemplating whether it will win or not -- would fully believe that the ticket will
lose. If Wittgenstein’s criterion were satisfied for this proposition, then no one would buy lottery tickets and there would be no lotteries. Lotteries exist because some people do not rule out the prospects of winning although the likelihood of this happening is close to zero. These people, by the way, are not irrational; they are simply more cautious that those who rule out themselves being the winner. That inquirers are allowed to differ with respect to how cautious they are in their acceptances has been part of Levi’s theory of induction from the start.\footnote{11}

Levi is committed to the counterintuitive view that a person can know that a given lottery ticket will lose. If I am right, the problem is to be located not in his theory of induction but in his account of knowledge which, although it was developed in opposition to mainstream epistemology, shares with that tradition of thought one of its main deficiencies, namely an individualistic conception of knowledge. Once it is acknowledged that knowledge, as opposed to belief, is essentially social, the conclusion that a person can know that her lottery ticket will lose is not forthcoming. This, however, does not mean that a person cannot become certain that her ticket will lose. She can if the chances of winning are dim as compared to the chances of losing and the informational value of accepting that the ticket will lose is not terribly low. But this will be her own personal certainty based on grounds that are not recognized as such by the members of her community.

Footnotes
4. That the lottery paradox is a central concern in Levi’s theorizing about inductive acceptance is witnessed by the frequency with which Levi returns to it in his writings. Here are some examples. The first treatment can be found in Levi (1965). The present description of Levi’s theory is based on Levi (1967), pp. 38--41, 92--95. For a brief criticism of Kyburg’s treatment focusing on the notion of acceptance, see Levi (1984), pp. 223--224, and Levi (1997), pp. 226--227. For a recent longer discussion, see Levi (1996), section 8.3.
5. See my “Levi on Potential Answers to Inductive Questions,” this volume.
6. For the details, see Levi (1967), pp. 92--93.
7. “Thus, in the lottery problem, it is possible to predict that ticket 1 will not win if the ultimate partition $U_1$, which consists of the sentences ‘ticket 1 will win’ and ‘ticket 1 will not win,’ is used.” (Levi, 1967, p. 92) By the ultimate partition is meant the set of (strongest) potential answers.
8. In their forthcoming paper, Gilbert Harman and Brett Sherman observe that “it is not intuitively correct that one can know using statistical reasoning that one’s ticket is not the winning ticket.”
9. Harman (1968), Shope (2002). Both Harman and Shope advance accounts of the social aspect of knowledge that differ from Wittgenstein’s. I have defended a social conception in Olsson (to appear).

11. The proposal to regard the degree of caution a contextual parameter was made for the first time in Levi (1962).

References


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